Electrical Engineering

May
1936

The Program and Full Particulars Concerning the

Summer Convention at Pasadena, California

Are Given in the News Section of This Issue. A Rare Opportunity Is Afforded for Combining the Allurements of a Pacific Coast Vacation With the Benefits of a Splendidly Balanced Technical Program.

Published Monthly by the
American Institute of Electrical Engineers

Published Monthly by

American Institute of Electrical Engineers

(Founded May 13, 1884)

Electrical Engineering

Registered U. S. Patent Office

May 1936
Volume 55
No. 5

The Official Monthly Journal and Transactions of the A.I.E.E.

JBLICATION OFFICE, 20th and North-
pton Streets, Easton, Pa.

DITORIAL AND ADVERTISING OFFICES,
West 39th Street, New York, N. Y.

AIL should be sent to New York address

NTERED as second class matter at the Post ffice, Easton, Pa., April 20, 1932, under the ct of Congress March 3, 1879. Accepted mailing at special postage rates provided in Section 1103, Act of October 3, 1917, thorized on August 3, 1918.

IBSCRIPTION RATES—$12 per year to ited States, Mexico, Cuba, Porto Rico, waii, and the Philippine Islands, Central merica, South America, Haiti, Spain, and anish Colonies, $13 to Canada, $14 to other countries. Single copy $1.50.

HANGE OF ADDRESS—requests must be eived by the fifteenth of the month to be ective with the succeeding issue. Copies delivered because of incorrect address can- t be replaced without charge. Both old d new addresses should be given, as well as change in business affiliation.

DVERTISING COPY—changes must be re- ved by the fifteenth of the month to be ective for the issue of the month succeeding.

ECTRICAL ENGINEERING is indexed an- lly by the Institute, weekly and monthly Engineering Index, and monthly by *Indus- l Arts Index;* abstracted monthly by *Science stracts* (London).

nted in the United States of America. mber of copies this issue—18,800

This Month—

Special Articles

A.I.E.E. Papers

—Turn to next page

In This Issue—

VIBRATION of transmission line conductors caused by wind has troubled transmission engineers ever since long-span lines first were built. Various phases of the problem are treated in 3 papers in this issue. In one paper, test methods and equipment developed for a laboratory investigation are described (*pages 490–3*). The second paper presents the result of a series of wind tunnel studies (*pages 543–7*). The third paper presents an analysis of the vibration of cables and dampers, representing the results of several years of research and field and laboratory experiments. This paper is being published in 2 parts; the first part is included in this issue (*pages 455–69*) and the second is scheduled for inclusion in the June issue.

PLANS for the A.I.E.E. 1936 summer convention to be held June 22–26 at Pasadena, Calif., are practically complete. The program will include an excellent variety of high class technical papers, social events, entertainment features, sports, and interesting inspection trips. Those attending from the East will have a splendid opportunity to visit many points of interest in the West and Far West by combining vacation trips with attendance at the convention. Those interested in taking advantage of any arrangements that may be made for a special train or special cars from the East should notify National Secretary H. H. Henline at once (*pages 554–9*).

AN experimental 50-kw synchronous-mechanical rectifier-inverter has been built since the introduction of the device in 1933. This machine is said to represent the first adaptation of the fundamental concepts of harmonic commutation to a practical rectifying unit. Tests show that the device has satisfactory characteristics and good efficiency (*pages 548–53*).

SYMMETRICAL COMPONENT method has been proved to be of considerable value in the analysis of a-c rotating machines under both steady state and transient conditions. By making use of suitably chosen components of current and voltage, the method readily can be extended to the unsymmetrical 2 phase or single phase machines (*pages 471–6*).

TESTS on many varieties of incandescent electric lamps, of both domestic and foreign manufacture, show a wide difference in quality between lamps complying with federal specifications and those that do not. Under no conditions can the use of inferior lamps be justified economically (*pages 516–24*).

ELECTRIC SHOCK may derange heart action causing ventricular fibrillation without damage to heart tissue, but resulting in death within a few minutes. This is one of the significant results of a joint investigation of the effects of electric shock on the heart, extending over a period of several years (*pages 498–515*).

AUTOTRANSFORMERS for 287.5 kv service on the Boulder Dam-Los Angeles transmission system, rated at 65,000 kva, are among the largest so far built. Special means of transporting these units from the factory to Los Angeles were required (*pages 438–44*).

MULTIRANGE precision instrument transformers of special design have been used successfully for calibrating metering equipment on a large western power system, with a noticeable improvement in the average accuracy of watt-hour meters on that system (*pages 480–9*).

NEUTRALIZING transformers are devices for protecting communication circuits against extraneous longitudinal voltages. They are proving particularly useful in protecting commercial telephone circuits serving electric power stations (*pages 524–30*).

THE magnetic vector potential is said to be an exceedingly useful mathematical tool in the analysis of electric circuits, of value not only in the solution of particular problems, but also in the derivation of important general relationships (*pages 534–42*).

STRAY LOAD loss tests on induction machines made by means of a recently developed highly accurate method confirm the validity of the "belted load-back method" previously developed (*pages 493–7*).

MEASUREMENT of high voltages by means of the sparkless sphere gap voltmeter, in which the force between a pair of spheres (instead of sparkover) denotes the magnitude of voltage, has been subjected to further study (*pages 444–7*).

HEAT TRANSFER with hydrogen cooling cannot be predicted on the same basis as with air cooling, because of the difference in the manner of flow (*pages 530–4*).

AN exact formula for transformer regulation confirms the present formula for 2-winding transformers in the A.I.E.E. test code; a similar formula is obtained for 3-winding transformers (*pages 466–71*).

SCIENTIFIC PROGRESS is not a great leap of imagination, but a steady process, like the advance of a great army, according to a noted scientist and engineer (*pages 436–7*).

DISCUSSIONS have been omitted from this issue in order to provide additional space for the advance publication of papers to be discussed at the Institute's 1936 summer convention.

AN occupational and geographical analysis of the readers of ELECTRICAL ENGINEERING recently has been compiled (*page 559*).

SPACE CHARGE is an important factor in determining the distribution of electric gradient about a conductor in corona (*pages 448–54*).

CURRENT and voltage loci in 3-phase delta-delta circuits have been determined by means of a method previously outlined (*pages 476–9*).

Institute Ideals

—A Message From the President

IN traveling around the country during the past 6 or 8 months, visiting various Sections and Branches throughout the West, South, and Southwest, I was very glad to see the entire membership taking such a lively interest in the activities of the Institute. Everyone with whom I talked seemed to approve the present policies and to be pleased with what is being done by the Institute as a whole.

The Institute, as you know, is divided into 10 geographical Districts, and these districts are again divided into many Sections. In addition, many Student Branches have been established in the colleges and universities all over the country.

The unusual opportunity that has been afforded me in visiting so many of the Sections and Branches has been greatly appreciated, and to me, as representing the Institute, the pleasant personal greetings and expressions of interest in its problems and loyalty to its purposes have been an inspiration. After all, the officers of the Institute, its board of directors, and the headquarters staff are only the means by which the various Sections are co-ordinated, and they are the means by which the Institute's Code of Principles of Professional Conduct is kept alive in the hearts and minds of the Section officers and members in general.

There have been many and evident instances that whatever touches the heart of the life and future of the Institute at the same time touches the heart and interest of its members.

The objects of the Institute are: "the advancement of the theory and practice of electrical engineering and of the allied arts and sciences, and the maintenance of a high professional standing among its members."

There are 4 items in the Code of Principles of Professional Conduct which to me are very important:

"In all of his relations the engineer should be guided by the highest principles of honor.

"It is the duty of the engineer to satisfy himself to the best of his ability that the enterprises with which he becomes identified are of legitimate character. If after becoming associated with an enterprise he finds it to be of questionable character, he should sever his connection with it as soon as practicable.

"The engineer should endeavor to assist the public to a fair and correct general understanding of engineering matters, to extend the general knowledge of engineering, and to discourage the appearance of untrue, unfair, or exaggerated statements on engineering subjects in the press or elsewhere, especially if these statements may lead to, or are made for the purpose of, inducing the public to participate in unworthy enterprises.

"The engineer should take an interest in and assist his fellow engineers by exchange of general information and experience, by instruction and similar aid, through the engineering societies or by other means. He should endeavor to protect all reputable engineers from misrepresentation."

These are the purposes for which the Sections and Branches were founded and are maintained. But, of course, it is the individual engineer, in the long run, who makes the Institute what it is—it is his Institute and the policies and codes formulated are merely reflections of his wishes. Based upon a common professional interest in electrical engineering, the addition of broad human sympathies applied throughout the years cements friendships of enduring character—"A man is known by his friends." The various Sections I have visited are keenly aware of this, and are to be congratulated on their adherence to the Code, and on the practical results that have followed.

The Branches of the Institute in the various colleges and universities throughout the country are endeavoring to instill these ideals in the minds of the engineering students. It is my belief, after visiting so many of them, that the branches are making a great success of it, and that the engineers of tomorrow will be leaders in the field of electrical engineering.

I feel that it has been an inestimable privilege to have had the opportunity of visiting so many of the Sections and Branches, and I hope that the many things I have seen and learned may be of some lasting benefit to the Institute.

E. B. Meyer

What's Next in Science?

By KARL T. COMPTON FELLOW A.I.E.E.

IT IS AN ALL TOO COMMON practice of scientific publicists and prognosticators to give free vent to their imaginations in a most unscientific way by picturing our descendants as flying through interplanetary space, after the manner of Buck Rogers—or operating a navy with a thimbleful of transmuting atoms—or banishing old age by administration of glandular extracts. No doubt many of the past triumphs of science were, relatively to the times, as spectacular and even more unexpected than these would be. However, scientific progress is not a great leap of imagination, but a steady process, like the advance of a great army; at times strategic positions are captured, as when the positive electron (positron) was discovered; at times there is a steady "mopping up" process all along the line, as when the systematic search for chemical isotopes followed the first discovery; at times there is retreat, as when a theory is proved untenable; at times a new powerful engine of this scientific war is invented, like the radio tube amplifier. While the scientific campaign is generally well planned in advance and directed toward certain main objectives, it also, on occasion, is opportunistic in that its center of activity may quickly be shifted by some new discovery or idea which discloses new territories to be conquered.

A better analogy to scientific progress is geographical exploration. Just as the discovery of the Great Lakes and the Ohio, Mississippi, and Missouri Rivers led to their use as avenues of colonization, so great scientific discoveries and theories are the channels for widespread advance of scientific knowledge. Similarly there are desert and mountain barriers in science where progress is slow because the oases and passes have not yet been discovered. But, as far as we can now see, there is no limit to the completeness with which man may expect to understand the materials, forces, and processes of nature. . . . . . There are many divisions to this [science] army—the mathematicians, engineers, astronomers, chemists, physicists, botanists, psychologists, medical men, biologists, sociologists, and many others. . . . .

I have been asked to discuss the topic "What's Next in Science?" To answer this question properly, we need to consider where we are now and what has been our path in the past decades. Nowhere has this been more strikingly and comprehensively stated than in the preamble to a resolution adopted just a year ago by the American Association for the Advancement of Science and submitted to the President and the Congress of the United States, urging "that aggressive governmental support of scientific work is essential to any sound program of building for the future national welfare, and is essential if this country is to do its full part in the further advance of civilization and if it is to enjoy its proper share in the benefits of this advance."

The preamble to this resolution summarizes the importance and achievements of science in the following words:

"Development and application of science have been basic to the economic and social progress of nations, making possible such movements as universal education, abolition of child labor and slavery, emancipation of women, insurance and pensions, moderate hours of labor and great improvement in the standards of health, comfort, and satisfaction in living.

"Scientific developments have not only conferred general benefits, but in particular have been largely effective in leading to recovery from previous depressions—as the railroad industry following the depression of 1870, the electric industry following that of 1896 and the automobile industry following that of 1907.

"Scientific research is a productive investment proved by experience to yield a high rate of return, as illustrated by the saving of $2,000,000,000 per year from the Bessemer steel process and of over $1,000,000 per day from the modern incandescent lamp, and as illustrated also by the entire chemical, electrical, communication, transportation, and metallurgical industries and by the enormous employment in such industries.

"Progressive foreign nations have recognized the importance of maintaining their scientific strength at a high productive level and have provided for this maintenance by allocation of funds to support scientific work on a national scale.

"There now exists in America a situation demanding as never before an intelligent use of our national resources.

"There are manifold problems in health, safety, agriculture, better use of resources, development of new products, and processes whose social value and urgent need are unquestioned but whose solution is being seriously hampered by lack of funds for research, which have been greatly curtailed at this time when properly directed scientific work is more than ever needed.

"The great national planning program, which is now under consideration for the use of our physical resources of soil, minerals, and crops, will be seriously deficient unless it includes provisions for utilizing the scientific resources of the country for creative work."

What, in the light of this background, are some of the lines in which we may expect increased scientific activity in the near future? Let me suggest a few scientific problems of incalculable importance to the country.

First. Agricultural research in the past has led to greater yields of improved farm products. The great problem of agriculture today is to discover

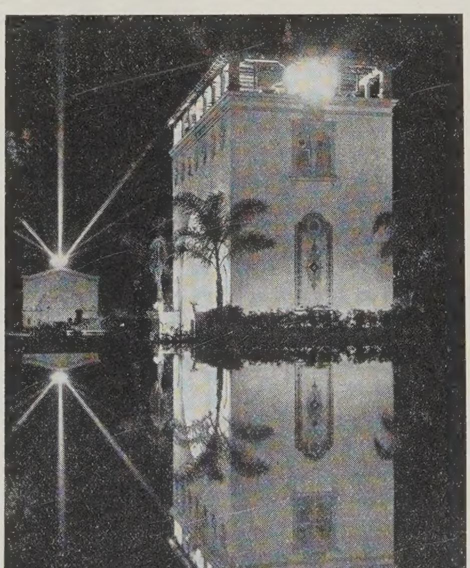
From an address delivered before the American Association for the Advancement of Science, St. Louis, Mo., Dec. 30, 1935, by Doctor Compton as president of the Massachusetts Institute of Technology, Cambridge. Published at the recommendation of the A.I.E.E. committee on education.

new uses for these products, uses that will create new social values or partially replace the consumption of our exhaustible natural resources. Silk from wood, rubber from weeds, and motor fuel (alcohol mixed with gasoline) from corn or potatoes are actual examples of what can be done. Experience justifies belief that, along such lines, science may create new demands for farm products which will provide a constructive and permanent solution of the agricultural problem. This would be an infinitely better solution than the present emergency expedient of paying huge sums to induce farmers to raise less—to plow under crops and slaughter stock, in order that the rest of us, who pay that bill, will also have to pay more for our food. I believe that no one, not even among its proponents, is enthusiastic about this destructive and temporary scheme to benefit agriculture. I call it temporary because there are already signs that it is doomed to failure in the rapidly mounting imports of food products from foreign countries—an inevitable situation which will not only kill the scheme but be a body blow at American agriculture itself. Cotton, meat, and wheat have been quick to respond to this invitation, by America, to foreign countries to seize not only America's foreign markets, but to invade the home markets as well. How much more satisfactory would be a positive solution, based on scientific developments, which will create new industrial demands for farm products, and thus stimulate instead of depress agricultural activity. Of all the expenditures authorized by the last Congress, the one which seems to me wisest was the appropriation of funds to the Department of Agriculture for use in research for developing new outlets for farm products. This is an encouraging sign, and presages important future scientific activity in this line—an activity which will probably be centered largely in chemistry and chemical engineering.

Second. I believe that a second line of increased activity in applied science will occur in industry—particularly in those industries which hitherto have depended largely on tariff protection, on monopolies, on exploitation of natural resources, on governmental subsidies or simply on momentum of past strength. These supports are temporary and precarious; sooner or later they fall before science, because no amount of artificial protection can permanently maintain an obsolete product, an inferior process, or a moribund organization against competitors which are based on scientifically improved products or methods. Furthermore, the general public is ill-served by industries that lean upon legislative favors rather than upon wide-awake technical policies for existence, and the public, when it knows the facts, is likely to take strenuous measures. We have splendid examples in the electrical, communication, chemical, and automotive industries, of industries that have met competition and continually have improved their service to the public through scientific research and the employment of technically trained men of highest caliber. And these industries are at the top of the list in prosperity. Contrast with these some other industries, which you can think of without my naming them, that have not built up their organizations with technically trained men, that depend more on lobbying than upon science for their prosperity, that are suffering from types of competition which they themselves might have developed and profited by, had they been alert to the opportunities offered by science. The contrast is striking. There are some signs that it has struck home even in boards of directors. I believe that the scientist and the engineer will be called upon as never before to lead all along the line in our industrial fields.

Third. In the medical field, we all know in a general way that there are great opportunities for scientific work. Few of us who are not close to this field realize how great the opportunities really are. For example, it is said that 20 per cent of all our state taxes go to the care of the mentally diseased. Think how great an investment, in money and in human happiness alike, it would be to pursue really actively those scientific leads which show any promise of prevention, alleviation, or cure. Tropical diseases are a tremendous drain, not only in the tropics but also in our healthier climates. It has been estimated that an alleviation of certain enervating diseases in the tropics would so raise the standards of living in these regions as to create an increased purchasing power that would well repay the probable cost of the medical research and practice necessary to improve the situation. Cancer, infantile paralysis, the common cold, influenza, and treatment by glandular extracts all suggest unsolved problems of medical science whose even partial solution would yield incalculable human as well as economic benefits.

Finally, the most important item of all is that the advancement of pure science should be fostered in every possible way. It is only as we learn about the materials, forces, and operations of the world in which we live that we can wisely adapt ourselves to life in it and use these materials and forces to our own advantage. Pure science seeks to gain this knowledge and applied science, or engineering, seeks to use it in desirable ways. These go hand in hand, and together their success epitomizes man's continual ascent to a richer, fuller, more satisfying life. . . . .


The San Diego Exposition beckons summer convention goers

Power Transformers for 287.5 Kv Service

Autotransformers rated at 65,000 kva recently built for use at the receiving end of the Boulder Dam-Los Angeles transmission lines are among the largest transformers so far constructed, and incorporate a number of new developments, such as a novel form of tank, an unusual arrangement of radiators and air blast equipment, and improved bushings. The design and method of shipment of these transformers are described in this paper.

By
W. G. JAMES
Membership Application Pending

F. J. VOGEL
ASSOCIATE A.I.E.E.

Both of Westinghouse Elec. and Mfg. Co., Sharon, Pa.

POWER TRANSFORMERS of unusual size and design recently were delivered to the City of Los Angeles, Calif., where they will be used to receive power from the 2 275 kv* Boulder Dam transmission lines and deliver it to the 132 kv transmission system of the city. The 7 single-phase 3-winding autotransformers to be described in this paper will be used to form 2 3-phase banks with one spare unit at substation *B*, and thus will serve as a very important link in the Boulder Dam generation, transmission, and distribution system, the details of which have been described by E. F. Scattergood.[1]

Aside from their association with this project and the fact that they have the highest voltage rating of any power transformer heretofore built in the United States, these transformers are of interest because of the many new developments incorporated in their design and manufacture. The design of the tanks has been made so as to provide the greatest strength consistent with minimum dimensions and weight. Methods of tank fabrication and welding have been improved both to strengthen the tank and to insure permanent tightness, and a new and efficient system of forced air cooling of transformers has been devised. The line voltage is the highest and the insulation test voltages are among the highest ever applied in the United States, and that these voltages can be successfully withstood has been made possible by the rapid progress in the art of insulating transformer windings. Improvements have also been made in capacitor bushing construction which should make the bushings permanently tight and further improve their almost perfect service record.

Rating of Transformers

Each of these 7 60 cycle transformers has a rating of 65,000 kva continuously, operating as a self-cooled unit with the air blast equipment in operation, or 48,750 kva continuously, operating as a self-cooled unit with the air blast equipment idle, without exceeding a temperature rise of 55 degrees centigrade. Also, they have a 2 hour safe temperature rating with air blast of 80,000 kva, following 40,000 kva continuously without air blast. Although the transmission voltage at the sending end of the Boulder Dam lines is 287,500 volts, line regulation made it necessary to design the high voltage winding turns ratio on the basis of 275,000 volts. The insulation class, however, is fixed at 287,500 volts in order to be co-ordinated with the insulation class of the transmission line and the receiving voltage which will be encountered under conditions of no load.

These transformers will operate as autotransformers, converting power from 275 kv to 132 kv, star connected, with the neutral solidly grounded and with a 16,000-kva 13.2-kv delta-connected tertiary winding provided for the purpose of controlling short-circuit currents, the circulation of third harmonic currents, and a possible future supply to a synchronous condenser for power factor correction. In anticipation of this possible future loading of the tertiary, these transformers are also required to operate continuously as self-cooled air-blast units with 65,000 kva at unity power factor being supplied to the 132,000 volt distribution system, 16,000 kva at zero power factor leading being supplied to the tertiary winding,

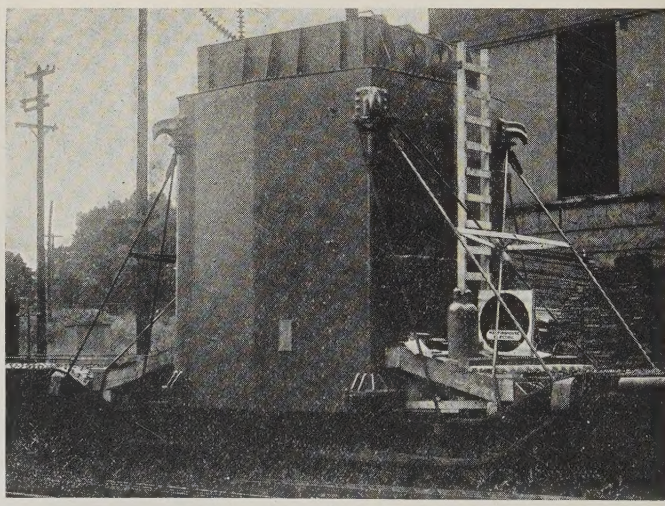


Fig. 1. 65,000 kva transformer loaded for shipment in lower section of tank, using a special shipping cover

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 9, 1936; released for publication March 18, 1936.

* Rated voltage at receiving terminal, 275 kv; at Boulder Dam, 287.5 kv.

1. For all numbered references see list at end of paper.

and with the resulting kilovolt-amperes being taken from the Boulder Dam line.

Method of Shipment

The shipment from the factory to Los Angeles of core, coils, and insulation assembly was made in the lower section of their own tank in dry nitrogen, so that drying out in the field might be avoided and so that field handling, with consequent risk of injury, might be minimized. At first it was feared that it might not be possible to ship these units in this manner because of their large size. In order to determine if it were possible railroad clearances were obtained, and it was found necessary to develop a new tank shape which would conform more closely to that of the core and coil assembly, to use a special shipping cover for the lower tank section, and to ship on a special depressed-bottom railway car. Bracing the core and coil inside the tank and the bracing of the entire assembly to the car for protection against shipping stresses common to such shipments presented many interesting problems. In fact, in the design of transformers such as these, the mechanical problem of fabrication, shipment, and installation in the field vie with those of electrical and thermal performance for degree of importance. One of these units is shown ready for shipment in figure 1, and

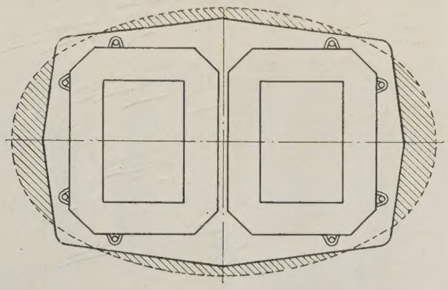


Fig. 2. Plan of core and coils in conventional octagonal and oval tanks

Crosshatched area indicates saving in space and oil with octagonal tank

it may be of interest to note that approximately 40 days time was required to reach the destination and that all-daylight movement at not over 30 miles per hour was required by the carriers.

Design of Tank

Inasmuch as the shell form of construction was used, a rectangular tank would be ideal because a transformer using such construction is rectangular in plan and could fit the tank closely. In order, therefore, to take full advantage of shipping clearances the tank should be essentially rectangular. However, a tank with large flat sides is weak, and large deflections would result upon subjection to pressure even though the pressure were moderate, such as that of the static head of oil, and even though the tank wall were quite thick. This inherent weakness of rectangular tanks has resulted in the past in the use of round or oval tanks and the sacrifice of such desirable characteristics as the material reductions of plan dimensions, oil content, and weight which have now been made possible. A simple modifica-

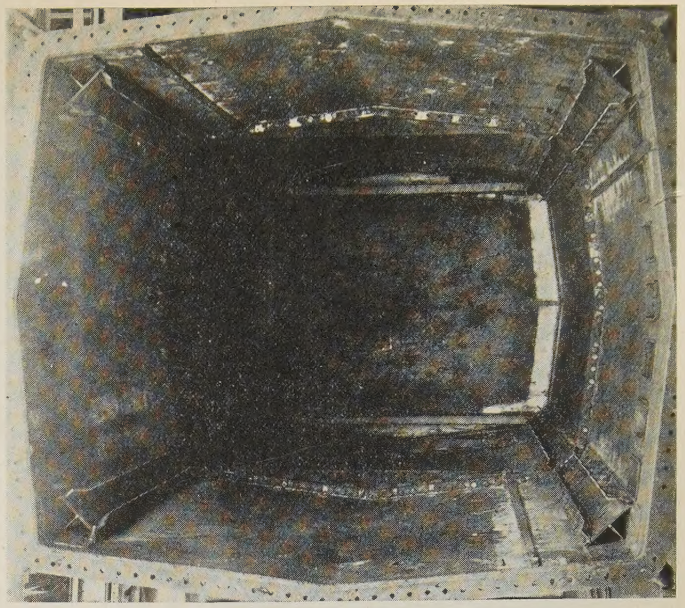


Fig. 3. View of interior of transformer tank, showing section flanges, guide rails in corners, and header valve openings

tion of the rectangular tank has transformed it into one having 8 sides, as shown in figures 2 and 3.

This new development is known as the "octagonal tank"[2] but actually it is essentially rectangular, having sides formed of 4 pairs of faces. This reduces the area of the flat sides to approximately half of what they otherwise would be and results in a very strong tank. In fact this tank is stronger, and will give less deflection under vacuum or pressure, than any other form for a given plate thickness and perimeter, the round tank excepted.

Transformers are usually shipped in their own tanks with the tank placed on the car in a vertical position, and if shipping clearances permit the tanks are made in one section. When clearances do not permit the use of such a manner of shipment, it becomes necessary either to ship the core and coils in a special shipping tank and to ship the regular tank on its side, or to sectionalize the regular tank and to ship the core and coils in the lower section. This latter practice, which usually requires a special shipping cover, is not new and was chosen for this application.

Mechanical Construction

When tanks have been sectionalized in the past, flanged surfaces and gaskets have been used to join the sections together. Such construction, alone, was not believed to be adequate for transformers such as these because with a gasketed joint there would still remain some risk of oil leakage after a long period of time. Consequently it was decided to weld these tank sections together in the field to make them permanently oil tight. This decision dictated that a suitable arrangement be found which was adapted not only for effective welding in the field but also for the usual assembly and processing in the factory, gas tight shipment, an oil tight assembly in the field

prior to welding, and last but not least which would present a good appearance. The construction finally selected is shown in figures 3, 4, and 5. In this construction the flanges are turned inward to make possible assembly with a gasket for factory processing and shipment as well as close conformity of top and bottom wall plates for the best type of position welding. This construction is neat in appearance and lends itself to testing the completed girth weld by

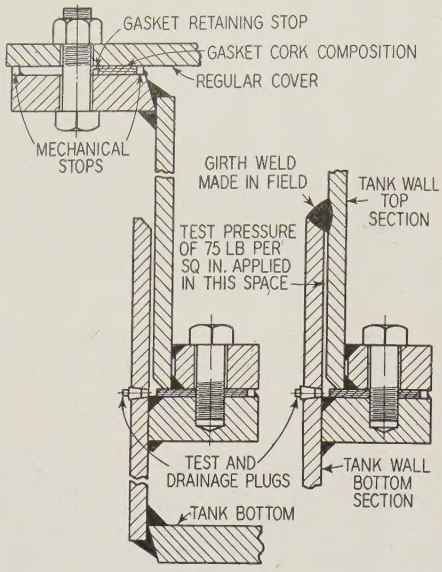


Fig. 4. Details of tank flange joints, showing at left the joint between tank sections as adopted for factory processing and at right as finally erected in the field

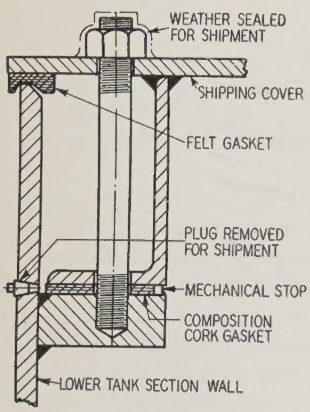


Fig. 5. Method of applying special shipping cover to lower section of transformer tank

applying air pressure to the space between the girth weld and the gasket. The transformer as prepared for shipment is shown in figure 1, and as completely assembled for test is shown in figure 6.

Although oil tightness of transformer tanks and fittings for large transformers has always been an important factor, it has received special emphasis during the past few years. Consequently it is now considered good practice to employ covered electrode welding for all oil tight welding on large transformer tanks because the weld metal is free from porosity and has a greater ductility. In using the covered electrode, welders must qualify for class II welding in accordance with A.S.M.E. "Boiler Code for Unfired Pressure Vessels." However, class III welding meets the requirements of the boiler code for transformers and is generally used. Should the thickness of the tank plate on very large transformers warrant it, double welds, as shown in figure 7, are used and a test pressure of 300 pounds per square inch is applied to the space between the welds. This latter practice was followed in fabricating the tanks for the transformers for Los Angeles in order to assure freedom from oil leaks or seepage in service.

Cooling System

New and novel means have been used in cooling these transformers as will be evident from a study of arrangements shown in figures 6 and 8. It will be noted that the perimeter of the new octagonal tank has been materially reduced over that of other tank forms as may be seen from figure 2. Even with the old tank forms, it was becoming increasingly difficult to find the space required to mount the radiators on the larger units. With the octagonal tank it is possible to mount all radiating surfaces direct from 2 sides of the tank wall by mounting 2 radiators in tandem from common headers. This arrangement results in the alignment of the radiator elements, and makes possible a new and unique air blast cooling system in which the air from the blowers traverses the entire length of the banked radiators, flowing parallel to the long axis of the streamlined elements of which the radiators are composed as shown in figure 9. An arrangement of this type gives more effective cooling with a great reduction in the number of blowers and blower losses without sacrificing the advantages to be gained by the dependability of a

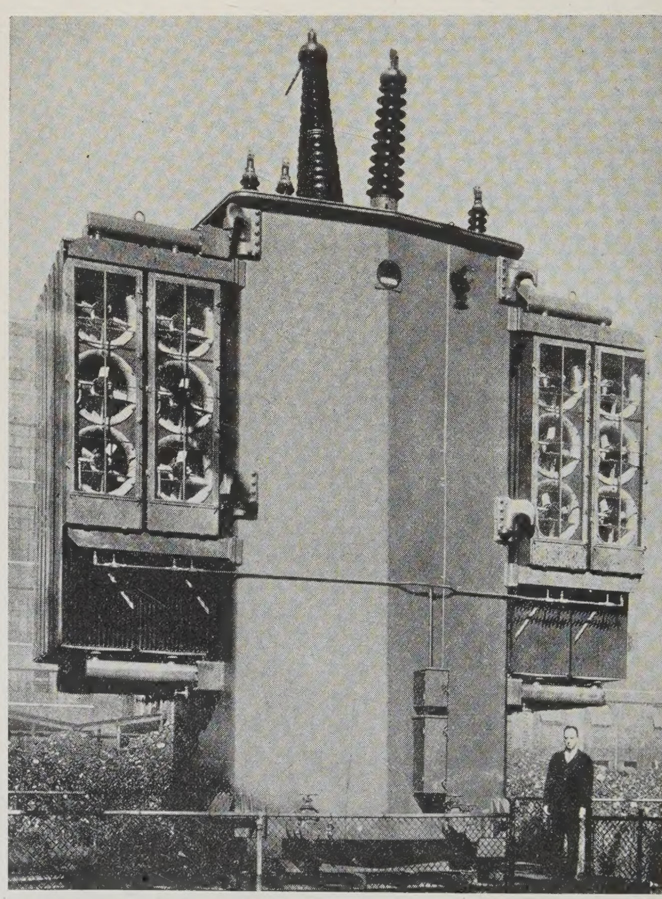


Fig. 6. Front view of one 65,000 kva transformer, completely assembled and ready for tests

nultiblower system. Also in such an arrangement, :he blower and radiator assemblies are independent of each other and require a minimum of wiring and conduit, resulting in a material simplification of in- stallation, repair, and maintenance. This design is :ermed the "streamlined air blast system." Special eak proof header valves are welded into the tank walls, both top and bottom, and provide mounting for the detachable radiator assemblies as shown in igure 9.

Other Mechanical Details

In addition to the new developments for these units which are more or less applicable to other simi- lar transformers, there are many other features of

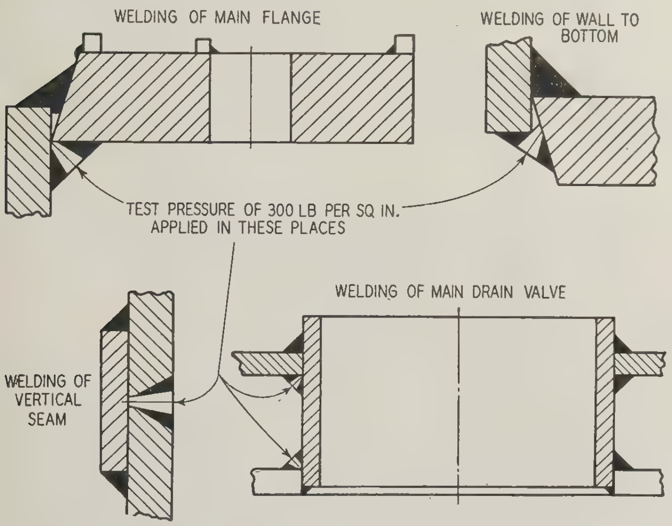


Fig. 7. Some methods of welding tank sections and fittings

minor interest, which were required specifically, such as a special base with wheels to withstand earth-quake shocks, bracing to withstand shipping stresses, guide rails as shown in figure 3 for tanking and un-tanking the core and coil assembly without removing the oil, and others.

Some idea of the tremendous size of these trans-formers may be secured by inspecting the weights and dimensions of a completely assembled unit as given in table I. The appearance of a core and coil assembly is shown in figure 10, a partial assembly view of a group of high voltage coils and insulation is shown in figure 11, and a view of one of the largest coils is shown in figure 12.

Electrical Design and Performance

Inasmuch as the performance to be obtained al-ways governs the electrical design it will no doubt be of interest to consider some of the salient factors affecting the electrical design of these units, such as insulation, reactance, efficiencies, bushings, noise, and radio interference.

The high level of line insulation, together with the associated problem of impulse voltages resulting

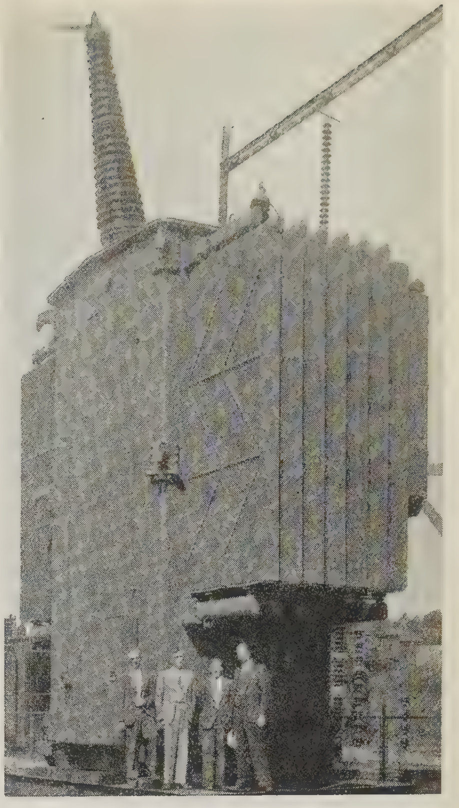


Fig. 8. Rear view of one 65,000 kva transformer, completely assembled and ready for tests

from lightning discharges, required special considera-tion. Unusually high voltage tests, both induced low frequency and impulse, were required to prove the adequacy of the insulation structure for service under these conditions.

The induced voltage test of 577 kv for 1 minute at 120 cycles is the highest ever used in the United States for testing power transformers, although approached at an earlier date by the tests imposed on the Roseland switching station transformers, built in 1929 for the Public Service Electric and Gas Company of New Jer-sey.[3] Also the impulse tests, which were of the highest voltage ever required on a commercial design, were materially complicated by the fact that these units were autotransformers. Five impulse tests were ap-

Table I—Transformer Weights and Dimensions

Weights of components of complete transformer, pounds	
Core and coils	160,000
Case and fittings	105,400
Oil	106,000
Total	371,400
Dimensions of complete transformer	
Elevation from tip of high voltage bushing to rail	35 feet 8 inches
Elevation from tip of low voltage bushing to rail	30 feet 0 inches
Plan—in a direction normal to a plane through high and low voltage bushings	23 feet 4 inches
Plan—in a direction parallel to a plane through high and low voltage bushings	12 feet 3 inches
Weights of components as shipped in lower section of tank, pounds	
Core and coils (complete)	160,000
Case and fittings (base and top section removed)	40,000
Oil (shipped in dry nitrogen gas)	0
Total (not including blocking)	200,000
Dimensions of transformer as shipped in lower section of tank	
Elevation from car rail	19 feet 0 inches
Plan—width as placed on car	10 feet 11 inches
Plan—length as placed on car	12 feet 4 inches

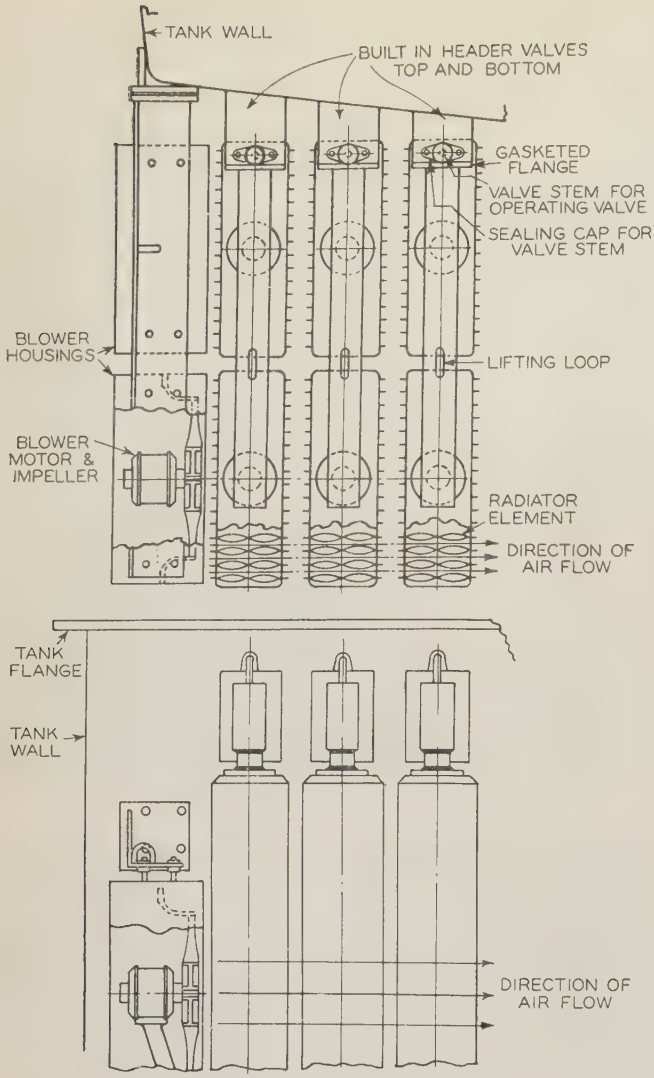


Fig. 9. Arrangement of radiators and blowers in air blast cooling system

Note direction of air flow in relation to radiator elements

plied in turn to both the high voltage and the low voltage line terminals, with the neutral solidly grounded, on each of the 7 transformers. These tests were those originally proposed by the A.I.E.E. transformer subcommittee,[4] using a standard test gap, as proposed by this committee, of 88 inches for testing the high voltage insulation and 42 inches for testing the low voltage insulation. The required impulse tests were as follows:

a. A wave having a voltage not more than 10 per cent less than the minimum permitted by the specified gap.

b. A wave sufficient to flash over the specified gap.

c. A wave having a crest voltage at least 10 per cent greater than the minimum flashover voltage of the test gap.

d. A wave having a voltage sufficient to flash over the bushing.

e. A wave having a voltage as great as that specified in test *c* or *d*, but with means for maintaining the excitation voltage across all parts of the winding.

Line stability required that the reactance of these transformers be much lower than would normally result without such a limitation. As a result of this requirement, reactance limits were set by the City of Los Angeles with the understanding that still lower values were desirable. It was found possible in the final design to secure reactances of the order of a third of those in normal designs. The following tabulation gives the average test values of reactance between windings at the loads given:

Windings, Ratios in Kilovolts	Kilovolt-amperes	Per Cent Reactance
275 to 13.2	65,000	6.42
275 to 13.2	16,000	3.53
132 to 13.2	16,000	2.75

The following tabulation of guaranteed efficiencies from high voltage to low voltage at 65,000 kva and unity power factor, with the tertiary idle, will be self-explanatory. A total loss of only 0.54 per cent at full load is indicated. All efficiency guarantees were met with ample margins.

Load, Kilovolt amperes	Efficiency, Per Cent
12,000	98.91
24,000	99.35
36,000	99.46
48,750	99.49
65,000	99.46
80,000	99.42

Bushing Design

The high voltage bushings for the Boulder Dam transformers have the highest flashover voltages of any yet made in the United States for power transmission service. The bushings have a 60 cycle dry flashover voltage of 820 kv and a 60 cycle wet flashover voltage of 635 kv, and are equipped with suitable gaps to make them equivalent to 92 inch test gaps for either positive or negative surges.

The line leads were brought through the cover of the tank by substantially improved capacitor bushings involving many advances in mechanical parts. Past experience has shown that it is extremely im-

Fig. 10. View of core and coils of one 65,000 kva transformer, showing rugged all-welded end frames for clamping and supporting coils, insulation, and core iron

portant to prevent the entrance of moisture into a high voltage bushing. In the new design of mechanical parts, provision has been made for the expansion and contraction of parts with changes in temperature without the necessity or possibility of breathing the surrounding air. This nonbreathing characteristic has been obtained by clamping the porcelain between the mounting flange and a special spring cap designed to maintain a positive pressure on the gaskets which in turn are protected from over-

Fig. 11. Partial assembly of a group of high voltage coils, showing coil, insulation, and lead arrangement. The bottom cable lead will go directly to the high voltage line bushing

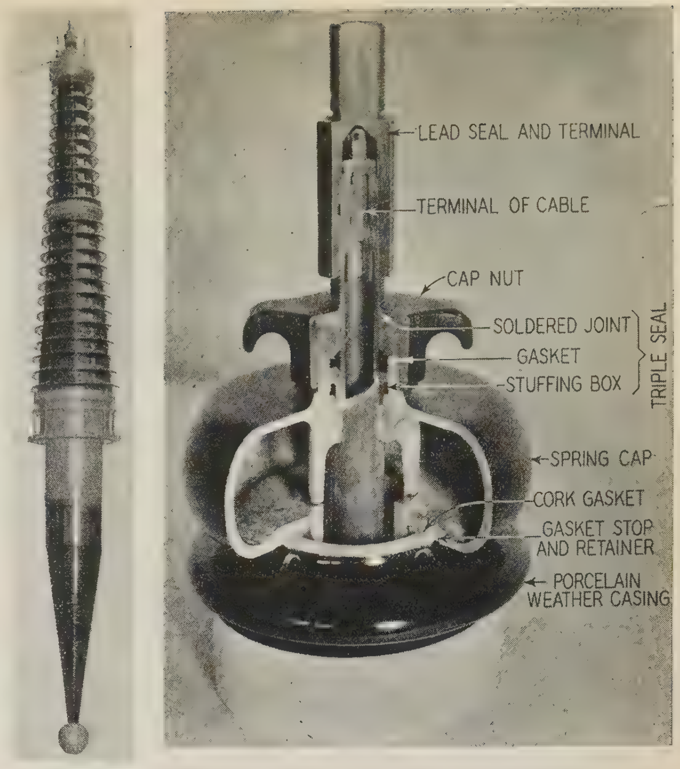


Fig. 13. A 287.5 kv capacitor bushing, and details of the spring cap construction

compression by mechanical stops. Figure 13 shows a 287.5 kv bushing and a sectional view of the spring cap. Barring mechanical damage to the bushings they should remain dry indefinitely. The tightness and strength of this new mechanical design has been proved by dropping the line end of a bushing 2 inches for 3,000 times without mechanical injury or loss of tightness, nor was there any loss of tightness after repeatedly heating the bushing to 95 degrees centigrade and cooling to −5 degrees centigrade. During manufacture all joints of all bushings are tested to insure tightness. Another proof of the ruggedness of design came quite by accident when a transformer was shipped with bushings of this type in place, the height then exceeding a bridge clearance. Although the tubes through the bushings were bent about 45 degrees, just above the cap, they were found to be tight and otherwise uninjured.

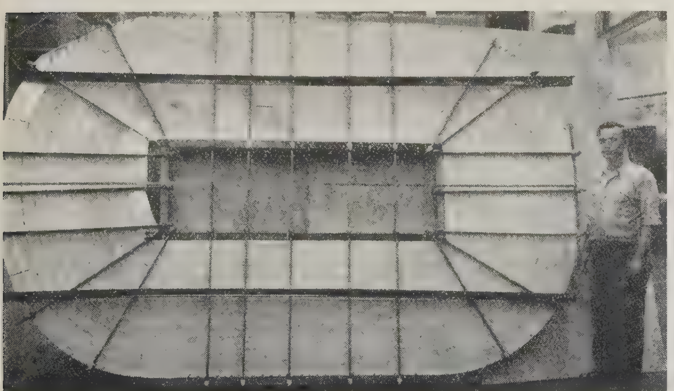


Fig. 12. One of the tertiary coils as formed and clamped for treatment

Other Considerations

Noise becomes important when transformers are to be located in a residential section of a city. Consequently it was an important consideration in the design of these units, and resulted in the use of a core density somewhat below that of a design where a reasonable amount of noise is permissible. Also, care was taken in the design of parts with particular regard for tightness of all mechanical members, whether inside the main tank or mounted external to the tank.

Like mechanical noise, radio interference is very objectionable in congested residential districts where the source of interference is in close proximity to the receiver. Although the attenuation of interference energy is very rapid with an increase in distance from the source, it becomes more and more necessary to consider this factor as the transmission voltage rises. Care was exercised in the design of these units to keep radio interference to a low level.

The authors believe that it may be stated safely that, although a power transformer is often considered a simple piece of static apparatus and hence worthy of relatively little note or attention, it is actually an exceedingly complicated piece of apparatus with design features embracing nearly all branches of engineering. Consequently the progress which has taken place in the arts of design and manufacture of transformers is to be expected. Certainly the mechanical and electrical design of these transformers exemplifies the rapid progress which

has been made in the general design and construction of transformers in recent years.

REFERENCES

1. ENGINEERING FEATURES OF THE BOULDER DAM-LOS ANGELES LINES, E. F. Scattergood. ELEC. ENGG. (A.I.E.E. TRANS.), v. 54, May 1935, p. 494–512.

2. "OCTAGONAL" TANKS FOR LARGE POWER TRANSFORMERS MEET SHIPPING AND SERVICE REQUIREMENTS, William J. Hart. *Steel*, v. 97, Sept. 30, 1935, p. 22.

3. DESIGN FEATURES OF THE WORLD'S LARGEST TRANSFORMERS, H. L. Cole and F. J. Vogel. *Elec. Wld.*, v. 97, Feb. 21, 1931, p. 374–77.

4. PROGRESS REPORT ON IMPULSE TESTING OF COMMERCIAL TRANSFORMERS, F. J. Vogel and V. M. Montsinger. A.I.E.E. TRANS., v. 52, June 1933, p. 409–10.

The Sparkless Sphere Gap Voltmeter—II

Further study of the method of measuring voltages by the force between spheres is reported in this paper, comparison being made of results obtained by means of 100 centimeter spheres with those obtained by other investigators and by a proposed* new A.I.E.E. standard for 50 centimeter spheres.

By
R. W. SORENSEN
FELLOW A.I.E.E.

SIMON RAMO
ENROLLED STUDENT A.I.E.E.

Both of California Institute of Technology, Pasadena

THE previous paper[1] presented the results of certain investigations made to determine the feasibility of using as a measure of voltage the force between a pair of spheres rather than the spark-over distance between them, and compared the results so obtained with spark-over voltages versus spacings for a pair of 100 centimeter spheres mounted along a horizontal axis.

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 3, 1936; released for publication March 28, 1936.

The authors acknowledge with appreciation the careful and large amount of laboratory work done by Gilbert McCann, Louis Rader, Leonard Patterson, F. V. Maloney, and George Kaneko in obtaining the results used in this paper.

* The A.I.E.E. committee on instruments and measurements appointed a subcommittee in 1933 under the chairmanship of E. J. Rutan to study sphere gap standards and the measurement of impulse voltages. The report of this subcommittee on the sphere gap spark-over voltages for 60 cycles and negative and positive impulses was submitted to the committee for letter ballot, and was approved on April 7, 1936. The recommendation now has been forwarded to the A.I.E.E. standards committee.

1. For numbered references see list at end of paper.

Those results were such as to warrant a further study of force between spheres as a means of measuring high voltages. This study as reported here comprises a comparison of voltage as determined by the force on the 100 centimeter horizontally mounted spheres used for the tests described in the previous paper with the spark-over voltage versus spacing for a proposed new A.I.E.E. standard 50 centimeter sphere gap, and a more complete study of force measurements between spheres as a basic or standard method of determining voltage.

In making this study, tests were made on the standard 50 centimeter sphere gap for vertical and horizontal sphere positions at different heights above the floor, the spheres for all tests being mounted in a framework having the recognized proper dimensions.

The following conclusions have been reached as a result of these tests:

1. The sparkless sphere gap voltmeter is worthy of serious consideration as a standard means for measuring high voltages.

2. The proposed new A.I.E.E. standard for the 50 centimeter sphere spark gap is a better calibration than the old standard, but probably indicates for given gap settings voltages which are too low for the larger gap spacings and too high for the smaller spacings.

3. There is no inherent difference between horizontally and vertically mounted sphere spark gaps and the influence of the floor or ground plans is very small for either position when the gaps are of the order of 6 sphere diameters from the ground plane and the gap spacings not over 3/4 the sphere diameter.

4. The sphere spark gap voltmeter is a convenience, but cannot be depended upon as a standard because atmospheric air does not have a constant breakdown voltage, nor does correcting for temperature and barometric pressure account for all variables.

5. The sphere spark gap voltmeter becomes increasingly inconsistent as the gap spacings exceeds 3/4 a sphere diameter.

CONDUCT OF TESTS

Voltage was obtained from the one million volt cascade transformer of the California Institute of Technology.[2] The effective value of the wave form was 99.25 per cent that of a true sine wave having the same crest value.[1] Voltage was supplied to the sphere gaps through a one megohm resistance connected as shown in figure 1.

The dimensions of the laboratory and the arrangement of equipment are shown in figure 1. The clearances quite evidently are such as to make the influence of building boundaries and equipment on sphere gap performance a reasonable minimum.

A special quick opening circuit breaker was used in the primary of the transformer, thus avoiding the necessity of running water in the resistance used to prevent pitting the spheres on flashover.

The test procedure was as follows: The 50 centimeter sphere gap was set at some desired spacing and a large number of spark-over tests made for each setting, the voltage on the tertiary or voltmeter coil being noted as each flashover occurred. For each test the voltage was started at zero and increased steadily at as rapid a rate as possible consistent with accuracy of tertiary voltmeter reading, the rate of rise being about 20 kv per second. Slower rates of voltage rise are productive of greater variations in spark-over voltage for any given gap setting, the slower rise frequently resulting in flashover at volt-

ages quite definitely below values specified by A.I.E.E. standards for given gap spacings.

These tests were followed by and interspersed with many tests for determining the relation between tertiary coil voltmeter readings and force measurements on the 100 centimeter sparkless sphere gap for appropriate selected gap spacings and voltmeter settings. Thus the tertiary voltmeter readings were made to serve wholly as comparative readings, making unnecessary a determination of the relation between tertiary voltage and the voltage of the transformer. Readings of transformer voltages as computed from the forces on the spheres were readily reproducible with an accuracy of better than 0.5 per cent for all tertiary voltmeter readings.

The tests were made under varying conditions as to temperature, humidity, and barometric pressure, but all readings have been corrected to 25 degrees centigrade and 760 millimeters barometric pressure.

Tests with the 50 centimeter gap were for the most part made with the gap in normal operating position, the axis of the gap vertical and the center of the upper or fixed sphere about 275 centimeters or $5^1/_2$ diameters above the floor. Two sets of tests were made with the gap horizontal, one at the same height, about 5 diameters above the floor, and the other at a distance of 3 diameters (150 centimeters) above the floor.

A fourth set of tests with the gap in a vertical position and over 10 diameters above the floor was also made to check the influence of the floor upon spark-over voltage.

The 2 sets of tests with the gaps in a vertical position gave identical readings for the 2 positions 5 and 10 diameters above the floor for gap spacings up to 50 per cent of the sphere diameter, but for greater spacings there was a definite indication that the higher the gap above the floor the greater the voltage required to produce flashover for given gap settings.

The sets of tests made with the gap horizontal, at standard distance 5 diameters above ground, gave readings which indicate that, with the spheres in this position, correct readings may be obtained for gaps not exceeding half the sphere diameter, but for larger spacings the gap when horizontal flashes over at voltages somewhat lower than for corresponding spacings when the gap is vertical. (See figures 2, 3, and 4.)

With the gap horizontal and only 3 diameters above ground, flashover occurred for all spacings at voltages definitely lower than for like spacings of the gap in its normal operating position. At a spacing of 40 centimeters the voltage in this position was less than 90 per cent the voltage required for flashover in normal position.

In figures 2 and 3 our test points and the results of other investigators are plotted so that the distances of the points from the reference line show the differences between the voltages indicated by the points and the voltages at which, according to a proposed new A.I.E.E. standard, the gap for a given setting should spark over. For each set of tests with the 50 centimeter sphere gap, the gap was conditioned by a number of flashovers before any readings were recorded, but all readings taken after the gaps indicated the ability to give consistent readings, barring, perhaps, a few quite evidently in error, have been plotted. The total spread of points shown for the various gap settings is not the spread found during any one test period, but is that which may be expected for tests extending over several months, and

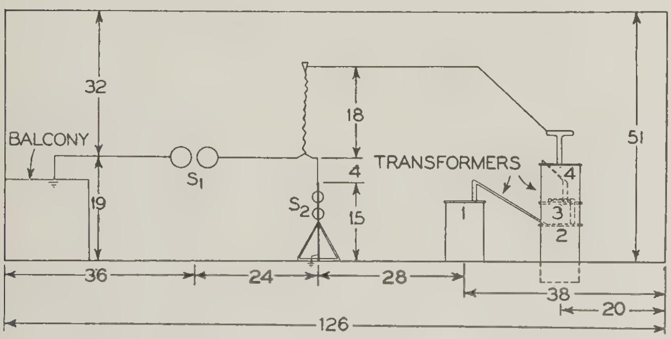


Fig. 1 (above). Arrangement of apparatus for tests in laboratory

S_1—Sparkless 100 centimeter spheres
S_2—50 centimeter spheres in standard frame
Laboratory is 64 feet wide with the test gaps approximately along the center of the building; all dimensions shown are in feet

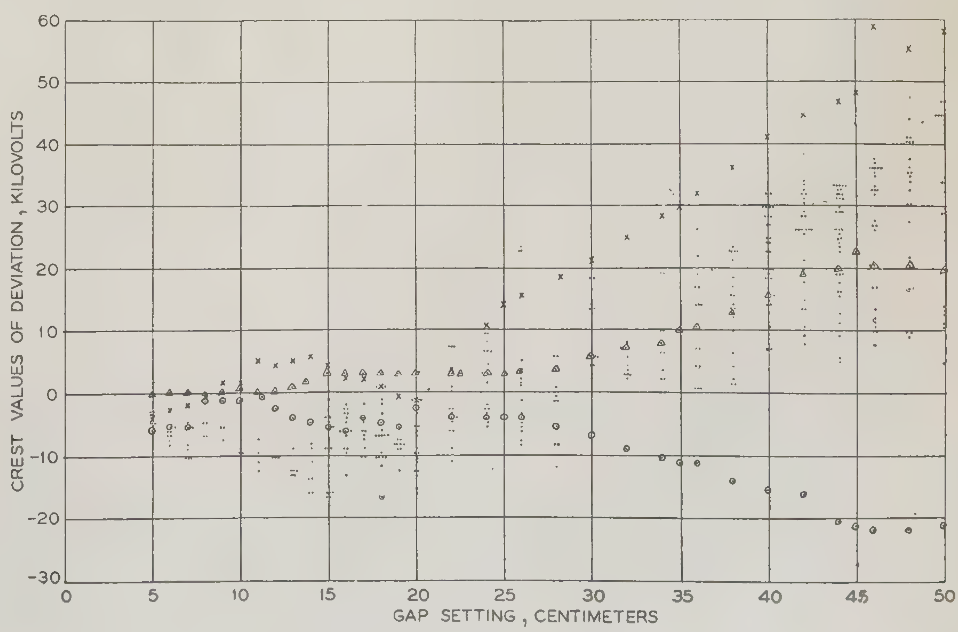


Fig. 2 (right). Deviation of test points from proposed new standard curves; spheres vertical, lower spheres grounded, upper sphere about $5^1/_2$ diameters from floor

Values obtained by other investigators shown as follows: X—Peek[3]; △—Bellaschi[4]; ⊙—Meador[5]

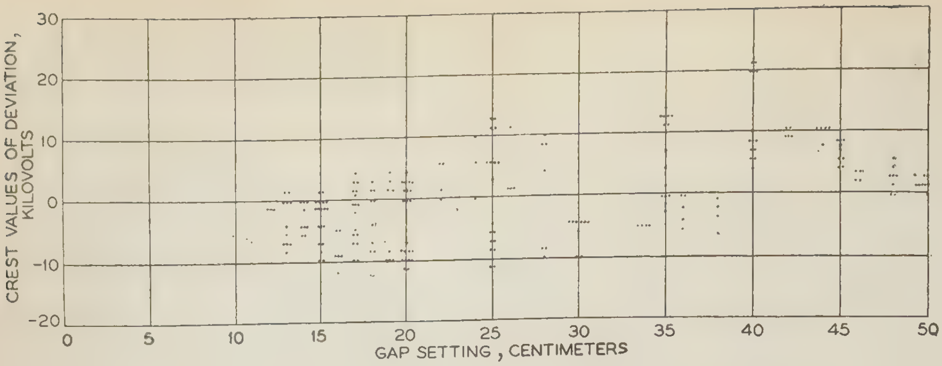


Fig. 3. Deviation of test points from proposed new standard curves for spheres horizontal about $5^1/_2$ diameters above floor

is a spread which apparently cannot be avoided by any amount of care and skill in performing the tests.

The accuracy of voltage determination by means of the sparkless sphere gap is a function of the accuracy used in determining the following:

1. Tertiary voltmeter reading
2. Sphere gap spacings
3. Force
4. Spacing factor S used in the equation $F = SV^2$ where F is the force, and V is the voltage impressed on the spheres.

The accuracy of single readings for items 1 to 3 was of the order of 0.2 per cent for tertiary voltmeter readings, $^1/_2$ millimeter for readings of sphere gap settings, and within a gram for force readings, the forces ranging from 100 to over 350 grams.

Mean values of numerous readings made from time to time by different observers and not single readings have been used in determining the relation between tertiary voltmeter readings and the forces between the sparkless spheres at various spacings. The average deviation of single reading values was not more than 0.5 per cent from a curve plotted through the mean values thus obtained.

The fourth item, spacing factor S, can be calculated accurately for isolated spheres. True isolated spheres are, of course, impossible and for actual conditions one must, as in the case of standard sphere gap measurements, take account of supporting shanks and adjacent bodies. The influence of shanks and adjacent bodies on the attractive force between the spheres may be determined in theory by test or by calculation, but in actual operation both methods present difficulties. One test method is that of measuring the capacitances at different spacings for the pair of spheres as a capacitor, and comparing the rates of change of capacitance values with distance to the calculated rates of change for isolated spheres. Such tests call for precise small difference determinations of electrical quantities for apparatus of large physical dimensions. Such determinations are not readily made by methods at our disposal and have not to date yielded satisfactory results.

Determination of Effect of Nearby Objects

The effect of shanks and nearby objects on the voltage required to exert a force on the spheres can be calculated exactly if enough image points are used. The voltage correction factor curves shown in figures 5 and 6 have been calculated with a sufficient number of image points to assure correction factors having a deviation of only about 5 per cent the absolute correction values which would be obtained with an infinite number of image points. Calculations made by Lord Kelvin, found useful in our image calculations and not readily available, are shown in convenient form in table I. By the use of this table the charge on each sphere and the force between them could be calculated for the condition of neither sphere at ground potential.

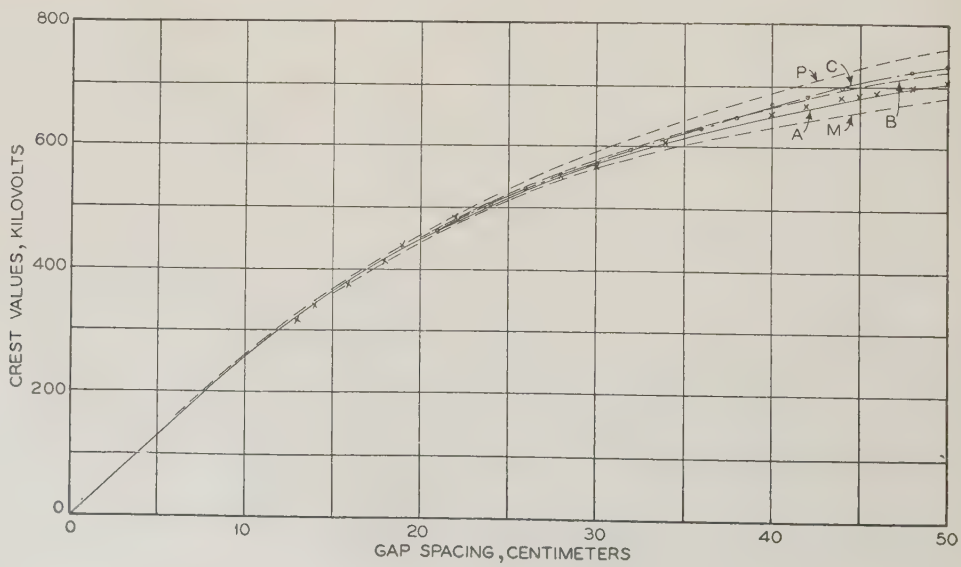


Fig. 4. Curves of sparking voltages for 50 centimeter spheres

A—Proposed new A.I.E.E. standard
B—From Bellaschi
C—Curve through means of points in figure 2
X—Means of points in figure 3
M—From Meador
P—From Peek

Table I—Calculations Made by Lord Kelvin for Sphere Gaps

Gap Setting in Per Cent of Sphere Diameter	$\frac{C_s}{\text{radius}}$	$-\frac{C_m}{\text{radius}}$	$\frac{1}{2}\frac{\partial C_s}{\partial x}$	$-\frac{\partial C_m}{\partial x}$
5	1.58396	0.88175	1.13844	2.34878
10	1.43131	0.72378	0.52852	1.12700
15	1.34827	0.63395	0.32917	0.72714
20	1.29316	0.57202	0.23159	0.52928
25	1.25324	0.52537	0.17432	0.41260
30	1.22218	0.48819	0.13696	0.33574
35	1.19755	0.45746	0.11082	0.28180
40	1.17738	0.43140	0.09174	0.24146
45	1.16056	0.40886	0.07720	0.21052
50	1.14629	0.38908	0.06592	0.18598
55	1.13404	0.37151	0.05693	0.16608
60	1.12340	0.35571	0.04963	0.14962
65	1.11410	0.34150	0.04363	0.13582
70	1.10588	0.32852	0.03863	0.12406
75	1.09859	0.31663	0.03441	0.11394
80	1.09208	0.30569	0.03084	0.10514
85	1.08623	0.29557	0.02775	0.09744
90	1.08095	0 28617	0.02509	0.09062
95	1.07617	0.27742	0.02278	0.08458
100	1.07182	0.26942	0.02075	0.07916

All values are in electrostatic units. The quantities C_s and C_m are the self- and mutual capacitances, respectively and $\frac{\partial C}{\partial x}$ denotes the rate of change of capacitance with respect to distance between centers. Hence, if the potentials of the 2 spheres are V_1 and V_2, then the total charges in each, Q_1 and Q_2, and the attractive force are given by:

$$Q_1 = C_sV_1 + C_mV_2 \quad Q_2 = C_sV_2 + C_mV_1$$

$$F = \frac{1}{2}\frac{\partial C_s}{\partial x}V_1^2 + \frac{\partial C_m}{\partial x}V_1V_2 + \frac{1}{2}\frac{\partial C_s}{\partial x}V_2^2$$

The values in table I in combination with the curves of figures 5 and 6 enable calculations to be made for the effect of combinations of planes such as intersecting floor and walls. The effect of these combinations is less than the sum of the influences of the individual planes.

A calculation of the effect of shanks was made for shanks 90 centimeters long and of varying diameter, approximately 5.5 centimeters. This calculation applied to the 100 centimeter spheres for a gap spacing of 25 centimeters showed that the voltage as determined by force measurement should be decreased 0.6 per cent. The actual shanks used in the laboratory are 5.5 centimeters in diameter and more than 90 centimeters long so the correction should be somewhat more than 0.6 per cent. The influence, however, of increased length is a rapidly decreasing increment.

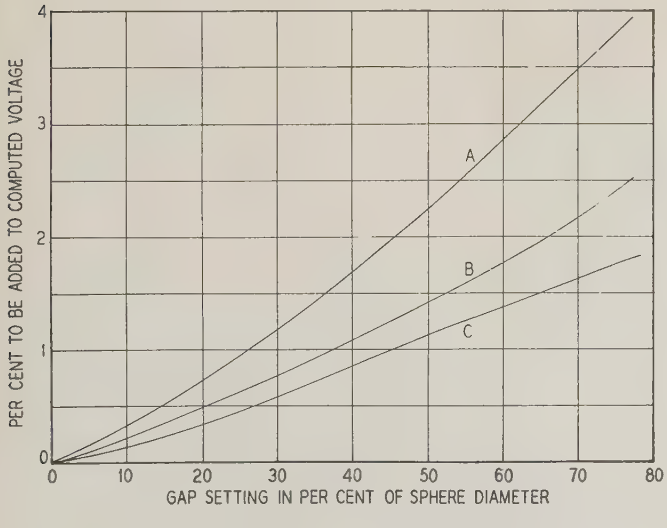


Fig. 5. Curves showing correction for effect of grounded plane parallel to the line of centers of the spheres on the voltage computed from force between spheres

Curves *A*, *B*, and *C* for distances above ground of 4, 6, and 8 diameters, respectively

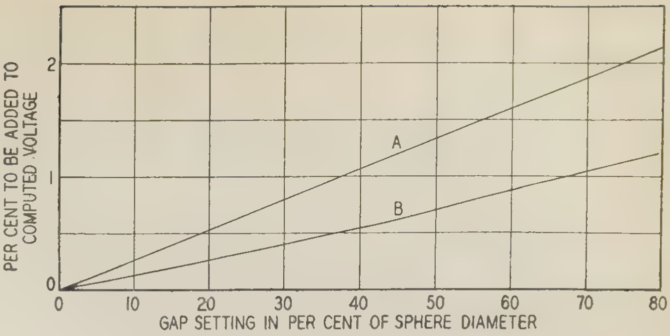


Fig. 6. Curves showing correction for effect of grounded plane perpendicular to the line of centers of the spheres on the voltage computed from force between spheres

Curves *A* and *B* for distances of the plane from the spheres of 5 and 10 diameters, respectively; sphere nearest the plane at plane potential

With the large clearances to floor and walls, and the total influence of the ground planes in a direction opposite that of the shanks and of the same order of magnitude for gap spacings of 25 centimeters or less on the sparkless 100 centimeter sphere gap, it has been considered advisable not to make corrections for these quantities. For greater spacings the influence of ground planes and shanks may not offset each other so nearly. Our large volume of test data for the range of spacing required to compare the readings of the 100 centimeter sparkless gap with the proposed new standard 50 centimeter spark gap shows that at the maximum required setting of the sparkless gap (35 centimeters) the correction for shanks and ground planes would not exceed 1.5 per cent; that is, the voltages indicated by the curves of figures 2, 3, and 4 would be decreased by an amount not exceeding this. This small correction, the only one which need be applied for sparkless sphere gaps, and which with further data available may be determined with excellent accuracy, indicates the possibility of using such sphere gaps for reference standards for high voltage measurements.

References

1. The Sparkless Sphere Gap Voltmeter, R. W. Sorensen, J. E. Hobson, and Simon Ramo. Elec. Engg. (A.I.E.E. Trans), v. 54, June 1935, p. 651–6.

2. Development and Characteristics of a 1,000,000-Volt Cascade Transformer at California Institute of Technology, R. W. Sorensen. A.I.E.E. Jl., v. 44, Apr. 1925, p. 373–8.

3. Dielectric Phenomena in High Voltage Engineering (a book), F. W. Peek, Jr. McGraw-Hill Book Co., New York, 1929, third edition.

4. Impulse Calibration of Sphere Gaps, P. L. Bellaschi and P. H. McAuley. *Elec. Jl.*, v. 31, June 1934, p. 228–32.

5. Calibration of the Sphere Gap, J. R. Meador. Elec. Engg. (A.I.E.E. Trans.), v. 53, June 1934, p. 942–8.

6. Discussion, J. R. Meador. Elec. Engg. (A.I.E.E. Trans.), v. 54, Sept. 1935, p. 1002–3.

Fields and Charges About a Conductor

An investigation of the electrostatic fields and space charges surrounding a conductor at high potentials, both alternating and continuous, is reported in this paper. The importance of space charge in determining the distribution of electric gradient is shown on experimental apparatus the dimensions of which approach those of many power transmission and related structures.

By
W. G. HOOVER
ASSOCIATE A.I.E.E.

Stanford Univ., Calif.

THE EXISTENCE of space charge is of importance in the mechanism of electrical discharge in gases and as such has a direct bearing upon many high voltage phenomena of engineering interest. A number of investigations of space charge effects have been made, mainly in recent years and with alternating voltages. To some extent a unity of theory and results has been obtained.

In studying the corona produced by continuous potentials, Farwell[1] and Warner and Kunz[2] determined potential distributions between parallel wires and between a wire and a surrounding cylinder by means of insulated probes. Considerable variations from conditions theoretically existing without corona are indicated. For the latter electrode arrangement, curves of space charge distribution were derived by means of Poisson's[3] equation. They indicate the original data to be not more than qualitatively correct, since to satisfy the charge distributions shown, ions of unreasonably high mobility would be required in certain parts of the field.

With alternating voltages applied to a wire at the center of a cylinder, Carroll and Ryan[4] employed an exploring wire to determine both the time and space variations of potential as a result of the presence of ions. Willis[5] used an experimental arrangement of a wire and plane, the distance between them being adjustable. Measurements of the charge arriving at the plane for various distances, voltages, and wire sizes apparently indicate a greater effect of diffusion in the movement of space charge than do those of Carroll and Ryan. This work demonstrates the influence of the excess of voltage above the corona starting value upon the limits of travel of the boundary of the charge. A study by Carroll and Lusignan[6] of the time rate of arrival of charge at both a plane and a cylinder from a wire in corona shows that the phase of the arrival of charge may be greatly different from that of the applied voltage, and indicates that the resultant field acting upon the ions is influenced largely by the space charge itself. Holm[7] has contributed to the theoretical knowledge of space charge through the presentation of a quantitative relationship for calculating corona loss. Though involving simplifying assumptions, the check with measured values of loss has been shown to be satisfactory, both by his own work and that of Waldorf.[8]

It has been the object of the present investigation to secure further information concerning the effects of space charge in regions surrounding a conductor in corona. The scope of the laboratory work was confined to the field between a wire and a cylinder, but the dimensions involved were of sufficient magnitude to approach those of many practical electrical structures. Both continuous and alternating voltages were applied to the wire so that a wide range of conditions might be attained.

Summary of Results

The results of this experimental investigation may be summarized as follows:

1. With both d-c and a-c corona, the importance of space charge as a factor in determining the distributions of electric gradient in the regions surrounding the discharge is demonstrated. The assumption has frequently been made, in engineering studies, that ions in the regions outside the discharge are acted upon by the field existing if there were no corona. That this may involve serious error is evident.

2. Exploration of steady electric fields of cylindrical symmetry is accomplished with a considerable degree of accuracy. The rotating exploring device is shown to be a practicable means for such measurements, whether or not the region contains space charge.

3. Though less accuracy was obtained in the measurements of alternating fields around the wire in corona, definite indications of the range of travel of the space charge and the time variation of the field as altered by this charge are obtained. The distinction between the main body of moving charge and the surplus charge that may reach the cylinder is clarified.

Methods and Theory of Experimental Procedure

The major portion of the experimental work consisted of the measurement of field intensity at various radial distances from the central conductor, with both continuous and alternating voltages applied to the wire. Analysis of the field intensity data with continuous voltages was made through the application of Poisson's equation. This may be expressed in cylindrical co-ordinates as

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho \qquad (1)$$

V represents the electric potential at any point, ρ the corresponding space charge density, and r,

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 2, 1936; released for publication March 23, 1936.

The author is indebted to Prof. J. S. Carroll of the department of electrical engineering for his co-operation and constructive criticisms.

1. For numbered references see list at end of paper.

ϕ, and z the co-ordinates of the point. As guard rings were used at the ends of the cylinder in this investigation, there was produced a nearly radial field from the wire to the central section; therefore in the application of this equation the derivatives with respect to z and ϕ were considered zero. The relation in simplified form becomes

$$\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} = -4\pi\rho \qquad (2)$$

Equation 2 was applied directly to the analysis of field strength measurements. Values proportional to the electric gradient were plotted, for any particular condition, as functions of the distance from the wire. The ordinate at any point was multiplied by a calibration constant for the determination of dV/dr, while the same constant times the slope of the curve gave d^2V/dr^2.

With d-c corona, the ions at all radial distances within the range of investigations were considered to be only of the same sign as that of the potential on the wire. The basis of this assumption was the fact that all field strengths measured were many times less than that required for the breakdown of air, and hence were not in regions of appreciable ionization. The numbers of ions per cubic centimeter were calculated by further assuming that they were predominantly singly charged.[9] The electron charge was taken as 4.77×10^{-10} electrostatic units.[10] Measurements of current to the central section of the cylinder provided a means of calculating ion velocities and mobilities. Let the current density in the gas be i (in the radial direction). The drift velocity v was found from the expression

$$v = \frac{i}{\rho} \qquad (3)$$

The mobility k was computed in terms of the velocity per unit field, from the relation

$$k = \frac{v}{dV/dr} \qquad (4)$$

The rotating field exploring device was adapted directly from the rotary voltmeter for measuring high potentials.[11] In the construction of the latter the rotating part usually consists of a split cylinder connected to a commutator, to the brushes of which is connected a galvanometer. The voltage to be measured is applied to plates, creating an electric field in the region of the cylinder. This field causes the appearance of bound charges on opposite sides of the rotating cylinder and the maintenance of this charge distribution requires the flow of current through the commutator and galvanometer circuit. In the actual construction used for field measurement in this investigation, 2 parallel wires were used instead of cylindrical segments. The operation is the same, however, commutation taking place when the plane of the wires is parallel to the field.

The total flow of charge through the galvanometer between successive commutation instants is equal to the difference in charge at the beginning and end of this interval and is independent of the manner of its variation between these limits. This device may be

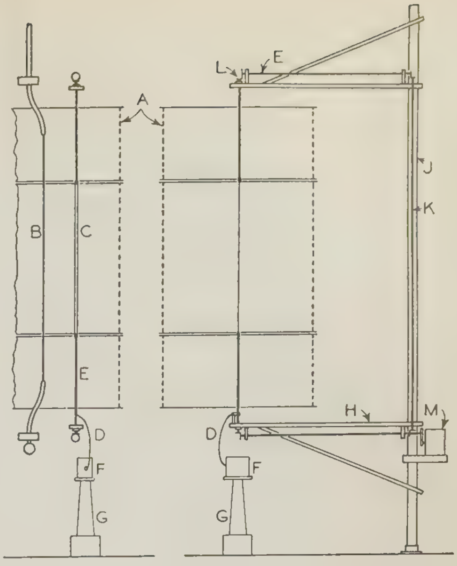


Fig. 1. Diagram illustrating arrangement of wire, cylinder, and exploring device

A—Cylinder
B—Central wire
C—Rotating wires
D—Commutator
E—Bakelite tube
F—Galvanometer shield cage
G—Porcelain pedestal
H—Wooden arm
J—Pipe
K—Drive shaft
L—Gears
M—Synchronous motor

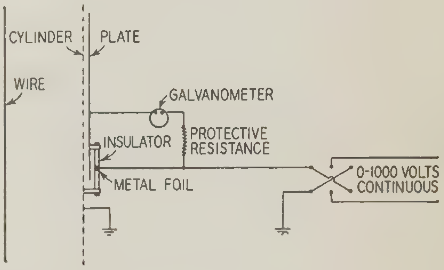


Fig. 2. Connections to plate for measurement of rectified charge

used with alternating voltages and fields, by driving the rotating structure with a synchronous motor at a speed such that the potential is the same at successive commutation times. A 4 pole motor will bring about this condition, providing commutation at points one cycle apart. The particular point of the voltage cycle at which the potential difference is measured may be controlled by advancing or retarding the rotor in its phase relation to the applied voltage. The complete wave form may be determined in this manner.

It may be shown that where ions are not present in the surrounding region, the average galvanometer current is independent of the potential of the rotating structure.[12] This follows from the application of the principle of the superposition of electrostatic fields. When the region surrounding the rotating device contains ions, difficulties may be encountered. In continuous fields, the drift of ions into the wires, allowed by insulation leakage, does not contribute to the average galvanometer current; and if the potential of the rotor is not too different from that of the adjacent space, its disturbance of the normal flow of ions in that region should be small. With alternating fields, the difference between the potential of the rotating element and that corresponding to its position may be considerable, mainly as a result of the capacitance of the galvanometer circuit to regions of different potential. A corresponding serious distortion of the field in the region surrounding the revolving wires is easily possible. The importance of this is that the motions of the space charge may be considerably disturbed and erroneous results obtained. In addition, leakage currents may

result in an average galvanometer current because of the time variation of surrounding ion density. The seriousness of these troubles can be reduced considerably by maintaining the exploring device at a potential corresponding approximately to that of the space in which it exists. A separate source of adjustable voltage and phase is required. In the experimental work, the test for the proper potential was made by means of an electroscope connected between the commutator and the lead from the galvanometer. The adjustment producing the minimum deflection of the electroscope leaf was used. Because of the effects of moving space charges upon the variation of the field, only a very approximate balance could be obtained near the wire with the higher voltages. A source of balancing potential containing controllable harmonic components would no doubt produce the most satisfactory results.

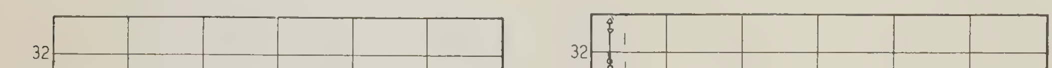
The arrival of charge at the cylinder, with a-c corona, was investigated over a wide range of voltages; first by measuring the net flow of charge, and second by separating this flow into its component parts. A galvanometer connected between the central section of the cylinder and ground indicated the former. The latter information was obtained by using a polarized plate placed just outside a section of the mesh constituting the cylinder.

The diameter of the main cylinder was made sufficiently large that ample room should be available for the free motion of the space charge when subjected to alternating fields. The size of wire employed for the corona electrode was such that the corona starting voltage was less than half the available continuous voltage. All a-c measurements were made at a frequency of 60 cycles. Variations in air density were small and were not corrected for in this work.

Description of Apparatus

The cylinder was 6 feet in diameter and consisted of 3 parts: a central section 6 feet long, and 2 end sections each 3 feet long. A separation of $^1/_2$ inch was maintained between these sections. The material used for the cylinder walls was $^1/_2$ inch mesh galvanized wire cloth. The complete assembly of central and end sections, 12 feet long, was suspended vertically from the upper end so that the clearance from the floor was 6 feet. A number 10 Brown and Sharpe gauge copper wire was used as the central electrode throughout all tests. The wire was polished and maintained in clean condition.

The rotating field exploring device consisted of 2 parallel number 20 copper wires 6 feet in length and spaced $^1/_2$ inch apart, arranged so that they could be rotated about their common axis. The wires were supported at each end on pieces of $^1/_2$ inch bakelite tubing. Twisted leads from the lower ends of the wires were brought through the tubing to a 2 segment commutator. Driving torque was supplied to each end of the rotating structure by means of shafts and right angle gears. Ball thrust bearings were used at the outer ends of the bakelite tubing in order that the wires might be operated under sufficient tension to secure their accurate alignment. To keep the wires from flying apart by centrifugal action, silk thread ties were placed at intervals of 6 inches. The mechanical operation attained was very satisfactory, the wires during rotation appearing to describe a uniform cylinder. The supports for the rotating structure were located at the ends of insulating wooden arms. These arms were fastened to a vertical pipe arranged to turn about its axis. By this means the exploring device could be placed at any desired radial distance between the wire and cylinder. Figure 1 shows the general arrangement of this construction. The exploring device was rotated at 1800 rpm by means of the gearing arrangement and the synchronous motor drive. When a-c corona was investigated, the motor was supplied with power through a phase shifter, so that the instant of commutation of the rotating device could be adjusted to occur at any



Figs. 3 and 4. Curves for field between wire and cylinder with continuous potential; figure 3 (left) with wire positive, and figure 4 (right) with wire negative

Dashed curves are calculated for 77,800 volts without corona

A—77,800 volts *B*—70,700 volts *C*—63,600 volts *D*—56,500 volts *E*—35,300 volts *F*—28,300 volts

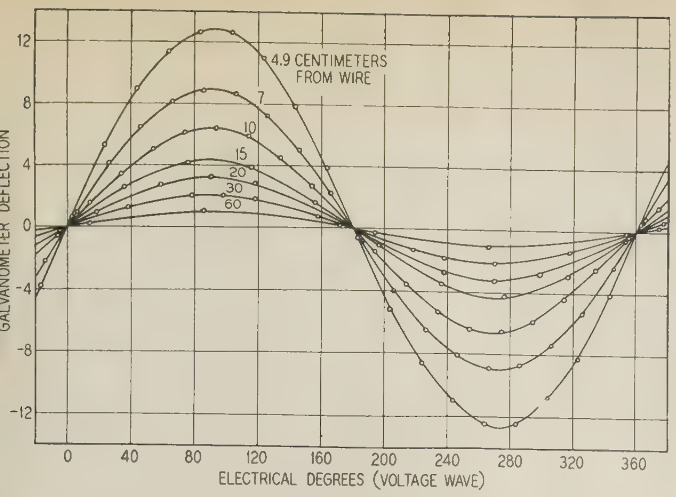


Fig. 5 (left). Curves for field between wire and cylinder with alternating potential of 25 kv effective value applied

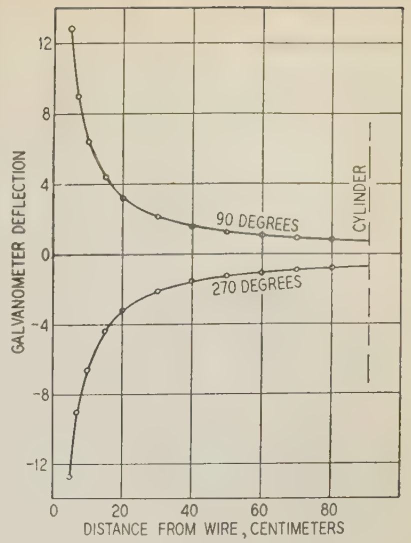


Fig. 6 (right). Curves derived from figure 5 showing relation of field intensity and distance from wire for 2 points in the voltage cycle

Positive deflections indicate field from wire; negative deflections indicate field toward wire

desired point in the applied voltage cycle. Small copper leaf brushes were used to make contact with the commutator. From these brushes a pair of flexible twisted leads was brought downward to a galvanometer just beneath. The galvanometer and shunt were placed in a copper screen shielding cage on the top of a porcelain pedestal having a leakage distance of approximately 3 feet.

The transformer used for the high voltage supply was equipped with a tertiary winding for operating a voltmeter for determining the secondary voltage. Ratio and phase angle errors have been found previously to be small, less than one per cent. One of the laboratory sine wave generators, rated at 1,500 kva, was used as the power supply for the transformer primary. The driving motor was of the synchronous type, receiving power from a large system maintained at a frequency constant between very narrow limits. This insured a definite relation between the phase of the generator voltage and that of the circuit supplying power to the phase shifter and the small motor driving the exploring device. The single phase load was but a small fraction of the generator rating and so did not affect the voltage wave form appreciably. Peak values of the alternating voltage on the secondary side of the transformer were calculated by multiplying the voltmeter reading by $\sqrt{2}$ times the ratio of turns.

A high vacuum tube rectifier was used to provide a supply of high continuous voltage. The capacitance placed across the rectifier output was such that the pulsation of output voltage was calculated to be negligibly small. Effects of the protective resistance in the high voltage circuit, either with or without the rectifier, were found to be too small to warrant correction for them. In the former case the voltage at the wire was taken as equal to the crest value of the alternating voltage.

Connections for the measurement of rectified charge are shown in figure 2. The plate was separated from the mesh cylinder approximately $^3/_8$ inch. It was necessary to operate the galvanometer with its most sensitive connection (10^{-9} ampere per centimeter deflection) for this test, the shunt being merely sufficient to provide suitable damping. Difficulty from leakage current over the insulation with the high sensitivity and with polarizing voltages up to 1,000 volts was eliminated by draining off such current as might exist without the necessity of its flow through the galvanometer. This was accomplished by wrapping bands of metal foil around the insulators and making connections to them as shown in the diagram.

Results Obtained With Continuous Voltages

The curves in figures 3 and 4 contain the results of the measurements of field intensity with continuous voltages applied to the wire. Factors for converting values of galvanometer deflections to values of field intensity were obtained from the curves for voltages below corona and the corresponding computed gradients. The latter were calculated from the relation

$$\frac{dV}{dr} = \frac{V}{r \log_e \left(\frac{b}{a}\right)} \tag{5}$$

where dV/dr is the field intensity, V the voltage between the wire and cylinder, r the distance from the wire to any point in the field, b the radius of the cylinder, and a the radius of the wire. This expression is based upon conditions in a purely radial field. For a constant voltage and a given wire and cylinder arrangement, equation 5 may be written

$$\frac{dV}{dr} = k\frac{1}{r}$$

where k is a constant. The graph of this relation is a rectangular hyperbola, in which the ordinate varies inversely as the abscissa.

The curves for voltages below the starting point of corona, 28,300 and 35,300 volts, may be seen to have very nearly the theoretical hyperbolic form. Two conclusions may be drawn from this: first, the field to the central section of the cylinder approached closely the desired radial distribution; second, the rotating device was very satisfactory in its operation.

The presence of ions in the field as the result of corona at the wire alters the field distribution from that existing in the absence of space charge. This

distortion is quite marked at the higher voltages. The dashed curves in figures 3 and 4 clearly indicate by comparison these differences for a wire voltage of 77,800. In general, the flow of ions outward from the vicinity of the wire tends to reduce the field intensity in this region and to increase it near the cylinder. At distances from the wire greater than about 30 centimeters, the gradient is maintained at an approximately uniform value at higher voltages. Equation 2 expresses a condition of constant field in the simplified form

$$\frac{1}{r}\frac{dV}{dr} = -4\pi\rho \tag{6}$$

since the quantity d^2V/dr^2 is zero. In this case the density of the space charge varies inversely as the distance from the wire for any value of dV/dr. It follows, then, that the same amount of charge exists in each cylindrical layer of equal thickness.

Table I contains calculated values of charge density, ion concentration, drift velocity, and mobility at the highest voltage used.

The density of the space charge, and the number of ions per unit volume, is seen to decrease with increasing distances from the wire, over the greater part of the space investigated. In regions close to the wire, where the quantity d^2V/dr^2 is large, this tendency is not followed to the same extent, the increased current density being partly supplied by the greater ion drift velocity.

Values of velocity and mobility were calculated by making use of field strength measurements and readings of the current to the central section of the cylinder. The requisite data, then, were obtained from sources independent of each other. The current was measured with good accuracy with a calibrated microammeter. The calculated mobilities of positive and negative ions agree very satisfactorily with those obtained by physicists. Thus it is shown that the distributions of electric gradient as represented by figures 3 and 4 are substantially correct.

Table I—Values Obtained by Calculation for 77,800 Volts

Distance From Wire, Centimeters	Field Strength, Volts Per Centimeter	Net Charge Density, Thousands of Electrostatic Units Per Cubic Centimeter	Ion Density, Millions Per Cubic Centimeter	Drift Velocity, Centimeters Per Second	Mobility, Centimeters Per Second Per Volt Per Centimeter
Wire Positive					
7.0	1060	4.3	9.0	2500	2.3
10.0	778	4.8	10.0	1550	2.0
20.0	515	4.4	9.3	850	1.6
40.0	442	2.6	5.5	740	1.7
60.0	442	2.2	4.4	590	1.3
Wire Negative					
7.0	1080	4.5	9.3	3200	2.9
10.0	800	4.3	9.3	2300	2.9
20.0	516	3.9	8.3	1200	2.4
40.0	441	2.7	5.6	920	2.1
60.0	431	1.9	4.0	860	2.0

Currents to central section of cylinder: wire positive, 30.0 microamperes; wire negative, 39.3 microamperes.

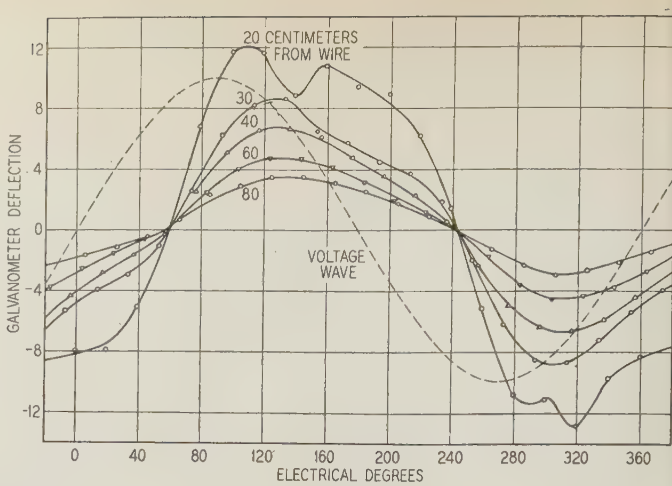


Fig. 7. Curves for field between wire and cylinder with alternating potential of 60 kv effective value applied

A number of interesting comparisons may be drawn from the results of the tests with continuous voltages. Values of field intensity recorded in table I are seen to be all very greatly below that required for the breakdown of air (about 30,000 volts per centimeter). It is probable, then, that the amount of ionization occurring within the region of investigation is negligible. This implies that only ions of one sign, that of the charge on the wire, may be present in appreciable numbers. In the case of the positive discharge, such ions must be ionized atoms and molecules or groups of molecules. The similarity between results for positive and negative voltages would indicate that the greater proportion of the negative ions also were in the atomic or molecular form.

The number of "natural" ions (mainly due to various ionizing radiations) per cubic centimeter in the atmosphere varies considerably, but is usually in the order of several thousand. Such ions exist in equal numbers, positive and negative, unless acted upon by an electric field. Even under such conditions, their direct effect in altering such a field would generally be insignificant. The concentration of ions in the region explored between the wire and cylinder in these measurements is found for the higher voltages to be several million per cubic centimeter, a value sufficiently large to modify the field greatly. Large as it is, however, only a very small fraction of the total number of molecules are ions. Avogadro's number is given as 6.06×10^{23} molecules per gram molecular weight,[13] corresponding to 2.7×10^{19} molecules per cubic centimeter in air at normal conditions. Thus it is evident that only about one molecule in 10 billion is an ion.

The average speed of random motions of molecules in air at ordinary temperatures and pressures is about 400 meters per second.[13] It is evident from table I that the progressive motion of ions toward the cylinder constitutes merely a drift at a much lower velocity, and that their kinetic energies are not sensibly altered by the influence of the electric field. The time required for the travel of an ion across the space from the wire to the cylinder should be in the

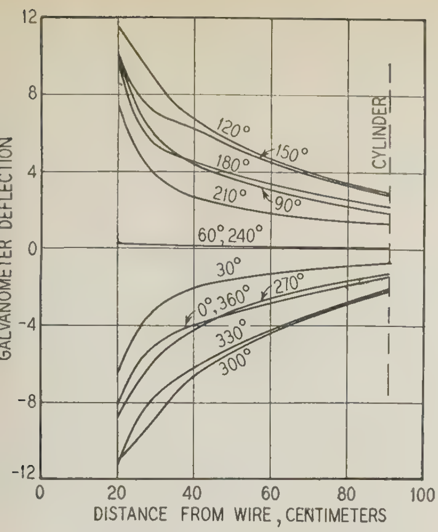


Fig. 8. Curves derived from figure 7 showing relation of field intensity and distance from wire for several points in the voltage cycle

Positive deflections indicate field from wire; negative deflections indicate field toward wire

order of 0.1 second for the higher voltages (assuming an average drift velocity of 1,000 centimeters per second).

In agreement with the results of more specialized measurements, the negative ions are found to have higher mobilities than the positive ions. Several reasons have been suggested for this difference. In air the ions probably consist largely of singly charged molecules of the reaction products of the corona discharge. In any complex gas, such as air, many kinds of chemical products may be formed in quantities during the process of ionization. It is probable that certain of these molecules may be attracted more strongly to ions of one sign than to another, with consequent varying effects on the mobility. An apparent average decrease in mobility as the distance from the wire is increased is also observed. Such may be due in part to aging effects, resulting from the formation of more complex ions during the time of passage from the wire to the cylinder. Changes in mobility at ages in the order of 0.1 second have been reported.[9]

Results Obtained With Alternating Voltages

The exploration of the electric field when alternating voltages are applied to the wire involves a consideration of the element of time as well as that of the radial distance from the wire. Graphical representation of the results of field measurements are contained in figures 5 to 8. The relation of galvanometer deflection to field strength is the same in all cases.

It is to be expected that the variation of field intensity with time for voltages below corona should have the same form and phase as that of the applied potential. Figure 5 shows that the indications of the exploring device are in accord with this principle. Correspondingly, in a radial field the intensity at any time should vary inversely as the distance from the wire, in a region not containing space charge. Figure 6, derived from the curves of figure 5, shows this condition for 2 points in the voltage cycle.

Figure 7 shows typical results of field exploration when the wire was in corona. Derived curves of the space variations of gradient for the same voltage are contained in figure 8. Several conclusions may be drawn from an inspection of these curves:

1. The presence of space charge causes the time variation of field intensity to depart in phase and wave form from that of the voltage applied between the wire and cylinder. From the principles of electrostatics,[8] where cylindrical symmetry of charge distribution exists, the electric field intensity at any distance from the axis of symmetry is proportional to the total (difference of positive and negative) charge within this distance according to the relation

$$\frac{dV}{dr} = \frac{2Q}{r} \tag{7}$$

for electrostatic units. Q is the total charge per centimeter axial length and r the radius to the point considered. Since the variations of field strength are not in direct accord with the voltage, space charge must be responsible for the differences.

2. For a given wire voltage, the changes of gradient with time are of very similar nature at most of the distances at which field explorations were made. This immediately suggests that amounts of space charge sufficient to appreciably distort the field do not penetrate to these points. Figure 8 bears out this conclusion. Consideration of the relation expressed by equation 7 shows that at any instant the field must vary inversely as the radius, in regions outside the space charge. Inside the space charge limits, however, this condition may be altered according to the variation of net charge within the distance from the wire.

3. Approximately 30,000 volts (effective value) was found necessary to produce the first indications of corona formation. At 60,000 volts, nearly twice the corona starting value, a sufficient amount of space charge to markedly modify the field apparently reaches at least 20 centimeters from the wire. The effects observed at 30 centimeters are small, so that it may be inferred that the maximum travel of the boundary of the main body of the charge is about this amount. At this voltage, the position nearest the wire at which measurements could be made satisfactorily was 20 centimeters, due to limitations of the equipment used for balancing the galvanometer cage potential. Though under these particular conditions a part of the space charge reaches a limit of travel within the range of exploration, the curves of figure 8 indicate that most of the space charge is at all times closer to the wire than 20 centimeters.

4. The significance of space charge in its effect upon the field is perhaps most strikingly indicated by the shift in phase of the field outside the charge with respect to the voltage. This may be seen readily by the dashed curve of the voltage wave in figure 7. The phase shifts were found to be approximately 20, 40, and 60 degrees at 40, 50, and 60 kv, respectively. Figure 8 shows that at the instant when the potential difference between the wire and cylinder is zero (both electrodes at ground potential) large values of electric gradient exist within the regions explored. This can be caused only by space charge, the sign of which is preponderantly that corresponding to the preceding half cycle of voltage. At the same time the

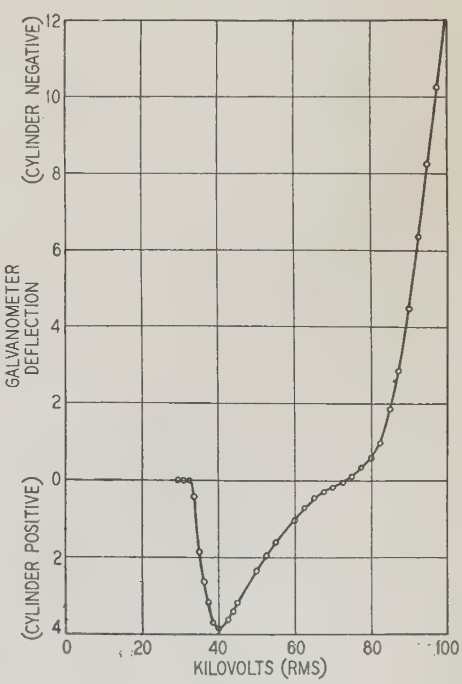


Fig. 9. Curve of net current to cylinder

Galvanometer deflection of 10 units indicates 0.253 × 10^{-6} ampere

field terminating on the wire due to the charge surrounding it is of the sign normally produced by the half cycle of voltage next to follow.

Although several investigators have studied the arrival of charge at a cylinder from a wire at its center, most of the work has been done with cylinders of much smaller diameter than was used in this work. The matter of diameter may be very important if the dimensions and voltages are such that the main body of space charge in moving under the influence of the field reaches the cylinder walls. With such conditions the charge that may be conducted from the cylinder is much greater than if the space charge motion is unimpeded by the boundary of the field. This has been pointed out by Waldorf.[8]

A galvanometer of the D'Arsonval type connected between the central section of the cylinder and ground indicated a small current when voltages slightly in excess of the corona starting value were applied to the wire. The direction of the current flow was such as to be explained by positive ions reaching the cylinder walls. As the voltage was raised, a point of maximum current was reached, beyond which the current decreased until the sign of the current changed to negative. Results of these measurements are shown in figure 9.

To determine separately the current resulting from ions of each sign arriving at the cylinder, use was made of the polarized plate electrode previously described. Figure 10 indicates that with about 1,000 volts polarizing voltage, almost all the ions of sign opposite to the potential of the plate and arriving in its vicinity are drawn past the mesh cylinder wall. In figure 11 is shown the current to the plate caused by the positive and negative ions. These values correspond closely, when consideration of the relation of cylinder to plate area is taken, to values in figure 9. It is evident that only ions of one sign

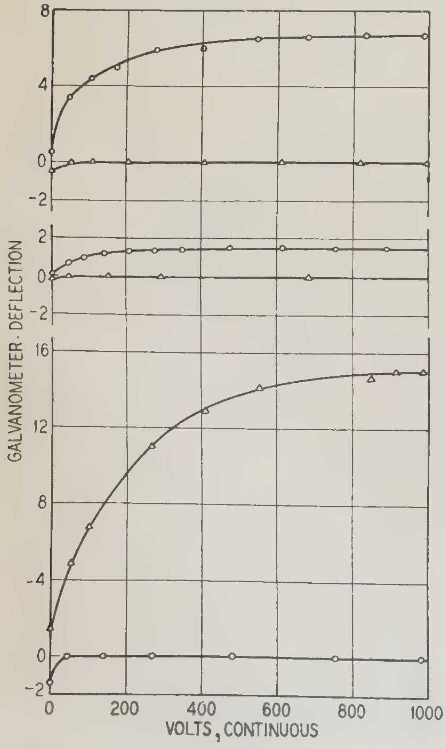


Fig. 10. Curves of current to plate

Top curves at 40 kv effective value
Middle curves at 70 kv effective value
Bottom curves at 95 kv effective value

O—Connection for positive charge
△—Connection for negative charge

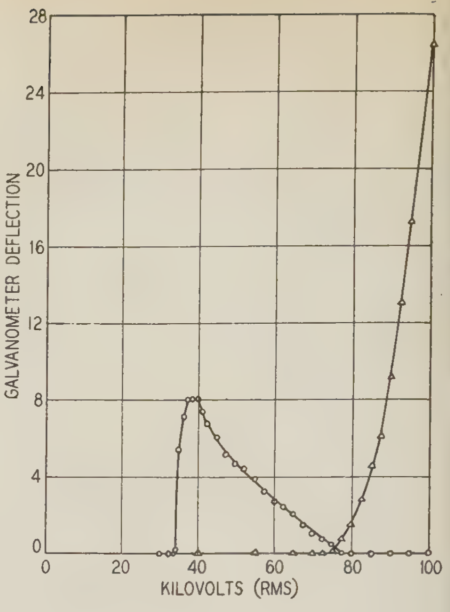
Galvanometer deflection of 10 units indicates 0.945 × 10^{-8} ampere

Fig. 11. Curves for currents to plate caused by positive and negative ions

Plate at 950 volts continuous; ratio of area of plate to area of cylinder 0.071

O—Plate positive; negative ions
△—Plate negative positive ions

Galvanometer deflection of 10 units indicates 0.945 × 10^{-8} ampere



reach the cylinder in significant numbers at any voltage.

The currents obtained from this rectified charge are small, in keeping with the apparently negligible space charge density in regions far from the wire. Distinction must be made between the main space charge moving in and out with the field and this small surplus charge of one sign. The latter may be considered to have a unidirectional drift superposed upon its oscillations with the field. An erroneous idea concerning the limits of travel of the main waves of space charge in alternating fields may be obtained, if care is not taken to separate these 2 effects. Current measurements to cylinders of different diameters or to a plane of adjustable distance from the wire may lead to the conclusion that the positive space charge is much more diffuse than the negative, if voltages below a certain value are used. At higher voltages the opposite impression may be gained.

References

1. The Corona Produced by Continuous Potentials, S. P. Farwell. A.I.E.E. Trans., v. 33, 1914, p. 1631–66.

2. Corona Discharge, E. H. Warner and J. Kunz. Bulletin No. 14, 1919, Univ. of Illinois Experiment Station.

3. Principles of Electricity (a book), Page and Adams. D. Van Nostrand Co., N. Y., 1932, or any standard reference on electrostatics.

4. The Space Charge That Surrounds a Conductor in Corona at 60 Cycles, J. S. Carroll and H. J. Ryan. A.I.E.E. Trans., v. 45, 1926, p. 1093–1101.

5. Space Charge and Current in Alternating Corona, C. H. Willis. A.I.E.E. Trans., v. 46, 1927, p. 271–287.

6. The Space Charge That Surrounds a Conductor in Corona, J. S. Carroll and J. T. Lusighan. A.I.E.E. Trans., v. 47, 1928, p. 50–7.

7. The Theory of Corona, R. Holm. *Archiv. f. Elek.*, v. 18, 1927, p. 567–82.

8. High Voltage Corona in Air, S. K. Waldorf. A.I.E.E. Trans., v. 49, 1930, p. 657–65.

9. Spectra of Ion Mobilities, L. B. Loeb and N. E. Bradbury. *Phys. Rev.*, v. 38, 1931, p. 1716–29.

10. International Critical Tables, McGraw-Hill Book Co., N. Y., 1926.

11. A Rotary Voltmeter, P. Kirkpatrick. Elec. Engg., v. 51, Dec. 1932, p. 863–5.

12. Generating Electrostatic Voltmeter, E. A. Schuchard. Bulletin No. 80, 1935, Univ. of Washington Experiment Station.

13. An Outline of Atomic Physics (a book), by members of the physics staff of the University of Pittsburgh. John Wiley and Sons, N. Y., 1933.

Vibration of Cables and Dampers—I

An analysis of the vibration of cables and dampers, representing the results of several years of research and laboratory and field testing, is presented in this paper. In Part I, presented herewith, an explanation of the causes and nature of free harmonic vibration of the cable is offered, followed by an analysis of the resulting stresses. In Part II, which is scheduled for publication in a subsequent issue, various ways of reducing the maximum stresses and means of controlling the vibration are discussed, including an analysis of the action of Stockbridge dampers. Applications of the formulas to specific cases are given, and comparisons are made with experimental data from laboratory and field.

By
R. G. STURM
Associate Am. Soc. of Civil Engrs.
Aluminum Research Labs.
New Kensington, Pa.

CABLES that are stretched tightly above ground are frequently subject to resonant vibration caused by the wind. The cause of this vibration has been explained by Theodore Varney[1] as the effect of the eddy currents of the steady wind as it blows over the cable. These eddy currents have a definite frequency of oscillation depending upon the velocity of the wind. Relf and Ower[2] have shown that this frequency for ordinary sizes of cable used in transmission lines may be expressed closely by the formula:

$$f = \frac{3.26V}{d} \tag{1}$$

in which

f = frequency in oscillations per second
V = velocity of the wind normal to the cable in miles per hour
d = diameter of the cable in inches

If the direction of the wind is not directly across the cable, the component of the wind perpendicular to the cable should be used.

These eddy currents formed on the leeward side of a cable cause partial vacuums to exist, first above the horizontal diameter, and then below the horizontal diameter with the result that minute impulses are given to the cable. If by chance a gust of wind or some other disturbance causes a tremor in the cable, these small forces actually will transmit energy to the cable. Should their frequency correspond to, or very nearly to, a frequency of resonant vibration for the cable, the cable gradually will start vibrating in a number of loops such that the frequency of vibration of the cable coincides with the frequency of the eddy currents. Since the wind seldom, if ever, blows at a uniform velocity and in a fixed direction, the eddy currents caused by the wind will vary somewhat in frequency, thereby, making it possible for the cable to vibrate with a frequency corresponding to the whole number of loops that fits in best with the sum total of varying eddy frequencies. In other words, the cable has a range of frequencies to select from, thereby making vibration of more frequent occurrence than would be the case if the wind were always absolutely uniform.

Under certain conditions this vibration may build up until the cable is vibrating at an amplitude that will cause alternating stresses large enough to produce fatigue failure in the strands of the cable, although the cable would be entirely safe without vibration. It is this phenomenon that makes the study of cable vibration of such great importance.

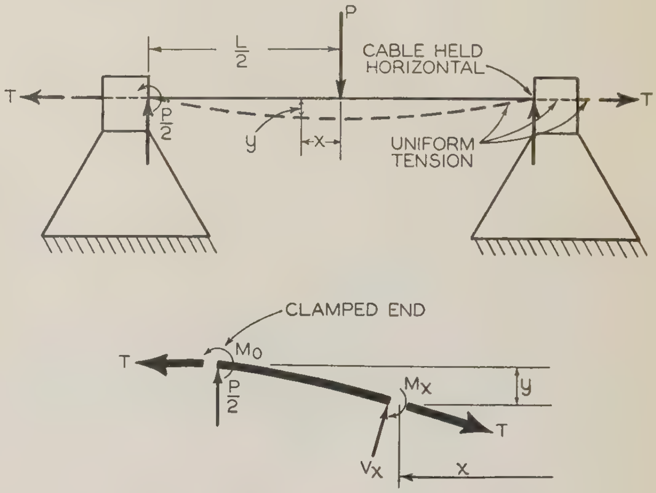


Fig. 1. Diagram of test setup of cable with concentrated load at midspan, and a free body diagram of a portion of the cable

The process involved in developing the condition of vibration in any cable is dependent upon the nature of the chance initial disturbances. Once vibration is started, each loop contributes a small impulse to maintain vibration. This vibration may be con-

A paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Jan. 17, 1936; released for publication March 28, 1936.

Acknowledgment is made to the following men connected with the Aluminum Company of America: R. L. Templin, under whose direction this work and the laboratory work was done and who was responsible for many of the methods of testing and for the design and construction of most of the testing equipment; G. W. Stickley, who is in immediate charge of the vibration laboratory at Massena, N. Y.; M. E. Noyes; R. A. Monroe, who was in charge of the field tests on vibrating cables in Texas; H. L. Anderson, who conducted the field tests at Royse City, Texas; and those who assisted with the computations.

1. For all numbered references, see list at end of paper.

sidered as the result of a great number of minute traveling waves[3] or as standing waves which build up gradually. Regardless of how the vibration is built up, a vibrating string soon assumes a form of displacement that has been shown to be practically that of a sine wave, provided the flexural rigidity of the string is relatively small in comparison with the length and the tension. Cables having a flexural rigidity that would be quite appreciable for short spans might be considered as strings when the span length becomes large. The limiting value of flexural rigidity that might be negligible for any given span will be discussed later.

Bechtold and Folkerts[4] show that for ordinary sizes and spans of cable a sinusoidal curve agrees very closely with the true shape of the cable. If a sinusoidal curve be assumed, the classical theory of vibrating strings may be applied and the frequency of vibration is found to be

$$f = \frac{1}{2L}\sqrt{\frac{Tg}{W}} \tag{2}$$

where

f = frequency in cycles per second
T = total tension in cable in pounds
W = weight in pounds per linear inch of cable
g = acceleration due to gravity in inches per second per second
L = length of one loop in inches, i. e., distance between node points

Also the deflection of the cable during steady vibration may be written as

$$y = \frac{A}{2}\sin\frac{\pi x}{L}\sin wt \tag{3}$$

in which

x = distance of point considered from the node point, in inches
y = deflection of any point x, in inches
$\frac{A}{2}$ = maximum displacement of the cable from its initial position, in inches; A then becomes the maximum throw of the cable, being a distance $A/2$ on each side of the center line. (The value A generally is referred to as the amplitude of vibration and will be used as such throughout this study)
$w = \frac{\pi}{L}\cdot\sqrt{\frac{Tg}{W}}$
t = time, in seconds, after the cable has passed through its initial position

A relationship between wind velocity and the loop lengths into which a given span will tend to vibrate may be obtained by equating the frequency of the eddy currents or impulses of the wind, equation 1, to the natural frequency of vibration of a cable into loops of length L, equation 2. This gives

$$\frac{3.26V}{d} = \frac{1}{2L}\sqrt{\frac{Tg}{W}}$$

which may be transposed into

$$L = \frac{d\sqrt{\frac{Tg}{W}}}{6.52V} \tag{4}$$

where the terms are as previously defined.

The wind velocities at which vibration of any span is most likely to occur are those giving rise to a loop length L, which can form an integral number of times in the span length L_s. In other words, when $L_s/L = n$ gives n equal to an integer, vibration is most likely to occur. If n be large, quite small variations in L still may give integral values of n, but if n be small, the changes in L must be correspondingly large.

Thus, it may be seen why spans vibrate most frequently in a relatively large number of loops. If n loops per span be chosen as a lower limit below which vibration of any particular span is rare, the minimum wind velocity that may be dangerous may be found as follows: The maximum loop length will be $L = L_s/n$. Substituting this value in equation 4 and designating the minimum wind velocity normal to the cable thus found as V_m,

$$V_m = \frac{nd\sqrt{\frac{Tg}{W}}}{6.52L_s} \tag{5}$$

where L_s is the span length in inches, and the other terms are as previously defined. From many observations of actual cables vibrating in the field it has been found that the number of loops rarely is less than 30.

In connection with the frequency of vibrating cables as given by the preceding formulas, it should be pointed out that the cable has been considered as a string with no flexural rigidity. For vibration where the loop length is short enough to give rise to considerable bending, the actual frequency would be expected to be higher than indicated by the formulas.

Stresses in Cables

As a cable vibrates, it passes into and through a series of curved forms as previously mentioned. If the span on which the cable is strung be of considerable magnitude, the cable will sag appreciably. These deviations from a straight cable give rise to variations in the stresses in the cable because, while it behaves very much as a flexible string, it does possess some flexural rigidity. The stresses set up in vibrating strings that have no rigidity can be only an increase or decrease of direct tension; but if the string have a certain amount of stiffness or flexural rigidity, bending stresses will be introduced. The flexural rigidity of a composite cable is something that cannot be determined satisfactorily from purely theoretical considerations because of the effects of the manufacturing processes upon it. It has been found that the flexural rigidity of the cable is affected also by the direct tension to which the cable is subjected.

Flexural Rigidity of Cable

For a solid bar, the stiffness factor or flexural rigidity may be expressed as EI, where E is the modulus of elasticity of the material and I is the moment of inertia of the cross section of the rod. For a stranded conductor, however, a working value representing the composite product of the equivalent of

these 2 values may be used. This composite value may be obtained by substituting measured values of load and deflection into the theoretical formulas much the same as is done frequently for ordinary beams.

The following analysis of the elastic behavior of a cable under tension and loaded with a concentrated load at midspan has been made for the purpose of determining the relationship between this EI value and factors that can be measured in tests. This analysis also gives a distribution of the bending moments along the cable throughout a span loaded in this manner.

Figure 1 represents the conditions of support and loading of the cable in the tests. The origin of coordinates is chosen as the midspan at the center line of the cable before transverse load is applied. The tension, T, is maintained constant throughout the test.

Figure 1 also shows a free body diagram of a portion of the cable. Referring to this free body diagram and applying the condition of equilibrium, the moments about the point x may be found to be

$$M_x = M_0 + T\cdot y - \frac{P}{2}\left(\frac{L}{2} - x\right)$$

or

$$M_x = M_0 + T\cdot y - \frac{PL}{4} + \frac{Px}{2} \tag{6}$$

where

M_x = bending moment in the cable at any point, at a distance x from the center line, in inch-pounds
M_0 = bending moment in the cable at the clamped end, in inch-pounds
L = length of span in inches
T = uniform tension in the cable, in pounds
P = concentrated load at the center of the span, in pounds
Y = deflection of the cable at a point x distance from the center line, in inches

From the theory of elasticity

$$M_x = EI\frac{d^2y}{dx^2} \tag{7}$$

Substituting this value for M_x,

$$EI\frac{d^2y}{dx^2} = M_0 + T\cdot y - \frac{PL}{4} + \frac{Px}{2}$$

and by rearranging

$$\frac{d^2y}{dx^2} - \frac{T}{EI}y = \frac{M_0}{EI} - \frac{PL}{4EI} + \frac{Px}{2EI} \tag{8}$$

A solution of this differential equation will give not only the shape of the deflected cable, but also, by differentiating twice, the equation for bending moments at every point in the cable. The solution of the equation must satisfy the boundary conditions which may be enumerated as follows:

1. The deflection of the clamped end is zero; whence $y]_{x=L/2} = 0$.
2. The slope of the tangent to the elastic curve of the cable must be zero at both ends and at the middle; whence $\left.\frac{dy}{dx}\right]_{x=\frac{L}{2}} = 0$, and $\left.\frac{dy}{dx}\right]_{x=0} = 0$.

The particular integral for equation 8 is

$$y_1 = \frac{-M_0}{T} + \frac{PL}{4T} - \frac{Px}{2T}$$

and the complementary function is

$$y_2 = Ae^{-Kx} + Be^{-K\left(\frac{L}{2}-x\right)}$$

in which A, B, and K are constants which may be determined from the original equation and the boundary conditions. The complete solution then, which is the sum of the particular integral and the complementary function may be written

$$y = Ae^{-Kx} + Be^{-K\left(\frac{L}{2}-x\right)} - \frac{M_0}{T} + \frac{PL}{4T} - \frac{Px}{2T} \tag{9}$$

Substituting this value in equation 8,

$$K^2 = \frac{T}{EI} \text{ or } K = \sqrt{\frac{T}{EI}} \tag{10}$$

From the boundary conditions, it is found that when $x = L/2$, $y = 0$, or

$$y]_{x=\frac{L}{2}} = Ae^{-K\frac{L}{2}} + B - \frac{M_0}{T} = 0 \tag{11}$$

Differentiating equation 9, the slope at any point x is found to be

$$\frac{dy}{dx} = -AKe^{-Kx} + BKe^{-K\left(\frac{L}{2}-x\right)} - \frac{P}{2T}$$

Again from the boundary conditions it follows that when $x = 0$, $dy/dx = 0$, or

$$\left.\frac{dy}{dx}\right]_{x=0} = -AK + BKe^{-K\frac{L}{2}} - \frac{P}{2T} = 0 \tag{12}$$

and when $x = L/2$, $dy/dx = 0$, or

$$\left.\frac{dy}{dx}\right]_{x=\frac{L}{2}} = -AKe^{-K\frac{L}{2}} + BK - \frac{P}{2T} = 0 \tag{13}$$

Using equations 12 and 13 to solve for the constants A and B,

$$A = -B = \frac{-P}{2TK(e^{-K\frac{L}{2}}+1)} \tag{14}$$

Substituting these values in equation 11 and solving for M_0 gives an expression for the bending moment at the edge of the clamp:

$$M_0 = \frac{P}{2K}\left[\frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}}\right] \tag{15}$$

Substituting these values in equation 9, the final equation for the deflection of the cable is found to be

$$y = \frac{P}{2KT(e^{-K\frac{L}{2}} + 1)}\left[-e^{-Kx} + e^{-K\left(\frac{L}{2}-x\right)} - 1 + e^{-K\frac{L}{2}}\right] + \frac{PL}{4T} - \frac{Px}{2T} \tag{16}$$

The maximum deflection at the center of the span (i. e., where $x = 0$) will be found by substituting $x =$

0 in equation 16. This gives an equation that may be reduced to

$$y_{\max} = \delta = \frac{P}{KT}\left[\frac{KL}{4} - \frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}}\right] \tag{17}$$

Solving this equation for K,

$$K\left(\frac{1 + e^{-K\frac{L}{2}}}{1 - e^{-K\frac{L}{2}}}\right) = \frac{4P}{PL - 4\delta T} = \frac{4}{L - 4\frac{\delta}{P}\cdot T} \tag{18}$$

Knowing the load P at midspan, the tension T, and the deflection δ for any given cable, it is possible to find the corresponding K value and then the equivalent EI value for the cable.

To solve equation 18 a process of successive approximations is the most satisfactory. First assume

$$\frac{1 + e^{-K\frac{L}{2}}}{1 - e^{-K\frac{L}{2}}} = 1$$

and solve for K. With this value of K evaluate the term $\left(1 + e^{-K\frac{L}{2}}\right) \Big/ \left(1 - e^{-K\frac{L}{2}}\right)$ and find a new

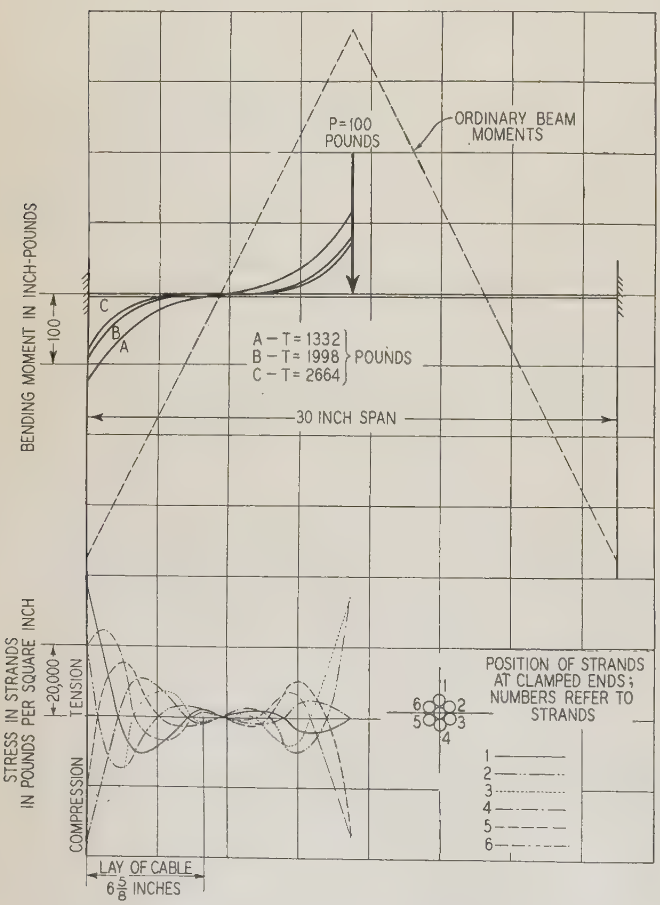


Fig. 2. Moment curves for a 7-strand steel-reinforced aluminum cable based upon equation 20 for a concentrated load at midspan; and computed bending stresses in the outermost fibers of the various strands

value of K, etc. Generally after the first or second correction no change in K can be detected. Knowing K, the value of EI may be found from equation 10.

The bending moment at any point, x, in the span may be found from equation 7. Using the value for y and differentiating twice,

$$M_x = \frac{EI\,PK^2}{2KT(e^{-K\frac{L}{2}} + 1)}\left[-e^{-Kx} + e^{-K\left(\frac{L}{2} - x\right)}\right] \tag{19}$$

Substituting the value $K^2 = T/(EI)$, this may be reduced to

$$M_x = \frac{P(e^{-K\left(\frac{L}{2} - x\right)} - e^{-Kx}}{2K(e^{-K\frac{L}{2}} + 1)} \tag{20}$$

An important conclusion that may be drawn from the foregoing mathematical derivation is that the bending moments decrease very rapidly toward the quarter point of the span.

Moment curves based upon equation 20 are shown in figure 2. It may be noted that the bending moment falls off very rapidly as the point considered moves from the clamped end or from the load point. A comparison with the ordinary bending moment in a simple beam also is shown. The computed bending stresses in the outermost fibers of the various strands are shown also in figure 2. The method of computation is outlined in a later section of this paper.

Table I shows the measured flexural rigidities of several types of cables at various tensions. These values were determined from test data using equations 18 and 10 together with test data obtained by the general method described by Monroe and Templin.[5] The effects of stranding as well as the effects of tension on the flexural rigidity are indicated by the values given in this table.

Stresses in a Vibrating Cable Near the Center of the Span

It may be expedient to consider the stresses caused by various forces separately. The stress resulting from the direct tension in the cable is considered first. For a single wire this stress, S_1, may be computed by the formula

$$S_1 = \frac{T}{a} \tag{21}$$

where

T = tension in the cable, in pounds
a = area of the cable, in square inches

For cables of only one metal, this same formula may be applied with fair accuracy, but for bimetallic cables this formula would not apply. Stickley's method[6] for computing the direct stresses in the strands of steel reinforced aluminum cable is based upon actual stress-strain curves and has been found to agree very well with measured stresses in individual strands.

Having the load-strain curve of the steel reinforced aluminum cable as a whole, the strain resulting from any tension may be read from it. Then from the stress-strain curve for the steel core the stress in the

Table I—Flexural Rigidities of Various Cables

Size and Type of Cable	Stranding of Cable ‡	Applied Tension, Pounds	Span Length Used, Inches	Ratio of Maximum Deflection to Span Length δ/L	EI Value From Tests, Pound-Inches²	Computed EI Value for All Strands Integral, Pound-Inches²	Computed EI Value for All Strands Separate, Pound-Inches²	Tensile Modulus, E_c, of Elasticity of Cable as a Unit, Pound-Inches²
795,000* A.C.S.R.	30 × 0.1628 (alum.) / 19 × 0.0977 (steel)	6,400	63	0.000†	370,000	651,000	12,900	10.6×10^6
				0.009	40,000			
		8,200	63	0.000	340,000			
				0.007	50,000			
		11,000	63	0.000	320,000			
				0.005	55,000			
Steel Core of 795,000 A.C.S.R.	19 × 0.0977	4,620	44	0.000	33,000	60,800	2,550	27×10^6
				0.013	10,700			
		7,580	44	0.000	35,000			
				0.008	16,000			
843,700* A.C.S.R.	6 × 0.375 (alum.) / 1 × 0.375 (steel)	6,500	72	0.000	240,000	553,000	87,400	11.1×10^6
				0.003	87,000			
		9,700	72	0.000	400,000			
				0.002	111,000			
984,400 all aluminum	7 × 0.375	2,300	72	0.000		534,000	68,000	8.34×10^6
				0.016	71,000			
		4,600	72	0.000				
				0.006	75,000			

* Cross-section area of aluminum, in circular mils, of steel reinforced aluminum cable.
† Values of EI for $\delta/L = 0$ are determined from the initial slope of the tangent to the load-deflection curves.
‡ Number of strands × diameter of each strand in inches.

steel corresponding to this strain may be read. The portion of the load carried by the steel core then may be computed and deducted from the total. The balance of the load must be carried by the aluminum.

If the equation of the shape of the cable during vibration be known, the curvature at any point may be determined by differentiating, and from this curvature the bending stresses set up in the cable may be found. If the amplitude of vibration be small compared with the loop length, the curvature may be expressed very closely as

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} \tag{22}$$

where

ρ = radius of curvature of the cable, in inches
y = displacement of the point considered from its original position, in inches
x = distance from the node point to the point considered, in inches

Out in the span where the flexural rigidity does not appreciably disturb the sinusoidal wave, the curve of a vibrating string may be used to determine the value of curvature; but at or near clamped ends where the curvature is great and the flexural rigidity enters vitally into the problem, a special analysis will have to be made. Within the span, however, the maximum curvature at the center of a loop at the instant of maximum displacement may be determined from equation 3. At that instant, equation 3 becomes

$$y = \frac{A}{2}\sin\frac{\pi x}{L} \tag{23}$$

and

$$\frac{d^2y}{dx^2} = -\frac{\pi^2}{L^2}\frac{A}{2}\sin\frac{\pi x}{L} \tag{24}$$

where

L = the loop length in inches
A = the amplitude of vibration in inches

and the other terms are as previously defined, from which there may be written for the center of the loop, i. e., where $x = L/2$ and $\sin(\pi x/L) = 1$,

$$\frac{1}{\rho} = \frac{-\pi^2 A}{2L^2} \tag{22a}$$

The moment at the center of the loop resulting from this curvature then is

$$M = \frac{EI}{\rho} = \frac{-EI\pi^2 A}{2L^2} \tag{25}$$

and for the usual amplitudes encountered, the corresponding stress S_2, resulting from bending alone, would be

$$S_2 = \frac{\pm McE_0}{EI} \tag{26}$$

where

c = half the diameter of the cable in inches
E_0 = modulus of elasticity of the material in the strands of the cable, in pounds per square inch
EI = flexural rigidity of the cable, in pound-inches²

and the other terms as previously defined. The value of $(EI)/E_0$ in equation 26 may be considered as the effective moment of inertia of the cable.

If, however, the amplitude increases to such a point that there is appreciable slipping between strands, the strands no longer will act integrally and the formula will not apply. In undamped cables the amplitude generally builds up to that point. In this event the stresses in the cable may be computed by introducing a new value for c. Test results thus far completed indicate that if the bending

is very much greater than that required to overcome friction, the value of c approaches half the diameter of a single strand. Substituting the value for the bending moment into equation 26

$$S_2 = \frac{\pm \pi^2 A c E_0}{2L^2} \tag{26a}$$

Still another stress will result from the length of the cable in the curved position being greater than the length of the cable when hanging free. This increase in length necessary to go around the curves will tend to increase the tension, which in turn will decrease the sag and so provide additional length. As a result, the increase in tension will be very much less than for a cable with practically no sag but vibrating between rigid nonyielding clamps.

The length of arc for any curve may be expressed as

$$s = \int ds = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \tag{27}$$

For vibrating cables the value of dy/dx at any point will be very small compared with unity; as a consequence, the radical may be approximated very closely by the first 2 terms of the binomial expansion. The resulting equation is

$$s = \int \left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right] dx \tag{27a}$$

Substituting in equation 27a the value of y given in equation 23,

$$s = \int \left[1 + \frac{A^2\pi^2}{8L^2} \cos^2 \frac{\pi x}{L}\right] dx$$

which for one loop gives

$$s_1 = \int_0^L \left[1 + \frac{A^2\pi^2}{8L^2} \cos^2 \frac{\pi x}{L}\right] dx = \left[1 + \frac{A^2\pi^2}{16L^2}\right] L \tag{28}$$

The unit elongation required for a cable having no sag and fixed between rigid supports would be the increase in length divided by the original length or

$$\epsilon_c = \frac{s - L}{L} = \frac{A^2\pi^2}{16L^2} \tag{29}$$

The stress caused by this elongation would be

$$S_v = E_c\,\epsilon_c = E_c \frac{A^2\pi^2}{16L^2} \tag{30}$$

where

E_c = tensile modulus of the cable as a whole, pounds per square inch

S_v = increase in direct tensile stress caused by vibration, pounds per square inch

The increase in tension, T_v, expressed in pounds, may be found from this increase in tensile stress, S_v, caused by vibration, by simply multiplying by the area of cross section of the cable for a monometallic cable, or by taking the sum of the stresses in the separate strands of a bimetallic cable.

Let the cable, when not vibrating, be subjected to a tensile force T; then the maximum tensile force during vibration is

$$T_{max} = T + T_v \tag{31}$$

The minimum tension T_{min} must be equal to T. The average tension in the cable then is

$$T_{avg} = \frac{T_{max} + T_{min}}{2} = T + \frac{T_v}{2} \tag{31a}$$

where

T_{avg} = resulting average tension in a cable during vibration, in pounds

T = tension in cable when not vibrating, in pounds

T_v = change in tension caused by formation of loops during vibration, in pounds

In an actual cable the foregoing conditions are not obtained. When the cable starts to vibrate an adjustment will take place so that as the average tension starts to increase, the sag of the cable will start to decrease and the supports to yield somewhat; consequently, instead of the average cable tension increasing to $(T + T_v/2)$ it will increase to a value only slightly greater than T, in which case the maximum tension will be this new average tension plus half of that caused by the increase in length. Expressed mathematically,

$$T_{max} = T_{avg} + \frac{T_v}{2} \tag{32}$$

Now the final average tension must lie between the initial tension T and the final tension $(T + T_v/2)$ for a rigidly supported cable with no sag. Thus, it may be said that the maximum tension must lie between $(T + T_v/2)$ and $(T + T_v)$. It follows, then, that the increment of tensile stress caused by vibration of a transmission line must lie between the 2 limits $S_v/2$ and S_v. This relation may be expressed mathematically as

$$\frac{\pi^2 A^2}{32L^2} E_c < S_3 < \frac{\pi^2 A^2}{16L^2} E_c \tag{33}$$

where

S_3 = the actual increase in tensile stress caused by vibration, in pounds per square inch

and the other terms are as previously defined.

In general, for transmission lines S_3 is very small so that either value may be used without appreciable error. However, in laboratory tests where short loop lengths and large amplitudes may be used, it is important to distinguish between these values.

If the tension in the test cable be maintained by means of levers and dead weights with the end free to move, the lower value should be used. In this case

$$S_3 = \frac{\pi^2 A^2}{32L^2} E_c \tag{33a}$$

If the tension be applied to the cable and the end rigidly clamped prior to vibration, the upper value of S_3 should be used. In this case

$$S_3 = \frac{\pi^2 A^2}{16L^2} E_c \tag{33b}$$

Stresses resulting from the dead weight of the cable throughout the central portion of the span will be practically pure tensile stresses set up in stringing. At the supports the stresses resulting from the bending caused by sagging of the cable may be very large and must be considered in addition to those already mentioned, while at points well removed from the clamped end these bending stresses resulting from the sag of the cable are negligible, as shown by the analysis in the next section.

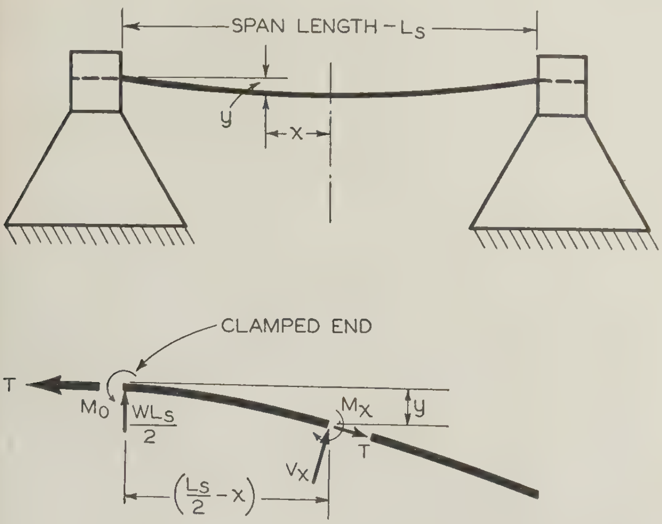


Fig. 3. Diagram of test setup of uniformly loaded cable, and a free body diagram of the portion of the cable

Stresses in the cable at points well removed from the end supports may be summarized as follows:

Maximum stress $= S_1 + S_2 + S_3$
Minimum stress $= S_1 - S_2 + S_3$

where

S_1 = the stress resulting from the direct tension in the cable and is equal to T/a for single rods or monometallic cables, in pounds per square inch; for bimetallic cables these direct stresses may be found by Stickley's method[6]

S_2 = the stress resulting from the bending of the cable into loops, in pounds per square inch, and may be calculated by means of equation 26a

S_3 = the stress resulting from the additional tension in the cable caused by the increased length of the curved form of the vibrating cable, in pounds per square inch, and may be calculated by means of equation 33a

Stresses in Cable at and Near Supports or Clamps

The 2 stresses S_1 and S_3 will exist at the clamp just the same as at points removed from the clamp and, therefore, will not be changed. The stresses resulting from the sag of the cable which are found to be negligible for the central part of the span are a very important factor in determining the total stresses at the clamp. An analysis leading to an evaluation of this stress follows.

Consider a portion of a cable as shown in figure 3. The sag under conditions encountered in practice is so small, generally less than $1/20$ of the span, that the weight of the cable may be considered as uniform along the span. Referring to the free body diagram in figure 3 and taking moments about point x, the following value is obtained:

$$M_x = -\frac{WL_s}{2}\left(\frac{L_s}{2} - x\right) + \frac{W}{2}\left(\frac{L_s}{2} - x\right)^2 + T\cdot y + M_0 \quad (34)$$

where

M_x = bending moment at any point x, in inch-pounds
x = distance of point from center of span, in inches
y = deflection of cable at point x, in inches

For the choice of co-ordinates indicated, the elastic relation between the bending moment and elastic curve is

$$M_x = +EI\frac{d^2y}{dx^2} \quad (35)$$

Substituting the value of M_x from equation 34 and transposing,

$$\frac{d^2y}{dx^2} - \frac{T}{EI}y = \frac{W}{2EI}\left(\frac{L_s}{2} - x\right)^2 - \frac{WL_s}{2EI}\left(\frac{L_s}{2} - x\right) + \frac{M_0}{EI} \quad (36)$$

which reduces to

$$\frac{d^2y}{dx^2} - \frac{T}{EI}y = \frac{-WL_s^2}{8EI} + \frac{Wx^2}{2EI} + \frac{M_0}{EI} \quad (37)$$

This is the general differential equation for the deflection of a cable with a uniform load along the span. Its solution will give the deflection of the cable at any point, and by differentiating, the bending moment at any point may be found. The particular integral of this equation is

$$y_2 = \frac{W}{2T}\left[\left(\frac{L_s}{2}\right)^2 - x^2\right] - \frac{WEI}{T^2} - \frac{M_0}{T} \quad (38)$$

in which all terms are as previously defined, and the complementary function is

$$y_1 = Ae^{-Kx} + Be^{+Kx} \quad (39)$$

Here A and B are arbitrary constants, and by substituting in the original equation (37) it is found that $K = \sqrt{T/(EI)}$. The complete solution then may be written

$$y = Ae^{-Kx} + Be^{+Kx} + \frac{W}{2T}\left[\left(\frac{L_s}{2}\right)^2 - x^2\right] - \frac{WEI}{T^2} - \frac{M_0}{T} \quad (40)$$

The determination of the arbitrary constants and the value of M_0 requires some definite assumption concerning the rigidity of the end clamps. To clarify the steps in the procedure of deriving a general expression for the moments, stresses, and deflections caused by the dead weight of a cable, the simplest case, that of rigid horizontal end clamps, is considered first.

Rigid Horizontal End Clamps

The arbitrary constants and the values of M_0 may be found from the boundary conditions, namely,

that the deflection is zero at the ends of the span, the tangent to the curve of the cable at midspan is horizontal, and the tangents at the supports are horizontal. Expressed mathematically, these are: When $x = L_s/2$, $y = 0$, from which

$$y\Big]_{x=\frac{L_s}{2}} = 0 = Ae^{-K\frac{L_s}{2}} + Be^{K\frac{L_s}{2}} - \frac{WEI}{T^2} - \frac{M_0}{T} \tag{41}$$

When $x = 0$, $dy/dx = 0$, from which

$$\frac{dy}{dx}\Big]_{x=0} = 0 = -AK + BK \tag{42}$$

When $x = L_s/2$, $dy/dx = 0$, from which

$$\frac{dy}{dx}\Big]_{x=\frac{L_s}{2}} = 0 = -AKe^{-K\frac{L_s}{2}} + BKe^{K\frac{L_s}{2}} - \frac{WL_s}{2T} \tag{43}$$

From equation 42,

$$A = B \tag{44}$$

Equation 43 then may be transposed as

$$+AK(e^{K\frac{L_s}{2}} - e^{-K\frac{L_s}{2}}) = \frac{WL_s}{2T} \tag{43a}$$

Solving for A and B, it follows that

$$A = B = \frac{+WL_s}{2KT}\left[\frac{1}{e^{K\frac{L_s}{2}} - e^{-K\frac{L_s}{2}}}\right] = \frac{+WL_s}{2KT}\,\frac{e^{-K\frac{L_s}{2}}}{1 - e^{-KL_s}} \tag{45}$$

Substituting this value in equation 41,

$$-\frac{M_0}{T} - \frac{WEI}{T^2} = \frac{-WL_s}{2KT}\left[\frac{1 + e^{-KL_s}}{1 - e^{-KL_s}}\right] \tag{46}$$

which may be transformed to

$$M_0 = +W\left[\frac{L_s}{2K}\cdot\frac{1 + e^{-KL_s}}{1 - e^{-KL_s}} - \frac{EI}{T}\right] \tag{47}$$

In general, the value of KL_s is so large that the factors of e^{-KL_s} may be neglected, whence the moment at the clamped end of a cable due to its own dead weight may be expressed as

$$M_0 = W\left[\frac{L_s}{2K} - \frac{EI}{T}\right] \tag{47a}$$

Substituting the values from equations 45 and 47 in equation 40,

$$y = \frac{WL_s}{2KT}\left[\frac{(e^{-Kx} + e^{+Kx})e^{-K\frac{L_s}{2}}}{1 - e^{-KL_s}}\right] + \frac{W}{2T}\left[\left(\frac{L_s}{2}\right)^2 - x^2\right] - \frac{WL_s}{2KT}\left[\frac{1 + e^{-KL_s}}{1 - e^{-KL_s}}\right] \tag{48}$$

which may be rewritten

$$y = \frac{WL_s}{2KT}\left[\frac{e^{-K\left(\frac{L_s}{2}+x\right)} + e^{-K\left(\frac{L_s}{2}-x\right)} - 1 - e^{-KL_s}}{1 - e^{-KL_s}}\right] + \frac{W}{2T}\left[\left(\frac{L_s}{2}\right)^2 - x^2\right] \tag{49}$$

This equation gives the deflection of a cable under its own dead weight provided the ends are clamped rigidly and the tension is great enough so that the total sag does not exceed $^1/_{10}$ of the span. In general, the value of KL_s is so large that factors of e^{-KL_s} may be neglected. The equation then may be reduced to

$$y = \frac{W}{2T}\left[\left(\frac{L_s}{2}\right)^2 - x^2\right] - \frac{WL_s}{2KT}\left[1 - e^{-K\left(\frac{L_s}{2}+x\right)} - e^{-K\left(\frac{L_s}{2}-x\right)}\right] \tag{50}$$

Thus the deflection is practically a parabola except near the ends.

The bending moment at any point is found from equation 7, or

$$M_x = \frac{-WEI}{T} + \frac{WL_s}{2K}\left[e^{-K\left(\frac{L_s}{2}+x\right)} + e^{-K\left(\frac{L_s}{2}-x\right)}\right] \tag{51}$$

Thus it may be seen that the moments in the cable will be very small except near the ends.

The alternating bending stresses set up during vibration near the clamp may be found by considering the cable deformed statically to give the same conformation of the cable as occurs during vibration. This may be derived as follows: Since the curvature of the cable when not vibrating is very slight a short distance from the clamped ends, the cable may be considered straight. From equation 16, for the deflection of a cable under a concentrated load, it may be seen that for spans equal to the loop lengths usually encountered in vibration, the cable will assume the form of 2 straight lines with curved portions at the ends and under the load. This then suggests that the shape of the cable near the clamp during vibration may be duplicated by producing the same slopes of the straight line portion as would be obtained at the node of a freely vibrating loop.

If 2 tangent lines be drawn to a sine curve at 2 adjacent node points, they will intersect at a distance from the line through the node points equal to $\pi/2$ times the maximum ordinate to the sine curve. Since the maximum ordinate from the line of nodes to the curved cable is $^1/_2$ the amplitude, the point of intersection of the lines lies 0.785 times the amplitude from the line of nodes. From a comparison of the cases actually worked out, it has been found that the curved portions near the ends and at the center of a cable deflected with concentrated loads reduce this factor somewhat.

Actual tests have shown that if a cable with a span equal to the loop length be deflected by a concentrated load by an amount equal to 0.70 to 0.75 times the amplitude, the curvatures at the clamped end are practically identical with those measured during vibration. Recent tests indicate that the value 0.70 gives the best agreement for the usual conditions in actual transmission lines.

The deflection at the center of a span of a length equal to the loop length L under any load P at midspan may be obtained from equation 17. If $\delta = 0.70A$, the value of P may be determined definitely for any given amplitude from equation 17. Then using this value of P in equation 20, the bending moment at any point at a distance x from the center may be found. For points near the clamped end of

the usual sizes of cables, equation 20 may be simplified to

$$M_x = \frac{P}{2K} \cdot e^{-K\left(\frac{L}{2}-x\right)} \qquad (51a)$$

without appreciable error.

Stresses in the extreme top and bottom portions of the cable may be found from an adapted form of the ordinary flexural formula. A study of the distribution of measured stresses in the individual strands of aluminum cable and steel reinforced aluminum cable under severe bending reveals a tendency of the strands to act individually rather than for them all to act monolithically. Actual stress measurements on 7 strand cable show that the stresses in the individual strands vary from tension to compression within a very short distance around the strand thereby indicating that each individual strand tends to bend about some axis within itself rather than about a common axis for the whole cable. This would indicate that the flexural rigidity of a 7 strand cable as a whole may be approximated by the sum of the flexural rigidities of the individual strands. In computing the moment of inertia of the cable in this way, it is assumed that the interaction between the strands of the cable is only sufficient to cause the strands to bend uniformly, that is, each strand to change curvature the same amount and in the same direction. Comparison with test results on 7 strand cables shows that this is a fair approximation for such cables. For cables having more than 7 strands, there are complicating factors, such as binding on intermediate layers of strands, which tend to increase the flexural rigidity. An investigation of the behavior of cables having multiple layers of strands is in progress, but thus far no definite conclusions can be given.

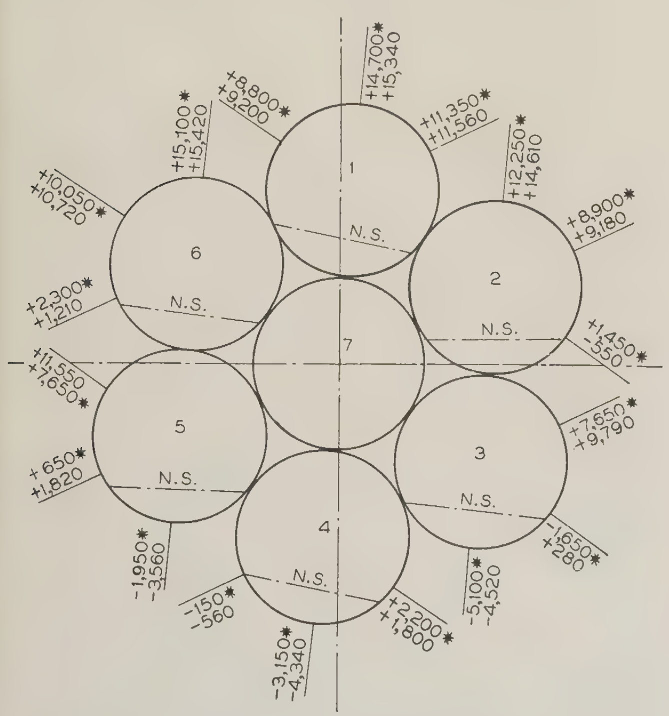


Fig. 4. Measured and computed stresses at points around the periphery of the individual strands of a cable, at a point $^5/_{16}$ inch from face of clamp

Measured stresses are indicated by an asterisk (*); all others are computed

N. S.—Neutral surface, or plane of zero stress

7-strand steel-reinforced aluminum cable containing 6 0.375 inch aluminum strands and one 0.375 inch steel strand

Total tension—8,000 pounds

Stress in aluminum—5,500 pounds per square inch

Span length—120 feet

Vibration equivalent—7 loops at $1^1/_8$ inch amplitude

End loop of cable considered at bottom-most position

Thus, the effective distance from the neutral axis to the extreme fiber of each strand would be the value to be used in the beam formula. For a 7 strand cable this would be $^1/_3$ of that for the whole cable since there are 3 strands in contact at all points.

The stresses resulting from bending severe enough to cause slipping of the strands on each other may be computed from

$$S_4 = \frac{McE_0}{3EI} \qquad (52)$$

where M is the bending moment in the cable, and the other terms are as previously defined. Using the value of M_x from equation 51, the maximum stress S_4 at the clamped end resulting from the dead weight of the cable may be found from equation 52.

Stresses in the outer fibers of the various strands around the cable may be found by multiplying the stress in the extreme top and bottom of the cable by the cosine of the angle between a vertical line and the radius drawn from the center of the strand to the point considered. This procedure assumes that the stresses vary as a straight line from the plane of no stress to the maximum at the extreme edge, whether there is a common plane of zero stress for all or a separate one for each. A comparison of the measured and computed stresses is shown in a previously published paper.[5]

If all the strands be forced to bend in a common vertical plane along the axis of the cable, some transverse forces between strands will be necessary to make the top strands, which lie at an angle to that plane, bend with the strands on the sides, which are in a plane parallel to such an axial plane. The strand then will act much as though it were subjected to first a bending moment in its own vertical plane and then a bending moment in a horizontal plane, the net result being that the neutral axis for bending will be tipped.

The angle of tip of the axis was assumed to be the same as that between the axis of the strand and the vertical plane through the axis of the cable, for computing the stresses around the various strands. Figure 4 shows the measured and computed stresses at various points around the periphery of the strands. The neutral surface shown is that computed on the basis of a direct stress of 5,500 pounds per square inch acting simultaneously with the bending stress. This is the tension used in the tests when the strains were measured.

It should be pointed out that the manufacturing process and the past history of the cable easily can influence the actual behavior of the strands sufficiently to cause variations as great as those indicated. Because of this, any computations for the variation

of the stresses around the individual strands of a cable should be considered as only approximate.

As might be expected, the maximum stresses always will occur at the extreme top or bottom fibers of the cable. In view of this, the stresses given by equation 52 are the significant ones to be considered. The maximum alternating stress, S_5, may be found by substituting the moments from equation 51a into equation 52 and following the same procedure as for the stresses resulting from the sag.

The stresses near the clamped ends of a 7 strand cable may be summarized as follows:

S_4 = stress resulting from the sag of the cable

$$S_4 = \frac{M_4 c E_0}{3EI}, \text{ from equation 52}$$

where M_4 is the bending moment resulting from the sag, and may be determined from equation 51.

S_5 = alterating stress at clamp caused by vibration

$$S_5 = \frac{M_5 c E_0}{3EI}, \text{ from equation 52}$$

where M_5 is the bending moment resulting from vibration, and may be determined from equation 51a.

S_1 = the stress throughout the cable caused by direct tension; this stress is equal to T/a for a monometallic cable, but for bimetallic cables it may be found by Stickley's method[6]

S_3 = stress resulting from the additional tension in the cable caused by the increased length of the curved form of the vibrating cable, and may be determined from equation 33a

The maximum and minimum combination of stresses for the outer fibers of the cable are as follows:

For the top of the cable:

$$S_{max} = S_1 + S_3 + S_4 + S_5 \quad (53)$$

$$S_{min} = S_1 + S_3 + S_4 - S_5 \quad (54)$$

For the bottom of the cable:

$$S_{max} = S_1 + S_3 - S_4 + S_5 \quad (55)$$

$$S_{min} = S_1 + S_3 - S_4 - S_5 \quad (56)$$

It may be pointed out that except for extremely large amplitudes of vibration, the top strands always will be in tension, whereas the bottom strands generally will be subjected to stresses extending well into compression as well as tension. A comparison of measured and computed stresses has been shown in a previously published paper.[5]

Nonrigid End Clamps

In actual practice, the clamps are not held perfectly rigid nor exactly horizontal. If the clamp at the end of the cable tips so that an angle is formed with the horizontal, one of the boundary conditions is changed. The slope of the tangent at the supports will not be zero but will be some amount β. Thus β = tangent of the angular position of the end clamp. Equation 43 then becomes

$$\beta = -AKe^{-K\frac{L_s}{2}} + BKe^{K\frac{L_s}{2}} - \frac{WL_s}{2T} \quad (57)$$

Using this value instead of equation 43, the bending moment at the clamp resulting from the sag of the cable becomes

$$M_0 = W\left[\left(\frac{\beta T}{KW} + \frac{L_s}{2K}\right)\frac{1 + e^{-KL_s}}{1 - e^{-KL_s}} - \frac{EI}{T}\right] \quad (58)$$

which for the usual spans may be simplified to

$$M_0 = W\left[\frac{L_s}{2K} - \frac{EI}{T}\right] + \frac{\beta T}{K} \quad (58a)$$

The stresses resulting from the sag then may be computed as before, except that this value of the moment is used.

It may be noted that if β has the proper negative value, that is, if the ends rotate so as to point down toward the center of the span, the end moment M_0 may be made zero. In designing end clamps for long spans in series it would be desirable to slope the 2 sides of the clamps by the amount

$$\beta = \frac{-WL_s}{2T} + \frac{KWEI}{T^2} = \frac{-WL_s}{2T} + \frac{W}{KT} \quad (59)$$

If the clamp be free to rock with the cable during vibration the bending moments resulting from vibration will be reduced. The moment at the clamp may be found by going back to equation 13 and considering that when $x = L/2$, $dy/dx = \beta_1$, where β_1 is the tangent of the angle of oscillation each side of the neutral position. Thus,

$$\beta_1 = -AKe^{-K\frac{L}{2}} + BK - \frac{P}{2T} \quad (60)$$

From equation 12 together with equation 60, the values of A and B are found to be:

$$A = \frac{-P}{2KT} \cdot \frac{1}{1 + e^{-K\frac{L}{2}}} + \frac{\beta_1}{K} \cdot \frac{e^{-K\frac{L}{2}}}{1 - e^{-KL}} \quad (61)$$

and

$$B = \frac{P}{2KT} \cdot \frac{1}{1 + e^{-K\frac{L}{2}}} + \frac{\beta_1}{K} \cdot \frac{1}{1 - e^{-KL}} \quad (62)$$

Substituting these values in equation 11 the resulting value for moment becomes

$$M_0 = \frac{P}{2K} \cdot \frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}} + \frac{\beta_1 T}{K} \cdot \frac{1 + e^{-KL}}{1 - e^{-KL}} \quad (63)$$

The value of P is determined, as previously, from a revised form of equation 17 which takes into account these slope changes.

Substituting the values for A and B from equations 61 and 62 and M_0 from equation 63 into equation 9, and letting $x = 0$ for $y_{max} = \delta$, it follows that

$$\delta = \frac{PL}{4T} - \left(\frac{\beta_1}{K} + \frac{P}{KT}\right) \cdot \frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}} \quad (64)$$

which may be written as

$$\delta = \frac{P}{KT}\left[\frac{KL}{4} - \frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}}\right] - \frac{\beta_1}{K} \cdot \frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}} \quad (64a)$$

Equation 64 may be used to find the value P for the value of δ corresponding to the amplitude. Now if an appreciable end rotation can take place so that there is little or no restraint at the end, the deflection will not be decreased as much as for fixed ends. Hence, when β is very small (less than $P/20T$) the value of δ may be taken as 0.70 times the amplitude. If β approaches the value $P/2T$ the value of δ should approach 0.74 times the amplitude. Since these values for δ are so nearly the same, the value of 0.70 may be used in both cases without appreciable error.

Using the value of P thus obtained, the bending moment may be computed by equation 63, and from equation 52 the stresses may be determined as in preceding cases.

Flexible Clamps and Elastically Supported Clamps

Clamps at the ends of intermediate spans of a series of spans may be restrained against rotation by the vibration of adjacent spans being out of phase. This can be overcome to some extent by using a flexible clamp. If the clamp be flexible or elastically supported so that the angular displacement is proportional to the bending moment M_0, the value of the slope β at the end may be determined as follows:

$$\beta = QM_0 \tag{65}$$

where

Q = a constant depending upon the flexibility of the support of the clamp. The value of Q always will be negative because of previous definition of β and M_0. This is made clear by equation 59 and the discussion pertaining to it.

If the clamp rotates any amount, β, the bending moment at the clamp caused by the sag is given by equation 58a. Substituting this value of M_0 into equation 65,

$$\beta = QW\left[\frac{L_s}{2K} - \frac{EI}{T}\right] + \frac{\beta QT}{K} \tag{66}$$

The elastic angular rotation of the clamp resulting from the sag may be found by solving for β, which gives

$$\beta = -\frac{QW\left[\frac{L_s}{2K} - \frac{EI}{T}\right]}{\frac{QT}{K} - 1} \tag{67}$$

The alternating moment at the clamp resulting from vibration is given by

$$M_0 = \frac{P}{2K}\cdot\frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}} + \frac{\beta_1 T}{K}\cdot\frac{1 + e^{-KL}}{1 - e^{-KL}} \tag{63}$$

Substituting this value into equation 65 and using the subscript to agree with equation 63,

$$\beta_1 = \frac{QP}{2K}\cdot\frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}} + \frac{\beta_1 QT}{K}\cdot\frac{1 + e^{-KL}}{1 - e^{-KL}} \tag{68}$$

The elastic angular rotation of the clamp resulting from vibration may be found by solving for β_1, which gives

$$\beta_1 = \frac{\frac{QP}{2K}\cdot\frac{1 - e^{-K\frac{L}{2}}}{1 + e^{-K\frac{L}{2}}}}{1 - \frac{QT}{K}\cdot\frac{1 + e^{-KL}}{1 - e^{-KL}}} \tag{69}$$

which for all practical cable spans may be reduced to

$$\beta_1 = \frac{\frac{QP}{2K}}{1 - \frac{QT}{K}} \tag{69a}$$

If the bending moment occurring at a perfectly rigid clamp and resulting from either the sag or vibration of the cable be indicated as $\overline{M}_0$, the value of β for an elastically supported clamp as given by equations 67 and 69a may be expressed by the single expression

$$\beta = \frac{-Q\overline{M}_0}{\frac{QT}{K} - 1} \tag{70}$$

Thus for an elastically supported clamp, the moments at the end of the cable may be obtained by substituting the value of β from equation 65 into equation 70 with the result that

$$M_0 = \frac{\overline{M}_0}{1 - \frac{QT}{K}} \tag{71}$$

The stresses may be computed as before, using this value for the bending moment at the elastically clamped end.

In addition to stresses that thus far have been considered, there may or may not exist local unknown stresses resulting from manufacture, handling, clamping, etc. Because of these unknowns, a substantial factor of safety must be used. The design stresses, computed as heretofore indicated, obviously should not exceed the fatigue strength for the material used and the stress range computed. In practice it will be desirable to use design values somewhat lower than the fatigue strength.

References

1. Notes on the Vibration of Transmission Line Conductors, Theodore Varney. A.I.E.E. Trans., v. 45, 1926, p. 791–9.

2. The Singing of Circular and Streamline Wires, E. F. Relf and E. Ower. Aueronautical Research Committee, Report No. 825, March 1921.

3. Vibration Analysis, W. B. Buchanan. Elec. Engg. (A.I.E.E. Trans.), v. 53, Nov. 1934, p. 1478–85, and subsequent discussion in Elec. Engg., (A.I.E.E. Trans.), v. 54, July 1935, p. 779.

4. Resistance of Cable to Transverse Vibration Especially Hollow Overhead Cable, K. Bechtold and H. Folkerts. *Elektrotechnik und Maschinenbau*, v. 47, July 14, 1929, p. 593–604.

5. Vibration of Overhead Transmission Lines, R. A. Monroe and R. L. Templin. A.I.E.E. Trans., v. 51, Dec. 1932, p. 1059–73.

6. Stress-Strain Studies of Transmission Line Conductors, G. W. Stickley. A.I.E.E. Trans., v. 51, Dec. 1932, p. 1052–8.

An Exact Formula for Transformer Regulation

An exact method for calculating the regulation of transformers is developed in this paper, and compared with approximate methods for both 2 winding and 3 winding transformers. The present formula for 2 winding transformers in the A.I.E.E. test code is confirmed, and a similar formula is obtained for 3 winding transformers.

By
J. E. CLEM
MEMBER A.I.E.E.

General Elec. Co.,
Schenectady, N. Y.

DURING the revision of the A.I.E.E. "Test Code for Transformers" (dated October 1931) the formulas for regulation were reviewed and an exact method developed. It is the purpose of this paper to present the results of the study, which resulted in the development of a simple approximation for the calculation of the regulation of 3 winding transformers, and which confirmed the present formula in the code for 2 winding transformers as the preferable approximation. The new formula for 3 winding transformers is selected to be similar to the one for 2 winding transformers, gives very close results, and is included in the revision of the test code being made by the test code subcommittee of the American Standards Association's sectional committee on transformers.

General Formula

The definition of transformer regulation now being considered by the A.S.A. is as follows:

"The regulation of a constant potential transformer is the change in secondary voltage, expressed in per cent of rated secondary voltage, which occurs when the rated kilovolt-ampere output at a specified power factor is reduced to zero, with the primary impressed voltage maintained constant.

"(Note: In the case of multi-winding transformers the loads on all windings, at specified power factors, are to be reduced from rated kilovolt-amperes to zero simultaneously.)"

The fundamental formula for the regulation of a transformer follows directly from the definition. It is

$$\text{per cent regulation} = \frac{E_p - E_s}{E_s} 100$$

in which

E_s = rated secondary voltage held with rated kilovolt-ampere output, at specified power factor

E_p = secondary voltage after load has been removed. Corresponds to primary voltage required to maintain rated secondary voltage with rated kilovolt-ampere output at specified power factor

It might be well to say a word of caution in regard to the regulation of multiwinding transformers. In a 2 winding transformer the regulation is directly proportional to the output. In a multi-winding transformer the regulation is not only dependent upon the total output but also upon the distribution of the output among the various secondaries.

Derivation of Exact Equation for the Regulation of a 2 Winding Transformer

In figure 1 let

E_s = secondary voltage; rated voltage at rated output at specified power factor

E_p = primary voltage; applied voltage required to maintain E_s

m = power factor of the rated output = $\cos \theta$

n = reactive factor of the rated output = $\sin \theta$

r = resistance factor = $\frac{IR}{E_s}$ (at the specified load)

x = reactance factor = $\frac{IX}{E_s}$ (at the specified load)

I = rated secondary current at specified power factor

R = transformer resistance, ohms

X = transformer reactance, ohms

θ = angle between E_s and I, i. e., power factor angle of rated output

ϕ = angle between E_s and E_p

Note that m, n, r, and x are to be expressed as decimals, i. e., on a per unit basis.

From the figure

$$\tan(\theta + \phi) = \frac{E_s \sin\theta + IX}{E_s \cos\theta + IR} \tag{1}$$

$$= \frac{n + x}{m + r} \tag{2}$$

From equation 2

$$\cos(\theta + \phi) = \frac{m + r}{\sqrt{(m + r)^2 + (n + x)^2}} \tag{3}$$

From the figure

$$E_p \cos(\theta + \phi) = E_s \cos\theta + IR \tag{4}$$

$$\frac{E_p}{E_s} = \frac{m + r}{\cos(\theta + \phi)} \tag{5}$$

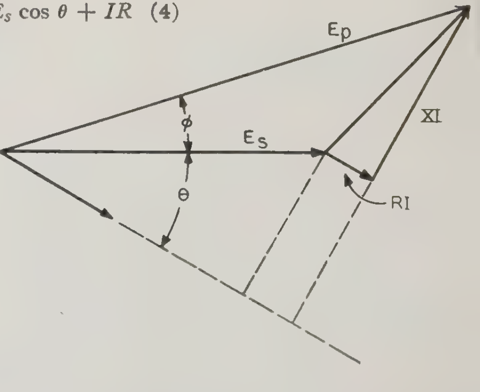


Fig. 1. Vector diagram for a 2 winding transformer

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Feb. 27, 1936; released for publication March 18, 1936.

Substitute equation 3 in equation 5

$$\frac{E_p}{E_s} = \sqrt{(m + r)^2 + (n + x)^2} \quad (6)$$

$$\frac{E_p}{E_s} - 1 = \sqrt{(m + r)^2 + (n + x)^2} - 1 \quad (7)$$

But

$$\frac{E_p - E_s}{E_s} 100 = \text{per cent regulation by definition}$$

so that

$$\text{per cent regulation} = \left\{\sqrt{(m + r)^2 + (n + x)^2} - 1\right\} 100 \quad (8)$$

Derivation of Exact Equation for the Regulation of a 3 Winding Transformer

In this case the power factors of the secondary and tertiary loads are given with respect to their own voltages. It should be noted at this point that in a 3 winding transformer it usually is possible to hold rated secondary voltage on only one secondary winding. It seems reasonable to consider that rated secondary voltage will be maintained on the secondary whose regulation is being determined and that rated load current at specified power factor will be held in both secondaries.

In figure 2 let E_i be the intermediate voltage, which is common to the 3 circuits. Let

E_s = secondary voltage
I_s = secondary current
θ_s = phase angle between secondary voltage and current
E_t = tertiary voltage
I_t = tertiary current
θ_t = phase angle between tertiary voltage and current
E_p = primary voltage
I_p = primary current
α = phase angle between secondary and tertiary currents
θ_p = phase angle between primary voltage and current
E_i = intermediate voltage
θ_i = phase angle between E_i and I_p
ϕ_i = phase angle between E_i and E_p

Note that I_s and θ_s, and I_t and θ_t, are given and they determine I_p and θ_p.

The factors m and n, r and x, and R and X are the same quantities as for single winding transformers, the subscript indicating the winding to which they apply. The factors r and x are on the basis of the current in the respective windings.

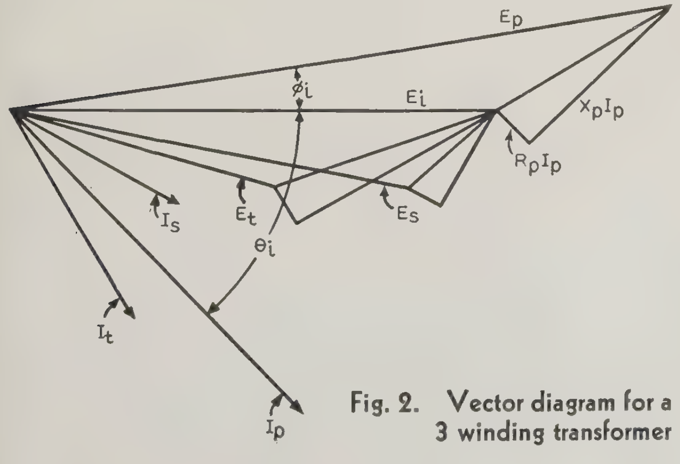


Fig. 2. Vector diagram for a 3 winding transformer

Write with the proper subscripts

$$K = \sqrt{(m + r)^2 + (n + x)^2} \quad (9)$$

By reference to figure 2,

$$\tan(\theta_i + \phi_i) = \frac{E_i \sin\theta_i + I_p X_p}{E_i \cos\theta_i + I_p R_p} \quad (10)$$

and from equations 6 and 9

$$E_i = K_s E_s \text{ and } E_i = K_t E_t \quad (11)$$

Then

$$\tan(\theta_i + \phi_i) = \frac{K_s n_i + x_p}{K_s m_i + r_p} \quad (12)$$

and also

$$= \frac{K_t n_i + x_p}{K_t m_i + r_p} \quad (13)$$

From the figure

$$E_p \cos(\theta_i + \phi_i) = E_i \cos\theta_i + I_p R_p \quad (14)$$

and when equations 11 and 12 or 13 are used in equation 14 there results

$$E_p = E_s \sqrt{K_s m_i + r_p)^2 + (K_s n_i + x_p)^2} \quad (15)$$

$$= K_{ps} E_s \quad (16)$$

and also

$$E_p = E_t \sqrt{(K_t m_i + r_p)^2 + (K_t n_i + x_p)^2} \quad (17)$$

$$= K_{pt} E_t$$

The regulation from primary to secondary or from primary to tertiary follows directly:

$$\text{per cent regulation}_{ps} = (K_{ps} - 1)\, 100 \quad (19)$$

$$\text{per cent regulation}_{pt} = (K_{pt} - 1)\, 100 \quad (20)$$

The factors m_i and n_i are found as follows: from figure 2

$$I_p = \sqrt{I_s^2 + I_t^2 + 2 I_s I_t \cos\alpha} \quad (21)$$

in which

$$\cos\alpha = \frac{(m_s + r_s)(m_t + r_t) + (n_s + x_s)(n_t + x_t)}{K_s K_t} \quad (22)$$

Again from figure 2

$$I_p \cos\theta_i = I_s \cos(\theta_s + \phi_s) + I_t \cos(\theta_t + \phi_t) \quad (23)$$

$$I_p \sin\theta_i = I_s \sin(\theta_s + \phi_s) + I_t \sin(\theta_t + \phi_t) \quad (24)$$

so that

$$m_i = \frac{I_s}{I_p}\frac{m_s + r_s}{K_s} + \frac{I_t}{I_p}\frac{m_t + r_t}{K_t} \quad (25)$$

$$n_i = \frac{I_s}{I_p}\frac{n_s + x_s}{K_s} + \frac{I_t}{I_p}\frac{n_t + x_t}{K_t} \quad (26)$$

The regulation of a 3 winding transformer may also be derived directly in terms of the formula for 2 winding transformers. Thus (see figure 2 and consider E_i and E_s)

$$E_i = K_s E_s \quad (27)$$

$$K_s = \sqrt{(m_s + r_s)^2 + (n_s + x_s)^2} \quad (9, 28)$$

Now consider E_p and E_i, then

$$E_p = K_i E_i \tag{29}$$

$$K_i = \sqrt{(m_i + r_i)^2 + (n_i + x_i)^2} \tag{9, 30}$$

and then

$$E_p = K_s K_i E_s \tag{31}$$

which determines the regulation.

It should be noted that the quantities r_i and x_i refer to the primary voltage E_p but the corresponding values of r_p and x_p cannot be used without first applying a correction factor.

The correction factor is found as follows:

$$r_i = \frac{R_p I_p}{E_i} \tag{32}$$

Reference to figure 1 will show why this is so.

$$r_i = \frac{R_p I_p}{E_s} \frac{E_s}{E_i} \tag{33}$$

It is necessary to use E_s instead of E_p because the per unit values are based on ratio voltages and the E_p as here used is the ratio voltage plus regulation.

But

$$r_p = \frac{R_p I_p}{E_s} \tag{34}$$

and from equation 27

$$\frac{E_s}{E_i} = \frac{1}{K_s} \tag{35}$$

so that

$$r_i = \frac{r_p}{K_s} \tag{36}$$

for calculating the regulation from primary to secondary.

For the tertiary regulation, in a similar manner

$$E_p = K_t K_i E_t \tag{37}$$

$$r_i = \frac{r_p}{K_t} \tag{38}$$

In both cases m_i and n_i are determined as in equations 25 and 26.

A little consideration of equations 15 and 16 will show that equation 31 is identical with equation 16.

Approximate Formula

Two Winding Transformer; Method A. The exact formula for the regulation of a 2 winding transformer is

$$\text{per cent regulation} = (\sqrt{(m + r)^2 + (n + x)^2} - 1)\,100 \tag{39}$$

and this may be written

$$\text{per cent regulation} = (\sqrt{1 + 2(mr + nx) + x^2 + r^2} - 1)\,100 \tag{40}$$

$$\text{per cent regulation} = (\sqrt{1 + 2d} - 1)\,100 \tag{41}$$

in which

$$d = mr + nx + \frac{x^2 + r^2}{2} \tag{42}$$

From equation 41 an approximation for per cent regulation is obtained as

$$\text{per cent regulation} = \left(d - \frac{d^2}{2}\right) 100 \tag{43}$$

$$d = mr + nx + \frac{x^2 + r^2}{2} \tag{42}$$

Two Winding Transformer; Method B. Equation 39 may also be transformed into the following form:

$$\text{per cent regulation} = (\sqrt{(1 + mr + nx)^2 + (mx - nr)^2} - 1)\,100 \tag{44}$$

and the approximation of this is

$$\text{per cent regulation} = \left(mr + nx + \frac{(mx - nr)^2}{2}\right) 100 \tag{45}$$

which is equivalent to the formula in the test code for 2 winding transformers.

Three Winding Transformer; Method A. An approximation for the regulation of a 3 winding transformer may be found as follows: Consider first the regulation from primary to secondary,

$$E_p/E_s = K_{ps} \tag{46}$$

$$= K_s \sqrt{\left(m_i + \frac{r_p}{K_s}\right)^2 + \left(n_i + \frac{x_p}{K_s}\right)^2} \tag{47}$$

Assume that the power factors are given for the secondary and tertiary loads and are both in reference to the primary voltage. Then since the primary current is the vector sum of the secondary and tertiary currents we have

$$I_p m_p = I_s m_s + I_t m_t \tag{48}$$

$$I_p n_p = I_s n_s + I_t n_t \tag{48a}$$

Also assume that the per cent resistance and reactance values for the various windings are all given on the same kilovolt-ampere base, corresponding to current I, then

$$I_p = L_p I \qquad I_s = L_s I \qquad I_t = L_t I \tag{49}$$

L being termed the loading factor.

Equations 48 and 48a become

$$L_p m_p = L_s m_s + L_t m_t \tag{50}$$

$$L_p n_p = L_s n_s + L_t n_t \tag{50a}$$

From equations 50 and 50a

$$L_p = \sqrt{L_s^2 + L_t^2 + 2 I_s I_t (m_s m_t + n_s n_t)} \tag{51}$$

$$m_p = \frac{L_s m_s + L_t m_t}{L_p} \tag{52}$$

$$n_p = \frac{L_s n_s + L_t n_t}{L_p} \tag{53}$$

$$E_p/E_s = K_s \sqrt{(m_p + r_p)^2 + (n_p + x_p)^2} \tag{54}$$

$$= K_s K_p \tag{54a}$$

and by reference to equation 41

$$\text{per cent regulation} = (\sqrt{1 + 2d_s}\ \sqrt{1 + 2d_p} - 1)\,100 \tag{55}$$

$$= \left(d_s + d_p - \frac{(d_s - d_p)^2}{2}\right) 100 \tag{56}$$

in which

$$d = mr + nx + \frac{x^2 + r^2}{2} \qquad (42)$$

Three Winding Transformer; Method B. If equation 54a is first transformed into the form of equation 44 there results

$$\text{per cent regulation}_{ps} = [(1 + D_s)(1 + D_p) - 1]\,100 \qquad (57)$$

and from this

$$\text{per cent regulation}_{ps} = (D_s + D_p + D_sD_p)\,100 \qquad (58)$$

in which

$$D = mr + nx + \frac{(mx - nr)^2}{2} \qquad (59)$$

for secondary and primary, respectively.

The regulation from primary to tertiary is found in a similar manner.

Application of Methods

Calculations will be made for a transformer having the following characteristics:

Secondary kilovolt-amperes = 12,500 at 0.8 power factor lag
Tertiary kilovolt-amperes = 6,000 at 0 power factor lead
Per cent impedances at 10,000 kva:

$Z_{ps} = 2.0 + j\,8.0 \qquad Z_{st} = 1.5 + j\,5.0 \qquad Z_{tp} = 3.0 + j\,15.0$

From these values there are obtained the following per cent and per unit impedances:

Per Cent Values	Per Unit Values
$Z_p = 1.75 + j\,9.0 = M_{st}$	$z_p = 0.0175 + j\,0.09$
$Z_s = 0.25 - j\,1.0 = M_{tp}$	$z_s = 0.0025 - j\,0.01$
$Z_t = 1.25 + j\,6.0 = M_{ps}$	$z_t = 0.0125 + j\,0.06$

Also,

$m_s = 0.8 \qquad n_s = 0.6 \qquad L_s = 1.25$
$m_t = 0 \qquad n_t = -1.0 \qquad L_t = 0.6$
L = loading factor = ratio of load carried by winding to load on which reactances are based

The calculations will be carried out to several decimal places for the purpose of checking the results of the approximate methods with the exact. There is no intention of intimating that this is necessary for routine work or that it would be supported by the accuracy of the test data.

Exact Calculations, 3 Winding Transformer

$$m_s + r_s = 0.8 + 1.25 \times 0.0025 = 0.803125$$
$$n_s + x_s = 0.6 + 1.25 \times (-0.01) = 0.5875$$
$$K_s = 0.99507086 \qquad (9)$$

$$m_t + r_t = 0 + 0.6 \times 0.0125 = 0.0075$$
$$n_t + x_t = -1 + 0.6 \times 0.06 = -0.964$$
$$K_t = 0.96402917 \qquad (9)$$

$$L_p = \sqrt{1.25^2 + 0.6^2 + 2\,\frac{0.803125 \times 0.0075 - 0.5875 \times 0.964}{0.99507086 \times 0.96402917}\,1.25 \times 0.6}$$
$$= 1.02290283 \qquad (21)$$

It is obvious that the loading factor of the primary winding may just as well be determined directly from the loading factor of the secondary and tertiary windings, as to calculate the current and then the load factor. It is obvious that

$$\frac{I_s}{I_p} = \frac{L_s}{L_p} \text{ and } \frac{I_t}{I_p} = \frac{L_t}{L_p}$$

$$m_i = \frac{1.25}{1.0229028}\,\frac{0.803125}{0.99507086} + \frac{0.6}{1.0229028}\,\frac{0.0075}{0.96402917} = 0.99085363 \qquad (25)$$

$$n_i = \frac{1.25}{1.0229028}\,\frac{0.5875}{0.99507086} + \frac{0.6}{1.0229028}\,\frac{-0.964}{0.96402917} = 0.13494034 \qquad (26)$$

$$K_sm_i + r_p = 0.99507086 \times 0.99085363 + 1.0229028 \times 0.0175 = 1.0038704$$

$$K_sn_i + x_p = 0.99507086 \times 0.13494034 + 1.0229028 \times 0.09 = 0.22633645$$

$$K_{ps} = 1.02907 \qquad (9)$$

Therefore per cent regulation$_{ps}$ = 2.91 drop

$$K_tm_i + r_p = 0.96402917 \times 0.99085363 + 1.0229028 \times 0.0175 = 0.973112602$$

$$K_tn_i + x_p = 0.96402917 \times 0.13494034 + 1.0229028 \times 0.09 = 0.22214769$$

$$K_{pt} = 0.99814714 \qquad (9)$$

Therefore per cent regulation$_{pt}$ = −0.19 boost

Approximate Calculations, 3 Winding Transformer

Method A

Secondary

$$Lr = 1.25 \times 0.0025 = 0.003125 \qquad Lx = 1.25 \times (-0.01) = -0.0125$$

$$d = mLr + nLx + \frac{(Lr)^2 + (Lx)^2}{2}$$
$$= 0.8 \times 0.003125 - 0.6 \times 0.0125 + \frac{0.003125^2 + 0.0125^2}{2} \qquad (42)$$

$$d_s = -0.004917$$

Tertiary

$$Lr = 0.6 \times 0.0125 = 0.0075 \qquad Lx = 0.6 \times 0.06 = 0.036$$

$$d = 0 \times 0.0075 - 1 \times 0.036 + \frac{0.0075^2 + 0.036^2}{2} \qquad (42)$$

$$d_t = -0.035324$$

Primary

$$L_pm_p = L_sm_s + L_tm_t = 1.25 \times 0.8 + 0.6 \times 0 = 1.0 \qquad (50)$$

$$L_pn_p = L_sn_s + L_tn_t = 1.25 \times 0.6 + 0.6(-1) = 0.15 \qquad (50a)$$

$$L_p = \sqrt{1.0^2 + 0.15^2} = 1.0111874 \qquad (51)$$

$$m_p = \frac{1}{\sqrt{1.0^2 + 0.15^2}} = 0.9889363 \qquad (52)$$

$$n_p = \frac{0.15}{\sqrt{1.0^2 + 0.15^2}} = 0.14834045 \qquad (53)$$

$$Lr = 1.0111874 \times 0.0175 = 0.01769578$$
$$Lx = 1.0111874 \times 0.09 = 0.09100687$$
$$d_p = 0.035298 \qquad (42)$$

$$\text{regulation}_{ps} = -0.004917 + 0.035298 - \frac{(-0.004917 - 0.035298)^2}{2} = 0.02957 \qquad (56)$$

per cent regulation $= 2.96$

$$\text{regulation}_{pt} = -0.035324 + 0.035298 - \frac{(-0.035324 - 0.035298)^2}{2} = -0.00252 \qquad (56)$$

per cent regulation $= -0.25$

Method B

SECONDARY

$$D_s = 0.0025 - 0.0075 + \frac{(-0.01 - 0.001875)^2}{2} = -0.0049295 \qquad (59)$$

TERTIARY

$$D_t = 0 - 0.036 + \frac{(0 - 0.0075)^2}{2} = -0.035972 \qquad (59)$$

PRIMARY

$$(Lm)_p = L_s m_s + L_t m_t = 1.25 \times 0.8 + 0.6 \times 0 = 1.0 \qquad (50)$$

$$(Ln)_p = L_s n_s + L_t n_t = 1.25 \times 0.6 - 0.6 \times 1.0 = 0.15 \qquad (50a)$$

$$D_p = 1.0 \times 0.0175 + 0.15 \times 0.09 + \frac{(1 \times 0.09 - 0.15 \times 0.0175)^2}{2} = 0.0348172 \qquad (59)$$

$$\text{regulation}_{ps} = 0.0348172 - 0.0049295 - 0.0348172 \times 0.0049295 = 0.029716 \qquad (58)$$

per cent regulation$_{ps}$ $= 2.97$

$$\text{regulation}_{pt} = 0.0348172 - 0.035972 - 0.0348172 \times 0.035972 = 0.0024072 \qquad (58)$$

per cent regulation$_{pt}$ $= -0.24$

EXACT CALCULATIONS, 2 WINDING TRANSFORMERS

All calculations will be made on the basis of 10,000 kva at 0.8 power factor.

$$K_{ps} = \sqrt{(0.8 + 0.02)^2 + (0.6 + 0.08)^2} = 1.0652699 \qquad (9, 7)$$

per cent regulation$_{ps}$ $= 6.53$

$$K_{pt} = \sqrt{(0.8 + 0.03)^2 + (0.6 + 0.15)^2} = 1.118659 \qquad (9, 7)$$

per cent regulation$_{pt}$ $= 11.87$

APPROXIMATE CALCULATIONS, 2 WINDING TRANSFORMERS

Method A

$$d_{ps} = 0.8 \times 0.02 + 0.6 \times 0.08 + \frac{0.02^2 + 0.08^2}{2} = 0.0674 \qquad (42)$$

$$\text{regulation} = 0.0674 - \frac{0.0674^2}{2} = 0.06513 \qquad (43)$$

per cent regulation $= 6.51$

$$d_{pt} = 0.8 \times 0.03 + 0.6 \times 0.15 + \frac{0.03^2 + 0.15^2}{2} = 0.1257 \qquad (42)$$

$$\text{regulation}_{pt} = 0.1257 - \frac{0.1257^2}{2} = 0.117800 \qquad (43)$$

per cent regulation$_{pt}$ $= 11.78$

Method B

$$\text{regulation}_{ps} = 0.8 \times 0.02 + 0.6 \times 0.08 + \frac{(0.8 \times 0.08 - 0.6 \times 0.02)^2}{2} = 0.06535 \qquad (45)$$

per cent regulation$_{ps}$ $= 6.54$

$$\text{regulation}_{pt} = 0.8 \times 0.03 + 0.6 \times 0.15 + \frac{(0.8 \times 0.15 - 0.6 \times 0.03)^2}{2} = 0.119202 \qquad (45)$$

per cent regulation$_{pt}$ $= 11.92$

COMPARISON OF RESULTS

The results of the preceding calculations are shown in the following tabulation:

Transformer		Exact Method	Approximate Methods A	Approximate Methods B
3 Winding	PS	2.91	2.96	2.97
	PT	−0.19	−0.25	−0.24
2 Winding	PS	6.53	6.51	6.54
	PT	11.87	11.78	11.92

It appears that there is little choice between the accuracy of method *A* or method *B*. However, method *B* for 2 winding transformers probably requires a little less work, and furthermore is the same as that already in use. This has been considered sufficient reason for the choice of method *B* for 3 winding transformers.

GENERAL METHOD

There is another method by which the regulation of a transformer of any number of windings may be calculated. It is as follows: Let E_p be the primary voltage, E_s the secondary voltage, E_t the tertiary voltage, etc.; then for self-impedance Z and mutual impedance M

$$E_p = E_s + I_s Z_{ps} + I_t M_{st} + I_q M_{sq}$$
$$E_p = I_s M_{st} + E_t + I_t Z_{pt} + I_q M_{tq}$$
$$E_p = I_s M_{sq} + I_t M_{tq} + E_q + I_q Z_{pq}$$

Assume that all the impedances are on the same base, then let L be the loading factor, i. e., the ratio of the actual load to the base load. Also let Z be the phase lag, or load impedance factor of the load on

any winding. Then a little consideration will show that

$$E_p = E_s + E_sL_s \frac{Z_{ps}}{Z_s} + E_tL_t \frac{M_{st}}{Z_t} + E_qL_q \frac{M_{sq}}{Z_q}$$

$$E_p = E_sL_s \frac{M_{st}}{Z_s} + E_t + E_tL_t \frac{Z_{pt}}{Z_t} + E_qL_q \frac{M_{tq}}{Z_q}$$

E_p = etc.

From this, by determinants,

$$E_s = E_p \frac{D_s}{D} \text{ and then } \frac{E_p}{E_s} = \frac{D}{D_s}$$

From this regulation follows directly.

This method does not lend itself to a simplifying approximation and has not been utilized for that reason.

Analysis of Unsymmetrical Machines

By
W. V. LYON
FELLOW A.I.E.E.
CHARLES KINGSLEY, JR.
ASSOCIATE A.I.E.E.

Massachusetts Institute of Technology, Cambridge

By making use of suitably chosen components of current and voltage, the method of symmetrical components, which is so useful in the analysis of symmetrically wound polyphase a-c machines, can be extended readily to the analysis of unsymmetrical 2 phase induction machines, such as the capacitor or split phase motor in which the windings are not in space quadrature. Such extensions to this method, illustrated by a typical numerical example, are presented in this paper.

THE method of symmetrical components originally was developed by C. L. Fortescue[1] in connection with the analysis of symmetrically wound polyphase induction machines operating under unbalanced conditions. Since its original development, the method has proved to be of inestimable value in the analysis of problems involving a-c rotating machines under both steady state and transient[2] conditions, and in connection with transmission lines and transformer banks under fault conditions. Recently the method has been shown to be useful when applied to the analysis of transformer banks in which the individual transformers have different ratios of transformation.[3]

The great advantage of the symmetrical component method of analysis as applied to problems involving rotating machines lies in the fact that unbalanced currents are resolved into 2 sets of components. One set produces a single sinusoidal positively rotating magnetic field of constant amplitude (if harmonics be neglected), while the other set produces a similar negatively rotating field. Thus the effect of unbalanced currents on the air gap flux can be calculated in terms of 2 simple rotating fields traveling in opposite directions. These rotating fields generate component voltages in the windings. When the windings are similar and symmetrically spaced, the component currents and voltages each form similar and symmetrical systems of opposite phase order.

In this paper it is shown that by making use of suitably chosen components of current and voltage, Fortescue's scheme can be extended readily to the analysis of unsymmetrical 2-phase or single-phase machines. This problem has been solved by other methods,[4,5] in which the machine is represented as 2 single-phase machines magnetically interrelated. When so treated, the problem involves 4 rotating magnetic fields. When analyzed by the method of

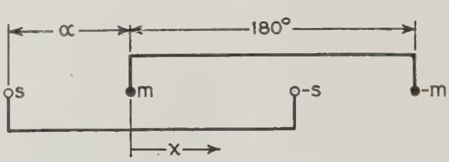


Fig. 1. Schematic diagram of stator windings of an unsymmetrical machine

symmetrical components as herein extended, only 2 such fields need be considered, and simpler expressions for torque result.

In what follows, only fundamental fluxes will be taken into account, the air gap will be assumed uniform, and the rotor will be assumed to have a symmetrical polyphase circuit of either the cage or wound type. These assumptions commonly are made in induction-motor analysis.

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Jan. 30, 1936; released for publication March 20, 1936.

1. For numbered references see list at end of paper.

Stator Magnetomotive Forces

Consider a stator having 2 windings, m and s, separated in space by α electrical radians, as shown diagrammatically in figure 1. Let the instantaneous currents in the 2 windings be

$$i_m = \sqrt{2}\,|I_m|\sin(\omega t + \theta_m) \quad (1)$$

$$i_s = \sqrt{2}\,|I_s|\sin(\omega t + \theta_m + \beta) \quad (2)$$

in which

$|I_m|, |I_s|$ = effective values of the 2 currents
ω = 2π times frequency
t = time in seconds
θ_m = phase angle of i_m with respect to zero time
β = phase angle by which i_s leads i_m

The space fundamental magnetomotive force H set up by the 2 windings will be

$$H = C[N_m i_m \sin x + N_s i_s \sin(x+\alpha)] \quad (3)$$

in which

N_m, N_s = effective fundamental turns of the 2 windings
x = distance in electrical radians, as shown in figure 1
C = a constant

Let the turn ratio $N_m/N_s = k$. Then equation 3 can be written

$$H = \sqrt{2}\,C\,N_m\left[|I_m|\sin x\sin(\omega t+\theta_m) + \frac{|I_s|}{k}\sin(x+\alpha)\sin(\omega t+\theta_m+\beta)\right] \quad (4)$$

$$= \frac{CN_m}{\sqrt{2}}\left[|I_m|\cos(x-\omega t-\theta_m) - |I_m|\cos(x+\omega t+\theta_m) + \frac{|I_s|}{k}\cos(x-\omega t-\theta_m+\alpha-\beta) - \frac{|I_s|}{k}\cos(x+\omega t+\theta m+\alpha+\beta)\right] \quad (5)$$

The first and third terms on the right-hand side of equation 5 are rotating fields traveling in the positive direction of x, while the second and fourth terms are negatively rotating fields. From equation 5 it can readily be seen that:

(*a*) If $|I_s| = k|I_m|$ and $\alpha+\beta = 180$ degrees, or i_s leads i_m by an angle $\beta = 180$ degrees $-\ \alpha$, the negatively rotating fields cancel, and only positively rotating fields result.

(*b*) If $|I_s| = k|I_m|$ and $\alpha-\beta = 180$ degrees, or i_s lags i_m by an angle $-\beta = 180$ degrees $-\ \alpha$, the positively rotating fields cancel, and only negatively rotating fields result.

Let $\boldsymbol{I}_m$ and $\boldsymbol{I}_s$ be the vectors representing the effective values of i_m and i_s. The conditions stated in (*a*) and (*b*) can be written conveniently in vector form as follows:

(*a*) Only positively rotating fields result if

$$\boldsymbol{I}_s = k\boldsymbol{I}_m\ \angle(180^\circ-\alpha) = -k\boldsymbol{I}_m\ \angle-\alpha \quad (6)$$

(*b*) Only negative rotating fields result if

$$\boldsymbol{I}_s = k\boldsymbol{I}_m\ \angle(-180^\circ+\alpha) = -k\boldsymbol{I}_m\ \angle\alpha \quad (7)$$

Components of Stator Currents

In general, $\boldsymbol{I}_m$ and $\boldsymbol{I}_s$ will set up both positively and negatively rotating fields. However, these currents can be resolved into components, $\boldsymbol{I}_{m1}$ and $\boldsymbol{I}_{m2}$, such that

	positive sequence components		negative sequence components	
$\boldsymbol{I}_m =$	$\boldsymbol{I}_{m1}$	$+$	$\boldsymbol{I}_{m2}$	(8)
$\boldsymbol{I}_s =$	$-k\boldsymbol{I}_{m1}\ \angle-\alpha$	$-$	$k\boldsymbol{I}_{m2}\angle\alpha$	(9)

From the simultaneous solution of these equations, the component currents can be expressed in terms of the actual currents as follows:

$$\boldsymbol{I}_{m1} = \frac{\boldsymbol{I}_m + \dfrac{\boldsymbol{I}_s\angle-\alpha}{k}}{2\sin\alpha\angle\delta} \quad (8a)$$

$$\boldsymbol{I}_{m2} = \frac{\boldsymbol{I}_m + \dfrac{\boldsymbol{I}_s\ \angle\alpha}{k}}{2\sin\alpha\ \angle-\delta} \quad (9a)$$

where $\delta = 90$ degrees $-\ \alpha$.

In equations 8 and 9, the currents $\boldsymbol{I}_m$ and $\boldsymbol{I}_s$ are resolved into components chosen so that the components $\boldsymbol{I}_{m1}$ and $-k\boldsymbol{I}_{m1}\angle-\alpha$ in the m and s windings set up only positively rotating fields, and the components $\boldsymbol{I}_{m2}$ and $-k\boldsymbol{I}_{m2}\angle\alpha$ set up only negatively rotating fields. Hence the component currents $\boldsymbol{I}_{m1}$ and $\boldsymbol{I}_{m2}$ defined by the foregoing equations will be called the positive and negative sequence components of $\boldsymbol{I}_m$. (According to Fortescue's notation, the subscript 3 should be used to denote negative sequence quantities in 2 phase systems. However, the subscript 2 is used in this paper in accordance with the notation in the more common 3 phase case.) In instantaneous form, the currents i_m and i_s are resolved into instantaneous positive sequence currents

$$i_{m1} = \sqrt{2}\,|I_{m1}|\sin(\omega t+\theta_{m1}) \quad (10)$$

$$i_{s1} = \sqrt{2}\,k\,|I_{m1}|\sin(\omega t+\theta_{m1}+180^\circ-\alpha) \quad (11)$$

and instantaneous negative sequence currents

$$i_{m2} = \sqrt{2}\,|I_{m2}|\sin(\omega t+\theta_{m2}) \quad (12)$$

$$i_{s2} = \sqrt{2}\,k\,|I_{m2}|\sin(\omega t+\theta_{m2}-180^\circ+\alpha) \quad (13)$$

in which

$|I_{m1}|, |I_{m2}|$ = the magnitudes of the vector currents $\boldsymbol{I}_{m1}$ and $\boldsymbol{I}_{m2}$
θ_{m1}, θ_{m2} = the phase angles of $\boldsymbol{I}_{m1}$ and $\boldsymbol{I}_{m2}$

The resultant positively rotating magnetomotive force $\boldsymbol{H}_1$, set up by the positive sequence stator currents i_{m1} and i_{s1} can be calculated by substituting equations 10 and 11 for i_m and i_s in equation 3. The resulting expression can be reduced to

$$\boldsymbol{H}_1 = \sqrt{2}\,C\,N_m\,|I_{m1}|\sin\alpha\sin(x-\omega t-\theta_{m1}+\alpha) \quad (14)$$

This resultant positively rotating stator magnetomotive force, H_1, now will be compared with the positively rotating stator magnetomotive force, H_1', that would be set up by balanced 2-phase positive-sequence currents i_{m1}' and i_{s1}' of the form

$$i_{m1}' = \sqrt{2}\,|I_{m1}'|\sin(\omega t+\theta_{m1}') \quad (15)$$

$$i_{s1}' = \sqrt{2}\,|I_{m1}'|\sin(\omega t+\theta_{m1}'+90^\circ) \quad (16)$$

in a symmetrical 2-phase stator winding of N_m effective turns per phase, the 2 windings being in space quadrature. By substituting $\alpha = 90$ degrees,

equation 14 reduces to

$$H_1' = \sqrt{2}\, C\, N_m \,|\, I_{m1}' \,|\, \sin(x - \omega t - \theta_{m1}' + 90°) \qquad (17)$$

Comparing equations 14 and 17, it readily can be seen that the unsymmetrical motor will set up exactly the same positive sequence stator magnetomotive force as an equivalent symmetrical 2-phase motor having N_m effective turns per phase, if

$$|\, I_{m1}' \,| = |\, I_{m1} \,| \sin \alpha \qquad (18)$$

and

$$-\theta_{m1} + \alpha = -\theta_{m1}' + 90° \qquad (19)$$

or i_{m1}' leads i_{m1} by an angle

$$\theta_{m1}' - \theta_{m1} = 90° - \alpha \qquad (20)$$

Vectorially, these conditions can be expressed as

$$\boldsymbol{I}_{m1}' = \boldsymbol{I}_{m1} \sin \alpha \quad \angle \delta \qquad (21)$$

in which $\boldsymbol{I}_{m1}'$ is the positive sequence current vector in the equivalent symmetrical 2-phase motor and $\delta = 90$ degrees $- \alpha$ is the angle by which the stator windings of the unsymmetrical motor differ from being in space quadrature.

In an exactly similar manner, it readily can be shown that the unsymmetrical motor will set up exactly the same negative sequence stator magnetomotive force as an equivalent symmetrical 2-phase motor having N_m effective turns per stator phase, if vectorially

$$\boldsymbol{I}_{m2}' = \boldsymbol{I}_{m2} \sin \alpha \quad \angle -\delta \qquad (22)$$

in which $\boldsymbol{I}_{m2}'$ is the negative sequence current vector in the equivalent symmetrical 2-phase motor.

Rotor Currents

So far as the internal reactions within the motor are concerned, it makes no difference how the positive and negative sequence stator magnetomotive forces are set up. That is, as viewed from the rotor it is impossible to tell whether the stator magnetomotive forces are due to the component currents given in equations 8 and 9 in the windings of the *unsymmetrical* motor, or are due to positive and negative sequence *symmetrical* 2-phase currents in a *symmetrical* stator winding, provided the *symmetrical* components of the equivalent 2-phase currents are related to the *unsymmetrical* currents as in equations 21 and 22. Hence the internal reactions within the unsymmetrical motor can be analyzed in terms of an equivalent symmetrical 2-phase motor if the resulting stator currents in the unsymmetrical motor are related to the equivalent symmetrical 2-phase currents by equations 21, 22, 8, and 9.

The internal reactions within a symmetrical 2-phase motor can be calculated readily from the well-known equivalent circuits shown in figure 2, in which all quantities are per phase referred to a symmetrical 2-phase stator winding of N_m effective turns per phase. In figure 2,

r_r = effective resistance of the rotor at slip frequency
r_r' = effective resistance of the rotor at (2-S) frequency
x_r = leakage reactance of the rotor at stator frequency

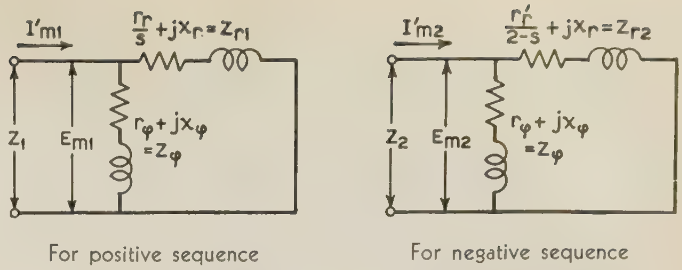


Fig. 2. Equivalent circuits for an unsymmetrical machine

r_φ = resistance of the magnetizing impedance
x_φ = reactance of the magnetizing impedance
S = slip expressed as a decimal
$\boldsymbol{E}_{m1}$ = voltage induced in stator phase m by the resultant positive-sequence air-gap flux
$\boldsymbol{E}_{m2}$ = voltage induced in stator phase m by the resultant negative-sequence air-gap flux

The constants for the equivalent circuits can be calculated from design formulas in common use in 2-phase motor design. From the manner in which these impedances are defined with respect to an equivalent symmetrical 2-phase stator, the rotor and magnetizing impedances here used are twice the corresponding impedances in references 4 and 5.

Since the positive and negative sequence rotating fields in the equivalent symmetrical motor are exactly the same as the corresponding fields in the unsymmetrical motor, and since both motors have the same number of effective turns in stator phase m, the induced voltages $\boldsymbol{E}_{m1}$ and $\boldsymbol{E}_{m2}$ given in the equivalent symmetrical motor of figure 2 will be equal also to the positive and negative sequence components of the induced voltage in phase m of the unsymmetrical motor.

The induced voltages can be expressed in terms of the terminal impedances Z_1 and Z_2 of the equivalent circuits of figure 2, as follows:

$$\boldsymbol{E}_{m1} = \boldsymbol{I}_{m1}'\boldsymbol{Z}_1 \qquad (23)$$

$$\boldsymbol{E}_{m2} = \boldsymbol{I}_{m2}'\boldsymbol{Z}_2 \qquad (24)$$

in which

$$\boldsymbol{Z}_1 = \frac{\boldsymbol{Z}_{r1}\boldsymbol{Z}_\varphi}{\boldsymbol{Z}_{r1} + \boldsymbol{Z}_\varphi} \qquad (25)$$

$$\boldsymbol{Z}_2 = \frac{\boldsymbol{Z}_{r2}\boldsymbol{Z}_\varphi}{\boldsymbol{Z}_{r2} + \boldsymbol{Z}_\varphi} \qquad (26)$$

$\boldsymbol{Z}_\varphi$, $\boldsymbol{Z}_{r1}$, and $\boldsymbol{Z}_{r2}$ = the vector impedances of the branches of the equivalent circuits shown in figure 2.

If $r_\varphi = 0$, (i. e., core loss is neglected) $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ can be expressed in the following forms which are useful in numerical calculations:

$$\boldsymbol{Z}_1 = \frac{[r_r/S]x_\varphi^2 + jx_\varphi\{[r_r/S]^2 + x_r[x_r + x_\varphi]\}}{[r_r/S]^2 + [x_r + x_\varphi]^2} \qquad (27)$$

$$\boldsymbol{Z}_2 = \frac{[r_r'/(2-S)]x_\varphi^2 + jx_\varphi\{[r_r'/(2-S)]^2 + x_r[x_r + x_\varphi]\}}{[r_r'/(2-S)]^2 + [x_r + x_\varphi]^2} \qquad (28)$$

Substituting equations 21 and 22 in 23 and 24:

$$\boldsymbol{E}_{m1} = \boldsymbol{I}_{m1}\boldsymbol{Z}_1 \sin \alpha \angle \delta \qquad (29)$$

$$\boldsymbol{E}_{m2} = \boldsymbol{I}_{m2}\boldsymbol{Z}_2 \sin \alpha \angle -\delta \qquad (30)$$

These equations express the symmetrical components of the induced voltage in the unsymmetrical motor in terms of the symmetrical components of its stator current and the rotor and magnetizing impedances of an equivalent symmetrical 2-phase motor having the same number of effective turns per stator phase as has phase m of the unsymmetrical motor.

Components of Voltages

The voltage induced in phase m is

$$\boldsymbol{E}_m = \boldsymbol{E}_{m1} + \boldsymbol{E}_{m2} \tag{31}$$

In phase s of the unsymmetrical motor, the voltage induced by the positive sequence field will be $1/k$ times the voltage induced in phase m and will lead this voltage by the angle α. Similarly, the voltage induced in phase s by the negative sequence field will be $1/k$ times the voltage induced in phase m and will lag this voltage by the angle α. Hence the voltage induced in phase s will be

$$\boldsymbol{E}_s = \frac{\boldsymbol{E}_{m1}\angle\alpha}{k} + \frac{\boldsymbol{E}_{m2}\angle -\alpha}{k} \tag{32}$$

It is therefore convenient to resolve the various voltages acting on the unsymmetrical motor into positive and negative sequence components in the manner shown in equations 31 and 32.

If these equations be solved for the component voltages $\boldsymbol{E}_{m1}$ and $\boldsymbol{E}_{m2}$ expressed in terms of the actual voltages $\boldsymbol{E}_m$ and $\boldsymbol{E}_s$, the following equations will be obtained:

$$\boldsymbol{E}_{m1} = \frac{\boldsymbol{E}_m - k\boldsymbol{E}_s\angle\alpha}{2\sin\alpha\angle -\delta} \tag{33}$$

$$\boldsymbol{E}_{m2} = \frac{\boldsymbol{E}_m - k\boldsymbol{E}_s\angle -\alpha}{2\sin\alpha\angle\delta} \tag{34}$$

It is interesting to compare equations 33 and 34 with equations 8a and 9a. Because of the dissymmetry of the stator windings, the resolution of voltages into symmetrical components is different from the resolution of currents.

Components of Stator Impedances

Let $\boldsymbol{Z}_m$ be the sum of the self-leakage impedance of winding m and any impedance external to the motor that may be connected in series with this winding. Let $\boldsymbol{Z}_s$ be the same quantity for winding s. If the 2 windings be not in space quadrature, there may be a mutual leakage reactance x_{ms} between these windings. (The mutual leakage reactance x_{ms} may be either a positive or negative quantity. It is positive when α is less than 90 degrees, and negative when α is greater than 90 degrees. Usually x_{ms} will be small.) The voltage drops in the 2 windings due to these impedances will be

$$\boldsymbol{V}_m' = \boldsymbol{I}_m\boldsymbol{Z}_m + j\,\boldsymbol{I}_s x_{ms} \tag{35}$$

$$\boldsymbol{V}_s' = \boldsymbol{I}_s\boldsymbol{Z}_s + j\,\boldsymbol{I}_m x_{ms} \tag{36}$$

It is necessary to resolve these voltage drops into symmetrical components. Let $\boldsymbol{V}_{m1}'$ and $\boldsymbol{V}_{m2}'$ be, respectively, the positive and negative sequence components of $\boldsymbol{V}_m'$. Then, from equations 33 and 34,

$$\boldsymbol{V}_{m1}' = \frac{\boldsymbol{V}_m' - k\,\boldsymbol{V}_s'\angle\alpha}{2\sin\alpha\angle -\delta} \tag{37}$$

$$\boldsymbol{V}_{m2}' = \frac{\boldsymbol{V}_m' - k\,\boldsymbol{V}_s'\angle -\alpha}{2\sin\alpha\angle\delta} \tag{38}$$

Substituting equations 35 and 36 in equations 37 and 38, and expressing the currents $\boldsymbol{I}_m$ and $\boldsymbol{I}_s$ in terms of their symmetrical components (equations 8 and 9) the symmetrical components of the impedance drops can be expressed as

$$\boldsymbol{V}_{m1}' = \frac{\boldsymbol{I}_{m1}\boldsymbol{Z}_0 + \boldsymbol{I}_{m2}\boldsymbol{Z}_{22}}{\sin\alpha\angle -\delta} \tag{39}$$

$$\boldsymbol{V}_{m2}' = \frac{\boldsymbol{I}_{m1}\boldsymbol{Z}_{21} + \boldsymbol{I}_{m2}\boldsymbol{Z}_0}{\sin\alpha\angle\delta} \tag{40}$$

in which

$$\boldsymbol{Z}_0 = \frac{1}{2}(\boldsymbol{Z}_m + k^2\boldsymbol{Z}_s) - j\,k\,x_{ms}\cos\alpha \tag{41}$$

$$\boldsymbol{Z}_{21} = \frac{1}{2}(\boldsymbol{Z}_m + k^2\boldsymbol{Z}_s\angle -2\alpha) - j\,k\,x_{ms}\angle -\alpha \tag{42}$$

$$\boldsymbol{Z}_{22} = \frac{1}{2}(\boldsymbol{Z}_m + k^2\boldsymbol{Z}_s\angle 2\alpha) - j\,k\,x_{ms}\angle\alpha \tag{43}$$

General Equations for the Unsymmetrical 2-Phase Motor

The general equations for a motor having a uniform air gap, a symmetrical rotor, and 2 unsymmetrical stator windings now can be written. Equating the positive sequence component of the applied potential to the sum of the positive sequence components of the voltage drops due to induced voltages, external impedances, and stator leakage impedances, and multiplying both sides of the equation by $\sin\alpha\angle -\delta$, there results:

$$\frac{1}{2}(\boldsymbol{V}_m - k\,\boldsymbol{V}_s\angle\alpha) = \boldsymbol{I}_{m1}(\boldsymbol{Z}_1\sin^2\alpha + \boldsymbol{Z}_0) + \boldsymbol{I}_{m2}\boldsymbol{Z}_{22} \tag{44}$$

in which $\boldsymbol{V}_m$ and $\boldsymbol{V}_s$ are the vector voltages applied to stator circuits m and s, respectively. The corresponding negative sequence equation is

$$\frac{1}{2}(\boldsymbol{V}_m - k\,\boldsymbol{V}_s\angle -\alpha) = \boldsymbol{I}_{m1}\boldsymbol{Z}_{21} + \boldsymbol{I}_{m2}(\boldsymbol{Z}_2\sin^2\alpha + \boldsymbol{Z}_0) \tag{45}$$

The solution of these equations for the positive and negative sequence currents gives:

$$\boldsymbol{I}_{m1} = \frac{1}{2}\,\frac{(\boldsymbol{V}_m - k\boldsymbol{V}_s\angle\alpha)(\boldsymbol{Z}_2\sin^2\alpha + \boldsymbol{Z}_0) - (\boldsymbol{V}_m - k\,\boldsymbol{V}_s\angle -\alpha)\boldsymbol{Z}_{22}}{(\boldsymbol{Z}_1\sin^2\alpha + \boldsymbol{Z}_0)(\boldsymbol{Z}_2\sin^2\alpha + \boldsymbol{Z}_0) - \boldsymbol{Z}_{21}\boldsymbol{Z}_{22}} \tag{46}$$

$$\boldsymbol{I}_{m2} = \frac{1}{2}\,\frac{(\boldsymbol{V}_m - k\boldsymbol{V}_s\angle -\alpha)(\boldsymbol{Z}_1\sin^2\alpha + \boldsymbol{Z}_0) - (\boldsymbol{V}_m - k\,\boldsymbol{V}_s\angle\alpha)\boldsymbol{Z}_{21}}{(\boldsymbol{Z}_1\sin^2\alpha + \boldsymbol{Z}_0)(\boldsymbol{Z}_2\sin^2\alpha + \boldsymbol{Z}_0) - \boldsymbol{Z}_{21}\boldsymbol{Z}_{22}} \tag{47}$$

Having determined these symmetrical components, the actual current in winding m is, by equation 8,

$$\boldsymbol{I}_m = \boldsymbol{I}_{m1} + \boldsymbol{I}_{m2} \tag{48}$$

The actual current in winding s is, from equation 9,

$$\boldsymbol{I}_s = k[-(\boldsymbol{I}_{m1} + \boldsymbol{I}_{m2})\cos\alpha + j(\boldsymbol{I}_{m1} - \boldsymbol{I}_{m2})\sin\alpha] \tag{49}$$

From well-known induction-motor theory, the torque in synchronous watts is

$$T = 2\,|Z_\varphi|^2 \sin^2\alpha \left[\frac{|I_{m1}|^2}{|Z_{r1}+Z_\varphi|^2}\,\frac{r_r}{S} - \frac{|I_{m2}|^2}{|Z_{r2}+Z_\varphi|^2}\,\frac{r_r'}{(2-S)} \right] \quad (50)$$

in which the vertical lines indicate the magnitude of the vector quantity they enclose. If r_φ be neglected, this equation reduces to the following simpler expression:

$$T = 2[\,|I_{m1}|^2 r_1 - |I_{m2}|^2 r_2]\sin^2\alpha \quad (51)$$

in which r_1 and r_2 are the resistance parts of the impedances Z_1 and Z_2, respectively, as given in equations 27 and 28. It can be shown that this expression can be expanded to the form given in equation 24 of reference 5.

These general equations apply to any motor having 2 stator windings and a symmetrical polyphase rotor winding, in the analysis of which only fundamental fluxes need be considered.

The Capacitor or Split-Phase Motor

For the capacitor motor shown in figure 3, the general equations can be simplified as follows:

$$\mathbf{I}_{m1} = \frac{\mathbf{V}}{2}\,\frac{(1-k\angle\alpha)(\mathbf{Z}_2\sin^2\alpha+\mathbf{Z}_0) - (1-k\angle-\alpha)\mathbf{Z}_{22}}{(\mathbf{Z}_1\sin^2\alpha+\mathbf{Z}_0)(\mathbf{Z}_2\sin^2\alpha+\mathbf{Z}_0) - \mathbf{Z}_{21}\mathbf{Z}_{22}} \quad (52)$$

$$\mathbf{I}_{m2} = \frac{\mathbf{V}}{2}\,\frac{(1-k\angle-\alpha)(\mathbf{Z}_1\sin^2\alpha+\mathbf{Z}_0) - (1-k\angle\alpha)\mathbf{Z}_{21}}{(\mathbf{Z}_1\sin^2\alpha+\mathbf{Z}_0)(\mathbf{Z}_2\sin^2\alpha+\mathbf{Z}_0) - \mathbf{Z}_{21}\mathbf{Z}_{22}} \quad (53)$$

in which $\mathbf{V}$ is the vector line voltage. The symmetrical components of the stator impedances are given by equations 41, 42, and 43, in which $\mathbf{Z}_m$ is the leakage impedance of the main winding and $\mathbf{Z}_s$ is the leakage impedance of the starting winding plus the impedance of the capacitor. If the mutual leakage reactance x_{ms} be neglected, the foregoing equations can be expressed in terms of the impedances $\mathbf{Z}_m$ and $\mathbf{Z}_s$, as follows:

$$\mathbf{I}_{m1} = \frac{\mathbf{V}}{2\sin\alpha}\left[\frac{\mathbf{Z}_2\sin\alpha - jk\mathbf{Z}_m - k(\mathbf{Z}_2\sin\alpha + jk\mathbf{Z}_s)\angle\alpha}{\mathbf{Z}_1\mathbf{Z}_2\sin^2\alpha + (\mathbf{Z}_1+\mathbf{Z}_2)\left(\dfrac{\mathbf{Z}_m+k^2\mathbf{Z}_s}{2}\right) + k^2\mathbf{Z}_m\mathbf{Z}_s}\right] \quad (54)$$

$$\mathbf{I}_{m2} = \frac{\mathbf{V}}{2\sin\alpha}\left[\frac{\mathbf{Z}_1\sin\alpha + jk\mathbf{Z}_m - k(\mathbf{Z}_1\sin\alpha - jk\mathbf{Z}_s)\angle-\alpha}{\mathbf{Z}_1\mathbf{Z}_2\sin^2\alpha + (\mathbf{Z}_1+\mathbf{Z}_2)\left(\dfrac{\mathbf{Z}_m+k^2\mathbf{Z}_s}{2}\right) + k^2\mathbf{Z}_m\mathbf{Z}_s}\right] \quad (55)$$

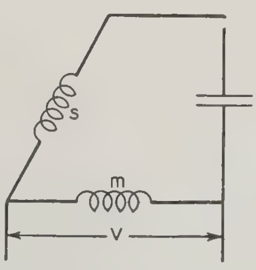


Fig. 3. Schematic diagram of capacitor motor

Knowing the symmetrical components of the stator currents, the actual currents in the stator windings are given by equations 48 and 49, the line current is given by the vector sum of the phase currents, and the torque is given by equation 50 or (if core loss be neglected) by equation 51.

At standstill, $\mathbf{Z}_1 = \mathbf{Z}_2$, as given by substituting $S = 1$ in the equivalent circuits of figure 2 or in equations 27 and 28. Letting $\mathbf{Z}$ be this impedance, the symmetrical components of the stator currents at starting are given by

$$\mathbf{I}_{m1} = \frac{\mathbf{V}}{2\sin\alpha}\left[\frac{\mathbf{Z}\sin\alpha - jk\mathbf{Z}_m - k(\mathbf{Z}\sin\alpha + jk\mathbf{Z}_s)\angle\alpha}{(\mathbf{Z}+\mathbf{Z}_m)(\mathbf{Z}+k^2\mathbf{Z}_s) - \mathbf{Z}^2\cos^2\alpha}\right] \quad (56)$$

$$\mathbf{I}_{m2} = \frac{\mathbf{V}}{2\sin\alpha}\left[\frac{\mathbf{Z}\sin\alpha + jk\mathbf{Z}_m - k(\mathbf{Z}\sin\alpha - jk\mathbf{Z}_s)\angle-\alpha}{(\mathbf{Z}+\mathbf{Z}_m)(\mathbf{Z}+k^2\mathbf{Z}_s) - \mathbf{Z}^2\cos^2\alpha}\right] \quad (57)$$

and the starting torque (neglecting core loss) is

$$T_{st} = 2\,r(|\,I_{m1}|^2 - |\,I_{m2}|^2)\sin^2\alpha \quad (58)$$

in which r is the resistance part of the impedance $\mathbf{Z}$ and $|I_{m1}|$ and $|I_{m2}|$ are the magnitudes of the symmetrical components of the starting currents given by equations 56 and 57.

If the windings be spaced symmetrically, $\alpha = 90$ degrees and obvious simplifications can be made in the preceding equations. It can be shown that when $\alpha = 90$ degrees the equations obtained by the methods of this paper can be expanded to the form given in reference 4.

The same equations that have been derived for the capacitor motor are also applicable to the split-phase motor, except that $\mathbf{Z}_s$ is the leakage impedance of the starting winding plus any external impedance that may be in series with it.

Numerical Example

The following constants, expressed in the notation of this paper, are given in references 4 and 5 for a 1/4-horsepower 110-volt capacitor motor:

Resistance of main winding = 2.02 ohms
Leakage reactance of main winding = 2.79 ohms
Hence $\mathbf{Z}_m = 2.02 + j\,2.79$ ohms
$k = 1/1.18 = 0.848 \qquad k^2 = 0.719$
Resistance of starting winding referred to main winding = 5.12 ohms
Leakage reactance of starting winding referred to main winding = 2.31 ohms
At start, capacitor resistance = 3.0 ohms; capacitor reactance = −14.5 ohms
Hence $k^2\mathbf{Z}_s = (5.12 + 0.719 \times 3.0) + j(2.31 - 0.719 \times 14.5)$
$= 7.28 - j\,8.10$
$r_r = 2.06 \times 2 = 4.12 \qquad x_r = 1.06 \times 2 = 2.12$
$x_\varphi = 33.4 \times 2 = 66.8$

Let it be required to calculate the starting characteristics of this motor when the angle between the windings is 105 degrees; core loss and mutual leakage reactance will be neglected. From equations 27 and 28, with $S = 1$,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z} = \frac{(4.12)(66.8)^2 + j\,66.8[(4.12)^2 + 2.12(2.12+66.8)]}{(4.12)^2 + (2.12+66.8)^2}$$

$$= 3.86 + j\,2.28$$

$$\mathbf{Z}^2 = 9.70 + j\,17.6$$

$\alpha = 105° \qquad \sin\alpha = 0.966 \qquad \cos\alpha = -0.259$
$\angle\alpha = -0.259 + j\,0.966 \qquad \angle-\alpha = -0.259 - j\,0.966$

Substituting numerical values of the complex quantities in equations 56 and 57, there results

$\mathbf{I}_{m1} = 11.42\angle-37.0° = 9.14 - j\,6.89$
$\mathbf{I}_{m2} = 3.40\angle-35.5° = 2.77 - j\,1.98$

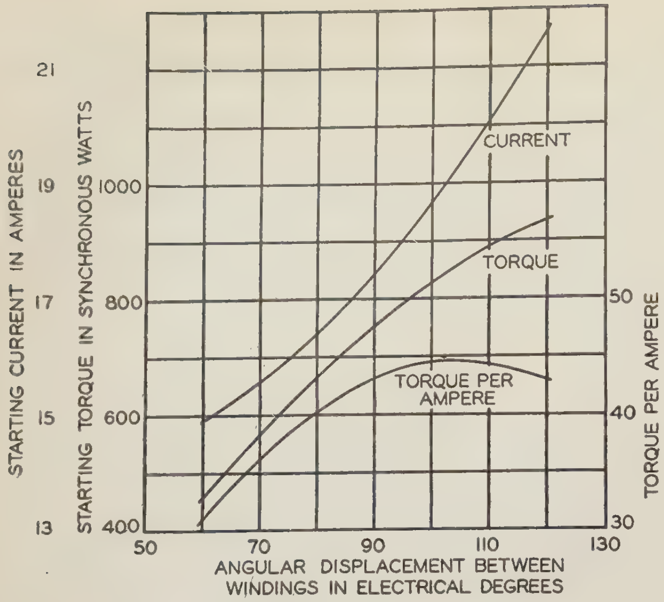


Fig. 4. Starting characteristics of a 1/4 horsepower capacitor motor as a function of the angle between the windings; see text for motor constants

Hence, by equation 48, the current in the main winding is

$$I_m = 11.91 - j\,8.87 = 14.85 \text{ amperes}$$

By equation 49, the current in the starting winding is

$$I_s = 6.64 + j\,3.28 = 7.41 \text{ amperes}$$

The line current is

$$I_m + I_s = 18.55 - j\,5.59 = 19.31 \text{ amperes}$$

By equation 58, the starting torque is

$$T_{st} = 2 \times 3.86[(11.42)^2 - (3.40)^2](0.966)^2$$
$$= 860 \text{ synchronous watts}$$

Figure 4 shows the effect on the starting characteristics of this motor as the angle between the windings is varied. The results at 60, 90, and 120 degrees agree with those given in reference 5. For this particular motor and starting capacitor, the maximum starting torque per ampere is obtained with an angle between the windings equal to 103 degrees. However, it is impossible to make any general statement based upon this particular case relative to the theoretical desirability of using angles other than 90 degrees.

References

1. Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks, C. L. Fortescue. A.I.E.E. Trans., v. 37, 1918, part 2, p. 1027–1115.

2. Sudden Short Circuit of an Alternator, S. Bekku. Researches of the Electrotechnical Laboratory, No. 203, Tokyo, Japan.

3. Loading a Bank of Dissimilar Transformers, W. V. Lyon and C. Kingsley, Jr. Letter to the Editor, Elec. Engg. (A.I.E.E. Trans.), v. 54, March 1935, p. 350.

4. The Revolving Field Theory of the Capacitor Motor, W. J. Morrill. A.I.E.E. Trans., v. 48, April 1929, p. 614–29.

5. Capacitor Motors With Windings Not in Quadrature, A. F. Puchstein and T. C. Lloyd. Elec. Engg. (A.I.E.E. Trans.), v. 54, Nov. 1935, p. 1235–9.

Current and Voltage Loci in 3-Phase Δ-Δ Circuits

Based on methods previously published for determining the loci of voltages and currents in a star-star circuit, the loci of voltages and currents in a general unbalanced delta-delta circuit are developed in this paper. A numerical example is given.

By
A. C. SELETZKY, MEMBER A.I.E.E.
K. F. SIBILA, ENROLLED STUDENT A.I.E.E.

Both of Case School of Applied Science, Cleveland, Ohio

IN A previous paper[1] there was presented a detailed exposition of the method for determining the circular loci of currents and voltages in a general star-star circuit together with an example illustrating the application to an actual circuit. Equations for the several currents were given in a convenient form, requiring only the substitution of numerical values to obtain the locus of any current or voltage existing in the system. This paper deals similarly with the delta-delta connection.

The general form of the delta-delta connection is shown in figure 1. Internal voltages of any magnitude and phase angle are assumed to exist in each phase of the load as well as the source. No equality is assumed among the several impedances of the source and load phases, and mutual elements of varying values are considered in the source and in the load. Likewise, the line impedances are considered

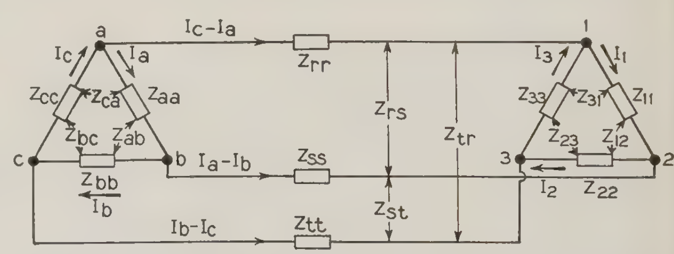


Fig. 1. Schematic diagram of general delta-delta circuit

to be unequal and to interact among each other through mutual elements.

All impedances are assumed to be linear and bilateral. Equations will be derived for the 6 phase

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Jan. 8, 1936; released for publication March 20, 1936.

1. For numbered references see list at end of paper.

currents. From these currents, the line currents may be obtained by addition and any voltage evaluated by summation of impedance drops and internal voltages.

By summing up the potential differences about meshes $a12ba$, $b23cb$, $c31ac$, and 1231 (figure 1) and the currents about junctions 1 and 2, the following 6 equations are obtained:

$$I_aA_1 + I_bA_2 + I_cA_3 - I_1Z_{11} - I_2Z_{12} - I_3Z_{13} = K_1 \tag{1}$$

$$I_aA_2 + I_bB_2 + I_cB_3 - I_1Z_{21} - I_2Z_{22} - I_3Z_{23} = K_2 \tag{2}$$

$$I_aA_3 + I_bB_3 + I_cC_3 - I_1Z_{31} - I_2Z_{32} - I_3Z_{33} = K_3 \tag{3}$$

$$I_1D_1 + I_2D_2 + I_3D_3 = K_4 \tag{4}$$

$$-I_a + I_c - I_1 + I_3 = 0 \tag{5}$$

$$I_a - I_b + I_1 - I_2 = 0 \tag{6}$$

in which

$$\left.\begin{aligned}
A_1 &= Z_{aa} + Z_{rr} + Z_{ss} - 2Z_{rs} & D_1 &= Z_{11} + Z_{12} + Z_{13}\\
B_2 &= Z_{bb} + Z_{ss} + Z_{tt} - 2Z_{st} & D_2 &= Z_{21} + Z_{22} + Z_{23}\\
C_3 &= Z_{cc} + Z_{tt} + Z_{rr} - 2Z_{tr} & D_3 &= Z_{31} + Z_{32} + Z_{33}\\
A_2 &= -Z_{ss} + Z_{ab} + Z_{rs} + Z_{st} - Z_{tr} & K_1 &= E_{ba} - E_{21}\\
B_3 &= -Z_{tt} + Z_{bc} - Z_{rs} + Z_{st} + Z_{tr} & K_2 &= E_{cb} - E_{32}\\
A_3 &= -Z_{rr} + Z_{ca} + Z_{rs} - Z_{st} + Z_{tr} & K_3 &= E_{ac} - E_{13}\\
& & K_4 &= E_{12} + E_{23} + E_{31}
\end{aligned}\right\} \tag{7}$$

By using Cramer's rule, the phase currents may be written as

$$\left.\begin{aligned}
I_a &= \frac{\Delta I_a}{\Delta} & I_b &= \frac{\Delta I_b}{\Delta} & I_c &= \frac{\Delta I_c}{\Delta}\\
I_1 &= \frac{\Delta I_1}{\Delta} & I_2 &= \frac{\Delta I_2}{\Delta} & I_3 &= \frac{\Delta I_3}{\Delta}
\end{aligned}\right\} \tag{8}$$

where Δ is the general determinant or the common denominator and the ΔI_a, ΔI_b . . . are the specific determinants or the numerators of the several current expressions.

The expansions of the common denominator and the numerators are given below. They are written as symmetrically as is possible with the number of groups of terms used. A still more apparent cyclic repetition of impedance elements could have been obtained at the expense of increasing the number of term groupings. Both symmetry and repetitive cycles are invaluable in expressions of this type for facilitating the check on the correctness of the analytical equations as well as the substitution of numerical values. Although the mutual elements are bilateral, i. e., $Z_{23} = Z_{32}$, both sequences of subscripts are used to make the cyclicism of terms more readily discernible.

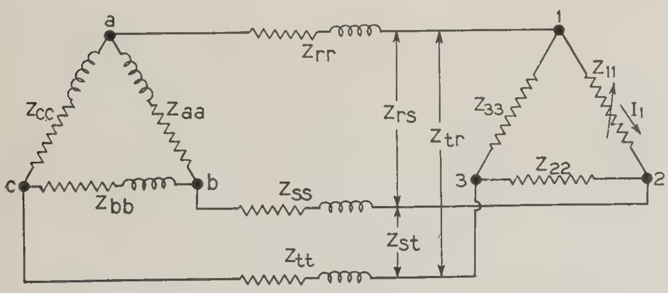


Fig. 2. Schematic diagram of specific delta-delta circuit used in numerical example

$$\begin{aligned}
\Delta = {}&\\
&(A_1B_2 - A_2^2)[Z_{33}(D_1 + D_2) - D_3(Z_{23} + Z_{31})] +\\
&(B_2C_3 - B_3^2)[Z_{11}(D_2 + D_3) - D_1(Z_{31} + Z_{12})] +\\
&(C_3A_1 - A_3^2)[Z_{22}(D_3 + D_1) - D_2(Z_{12} + Z_{23})] +\\
&(A_1B_3 - A_2A_3)[D_3(Z_{21} + Z_{22} - Z_{23}) + D_2(Z_{31} - Z_{32} + Z_{33}) - 2Z_{23}D_1] +\\
&(A_2C_3 - A_3B_3)[D_2(Z_{11} - Z_{12} + Z_{13}) + D_1(-Z_{21} + Z_{22} + Z_{23}) - 2Z_{12}D_3] +\\
&(A_3B_2 - A_2B_3)[D_1(-Z_{31} + Z_{32} + Z_{33}) + D_3(Z_{11} + Z_{12} - Z_{13}) - 2Z_{31}D_2] +\\
&(D_1 + D_2 + D_3)[A_1(B_2C_3 - B_3^2) + A_2(A_3B_3 - A_2C_3) + A_3(A_2B_3 - A_3B_2)] +\\
&(A_1 + A_2 + A_3)[D_1(Z_{22}Z_{33} - Z_{23}^2) + D_2(Z_{23}Z_{31} - Z_{12}Z_{33}) +\\
&\qquad D_3(Z_{12}Z_{23} - Z_{31}Z_{22})] +\\
&(A_2 + B_2 + B_3)[D_2(Z_{33}Z_{11} - Z_{31}^2) + D_3(Z_{31}Z_{12} - Z_{23}Z_{11}) +\\
&\qquad D_1(Z_{23}Z_{31} - Z_{12}Z_{33})] +\\
&(A_3 + B_3 + C_3)[D_3(Z_{11}Z_{22} - Z_{12}^2) + D_1(Z_{12}Z_{23} - Z_{31}Z_{22}) +\\
&\qquad D_2(Z_{31}Z_{12} - Z_{23}Z_{11})]
\end{aligned} \tag{9}$$

$$\begin{aligned}
\Delta I_a = {}&\\
&(K_3Z_{23} - K_2Z_{33})[D_1A_2 + D_2(A_2 + A_3)] - (K_3Z_{13} - K_1Z_{33}) \times\\
&\quad [D_1B_2 + D_2(B_2 + B_3)] + (K_2Z_{31} - K_1Z_{23})[D_1B_3 + D_2(C_3 + B_3)] -\\
&(K_3Z_{22} - K_2Z_{32})[D_1A_3 + D_3(A_2 + A_3)] + (K_3Z_{12} - K_1Z_{32}) \times\\
&\quad [D_1B_3 + D_3(B_2 + B_3)] - (K_2Z_{12} - K_1Z_{22})[D_1C_3 + D_3(C_3 + B_3)] +\\
&(K_3Z_{21} - K_2Z_{31})(D_2A_3 - D_3A_2) - (K_3Z_{11} - K_1Z_{31})(D_2B_3 - D_3B_2) +\\
&\qquad (K_2Z_{11} - K_1Z_{21})(D_2C_3 - D_3B_3) +\\
&K_4[(Z_{11} + Z_{12} + Z_{13})(B_2C_3 - B_3^2) + (Z_{21} + Z_{22} + Z_{23})(A_3B_3 - A_2C_3) +\\
&\qquad (Z_{31} + Z_{32} + Z_{33})(A_2B_3 - A_3B_2)] +\\
&(D_1 + D_2 + D_3)[K_1(B_2C_3 - B_3^2) + K_2(A_3B_3 - A_2C_3) + K_3(A_2B_3 - A_3B_2)] +\\
&(Z_{11}Z_{22} - Z_{12}^2)(K_3D_3 + K_4C_3) +\\
&(Z_{22}Z_{33} - Z_{23}^2)[K_1D_1 + K_4(Z_{11} - A_2 - A_3)] +\\
&(Z_{33}Z_{11} - Z_{31}^2)(K_2D_2 + K_4B_2) +\\
&(Z_{12}Z_{23} - Z_{31}Z_{22})[K_3D_1 + K_1D_3 + K_4(Z_{13} + A_3 - B_3 - C_3)] +\\
&(Z_{31}Z_{12} - Z_{23}Z_{11})(K_2D_3 + K_3D_2 + 2K_4B_3) +\\
&(Z_{23}Z_{31} - Z_{12}Z_{33})[K_1D_2 + K_2D_1 + K_4(Z_{12} + A_2 - B_2 - B_3)]
\end{aligned} \tag{10}$$

$$\begin{aligned}
\Delta I_b = {}&\\
&(K_3Z_{21} - K_2Z_{31})[D_2A_3 + D_3(A_1 + A_3)] - (K_3Z_{11} - K_1Z_{31}) \times\\
&\quad [D_2B_3 + D_3(A_2 + B_3)] + (K_2Z_{11} - K_1Z_{21})[D_2C_3 + D_3(A_3 + C_3)] -\\
&(K_2Z_{23} - K_2Z_{33})[D_2A_1 + D_1(A_1 + A_3)] + (K_3Z_{13} - K_1Z_{33}) \times\\
&\quad [D_2A_2 + D_1(A_2 + B_3)] - (K_2Z_{13} - K_1Z_{23})[D_2A_3 + D_1(A_3 + C_3)] +\\
&(K_3Z_{22} - K_2Z_{32})(D_3A_1 - D_1A_3) - (K_3Z_{12} - K_1Z_{32})(D_3A_2 - D_1B_3) +\\
&\qquad (K_2Z_{12} - K_1Z_{22})(D_3A_3 - D_1C_3) +\\
&K_4[(Z_{11} + Z_{12} + Z_{13})(B_3A_3 - A_2C_3) + (Z_{21} + Z_{22} + Z_{23}) \times\\
&\qquad (A_1C_3 - A_3^2) + (Z_{31} + Z_{32} + Z_{33})(A_2A_3 - A_1B_3)] +\\
&(D_1 + D_2 + D_3)[K_1(B_3A_3 - A_2C_3) + K_2(A_1C_3 - A_3^2) +\\
&\qquad K_3(A_2A_3 - A_1B_3)] +\\
&(Z_{11}Z_{22} - Z_{12}^2)(K_3D_3 + K_4C_3) +\\
&(Z_{22}Z_{33} - Z_{23}^2)(K_1D_1 + K_4A_1) +\\
&(Z_{33}Z_{11} - Z_{31}^2)[K_2D_2 + K_4(Z_{22} - A_2 - B_3)] +\\
&(Z_{12}Z_{23} - Z_{31}Z_{22})(K_3D_1 + K_1D_3 + 2K_4A_3) +\\
&(Z_{31}Z_{12} - Z_{23}Z_{11})[K_2D_3 + K_3D_2 + K_4(Z_{23} + B_3 - A_3 - C_3)] +\\
&(Z_{23}Z_{31} - Z_{12}Z_{33})[K_1D_2 + K_2D_1 + K_4(Z_{21} + A_2 - A_1 - A_3)]
\end{aligned} \tag{11}$$

$$\begin{aligned}
\Delta I_c = {}&\\
&(K_3Z_{22} - K_2Z_{32})[D_3A_1 + D_1(A_1 + A_2)] - (K_3Z_{12} - K_1Z_{32}) \times\\
&\quad [D_3A_2 + D_1(A_2 + B_2)] + (K_2Z_{12} - K_1Z_{22})[D_3A_3 + D_1(B_3 + A_3)] -\\
&(K_3Z_{21} - K_2Z_{31})[D_3A_2 + D_2(A_1 + A_2)] + (K_3Z_{11} - K_1Z_{31}) \times\\
&\quad [D_3B_2 + D_2(A_2 + B_2)] - (K_2Z_{11} - K_1Z_{21})[D_3B_3 + D_2(B_3 + A_3)] +\\
&(K_3Z_{23} - K_2Z_{33})(D_1A_2 - D_2A_1) - (K_3Z_{13} - K_1Z_{33})(D_1B_2 - D_2A_2) +\\
&\qquad (K_2Z_{13} - K_1Z_{23})(D_1B_3 - D_2A_3) +\\
&K_4[(Z_{11} + Z_{12} + Z_{13})(A_2B_3 - A_3B_2) + (Z_{21} + Z_{22} + Z_{23}) \times\\
&\qquad (A_2A_3 - A_1B_3) + (Z_{31} + Z_{32} + Z_{33})(A_1B_2 - A_2^2)] +\\
&(D_1 + D_2 + D_3)[K_1(A_2B_3 - A_3B_2) + K_2(A_2A_3 - A_1B_3) +\\
&\qquad K_3(A_1B_2 - A_2^2)] +\\
&(Z_{11}Z_{22} - Z_{12}^2)[K_3D_3 + K_4(Z_{33} - A_3 - B_3)] +\\
&(Z_{22}Z_{33} - Z_{23}^2)(K_1D_1 + K_4A_1) +\\
&(Z_{33}Z_{11} - Z_{31}^2)(K_2D_2 + K_4B_2) +\\
&(Z_{12}Z_{23} - Z_{31}Z_{22})[K_3D_1 + K_1D_3 + K_4(Z_{31} + A_3 - A_1 - A_2)] +\\
&(Z_{31}Z_{12} - Z_{23}Z_{11})[K_2D_3 + K_3D_2 + K_4(Z_{32} + B_3 - A_2 - B_2)] +\\
&(Z_{23}Z_{31} - Z_{13}Z_{33})(K_1D_2 + K_2D_1 + 2K_4A_2)
\end{aligned} \tag{12}$$

$$
\begin{aligned}
&\Delta I_1 = \\
&(A_1 + A_2 + A_3)[K_2(D_2Z_{33} - D_3Z_{23}) + K_3(D_3Z_{22} - D_2Z_{32}) + \\
&\qquad K_4(Z_{22}Z_{33} - Z_{32}^2)] + \\
&(A_2 + B_2 + B_3)[K_3(D_2Z_{31} - D_3Z_{21}) + K_1(D_3Z_{23} - D_2Z_{33}) + \\
&\qquad K_4(Z_{23}Z_{31} - Z_{21}Z_{33})] + \\
&(A_3 + B_3 + C_3)[K_1(D_2Z_{32} - D_3Z_{22}) + K_2(D_3Z_{21} - D_2Z_{31}) + \\
&\qquad K_4(Z_{21}Z_{32} - Z_{22}Z_{31})] + \\
&(A_1B_2 - A_2^2)(K_3D_3 + K_4Z_{33}) - \\
&(B_2C_3 - B_3^2)[K_1(D_2 + D_3) + K_4(Z_{12} + Z_{13} - A_1)] + \\
&(C_3A_1 - A_3^2)(K_2D_2 + K_4Z_{22}) - \\
&(A_1B_3 - A_2A_3)(K_2D_3 + K_3D_2 + 2K_4Z_{23}) + \\
&(A_2C_3 - A_3B_3)[K_2(D_2 + D_3) - K_1D_2 + K_4(-Z_{21} + Z_{22} + Z_{23} - A_2)] + \\
&(A_3B_2 - A_2B_3)[K_3(D_2 + D_3) - K_1D_3 + K_4(-Z_{31} + Z_{32} + Z_{33} - A_3)]
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&\Delta I_2 = \\
&(A_1 + A_2 + A_3)[K_2(D_3Z_{13} - D_1Z_{33}) + K_3(D_1Z_{32} - D_3Z_{12}) + \\
&\qquad K_4(Z_{13}Z_{32} - Z_{12}Z_{33})] + \\
&(A_2 + B_2 + B_3)[K_3(D_3Z_{11} - D_1Z_{31}) + K_1(D_1Z_{33} - D_3Z_{13}) + \\
&\qquad K_4(Z_{11}Z_{33} - Z_{13}^2)] + \\
&(A_3 + B_3 + C_3)[K_1(D_3Z_{12} - D_1Z_{32}) + K_2(D_1Z_{31} - D_3Z_{11}) + \\
&\qquad K_4(Z_{12}Z_{31} - Z_{11}Z_{32})] + \\
&(A_1B_2 - A_2^2)(K_3D_3 + K_4Z_{33}) + \\
&(B_2C_2 - B_3^2)(K_1D_1 + K_4Z_{11}) - \\
&(C_3A_1 - A_3^2)[K_2(D_1 + D_3) + K_4(Z_{12} + Z_{23} - B_2)] + \\
&(A_1B_3 - A_2A_3)[K_3(D_1 + D_3) - K_2D_3 + K_4(Z_{31} - Z_{32} + Z_{33} - B_3)] + \\
&(A_2C_3 - A_3B_3)[K_1(D_1 + D_3) - K_2D_1 + K_4(Z_{11} - Z_{12} + Z_{13} - A_2)] - \\
&(A_3B_2 - A_2B_3)(K_3D_1 + K_1D_3 + 2K_4Z_{31})
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&\Delta I_3 = \\
&(A_1 + A_2 + A_3)[K_2(D_1Z_{23} - D_2Z_{13}) + K_3(D_2Z_{12} - D_1Z_{22}) + \\
&\qquad K_4(Z_{12}Z_{23} - Z_{13}Z_{22})] + \\
&(A_2 + B_2 + B_3)[K_3(D_1Z_{21} - D_2Z_{11}) + K_1(D_2Z_{13} - D_1Z_{23}) + \\
&\qquad K_4(Z_{13}Z_{21} - Z_{11}Z_{23})] + \\
&(A_3 + B_3 + C_3)[K_1(D_1Z_{22} - D_2Z_{12}) + K_2(D_2Z_{11} - D_1Z_{21}) + \\
&\qquad K_4(Z_{11}Z_{22} - Z_{12}^2)] - \\
&(A_1B_2 - A_2^2)[K_3(D_1 + D_2) + K_4(Z_{31} + Z_{32} - C_3)] + \\
&(B_2C_3 - B_3^2)(K_1D_1 + K_4Z_{11}) + \\
&(C_3A_1 - A_3^2)(K_2D_2 + K_4Z_{22}) + \\
&(A_1B_3 - A_2A_3)[K_2(D_1 + D_2) - K_3D_2 + K_4(Z_{21} + Z_{22} - Z_{23} - B_3)] - \\
&(A_2C_3 - A_3B_3)(K_2D_1 + K_1D_2 + 2K_4Z_{12}) + \\
&(A_3B_2 - A_2B_3)[K_1(D_1 + D_2) - K_3D_1 + K_4(Z_{11} + Z_{12} - Z_{13} - A_3)]
\end{aligned} \tag{15}
$$

In another paper[2] there was developed a theorem concerning the circularity of loci in a general network. This theorem is as follows: In a network consisting of any number of linear and bilateral self- and mutual-impedance elements connected in any manner with constant sinusoidal electromotive forces of like frequency connected in any arms, all currents and voltages existing in the network follow circular loci when any one self-impedance is varied along a straight line in the complex plane.

This means that if any numerator and the denominator be expanded term by term it will be found that all self-impedances occur only to the first power. Thus if any self-impedance be chosen as a variable, the terms of the denominator Δ may be collected into 2 parts; γ the summation of terms free of the variable, and δ the summation of the terms involving the variable. The denominator then becomes $\Delta = \gamma + \delta\rho$ in which ρ is the scalar variable ranging from zero to infinity as the variable self-impedance varies from short circuit to open circuit.

Similarly, the numerator of any current such as ΔI_a may be separated into 2 groups of terms, the first (α) being the summation of terms free of the variable and the second (β) being the summation of terms multiplied by the variable. The numerator then becomes $\Delta I_a = \alpha + \beta\rho$. The current I_a is then

$$I_a = \frac{\alpha + \beta\rho}{\gamma + \delta\rho}$$

which follows a circular locus as ρ varies.

The general theorem of circularity of loci also states that if any self-impedance be varied, all voltages follow circular loci. This condition may likewise be ascertained by inspection of the various current equations. Examination of any current numerator, term by term, reveals that it is free of all impedance elements through which the current flows. The product of any current and impedances to obtain the difference of potential between 2 points then remains linear with respect to the variable self-impedance.

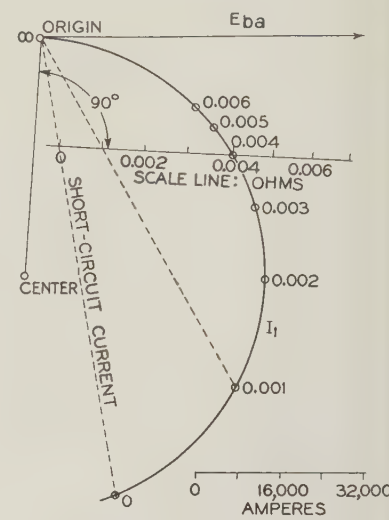


Fig. 3. Locus of phase current I_1 of figure 2

For example, referring to figure 1, the voltage V_{21} a cross-phase 12 of the load would be

$$V_{21} = E_{21} + I_1Z_{11} + I_2Z_{12} + I_3Z_{13} = \frac{\Delta \times E_{21} + \Delta I_1Z_{11} + \Delta I_2Z_{12} + \Delta I_3Z_{13}}{\Delta}$$

The constant portion of the numerator may be added as α' and the remaining terms involving the variable as β' giving

$$V_{21} = \frac{\alpha' + \beta'\rho}{\gamma + \delta\rho}$$

which is a circular locus.

It should be remembered that the variation of the self-impedance must not entail variation of any of the mutual impedances, since the equations are not linear with respect to mutual impedances. The loci are determined most readily by the 3 point method, that is, evaluating the desired quantity at the 2 invariant points, $\rho = 0$ and $\rho = \infty$, and at some third convenient value of ρ. It is necessary to consider only the current equations in analytical form. After the necessary current equations have been set up in numerical form as linear fractional transfor-

mations, all voltage loci may be computed directly by operating on the currents with impedances.

Because all the currents and voltages in the system may be expressed as linear fractional transformations having a common denominator, it is evident that any summation of currents or voltages, in which each of the currents or voltages is multiplied by a constant, will result in another linear fractional transformation. Thus the loci of all symmetrical components of currents and voltages are circular.

The current equations developed for the delta-delta circuit are long. This is necessarily so in view of the completely general character of the circuit assumed, that is, unbalanced internal voltages and impedances including mutual elements in both source and load phases as well as unbalanced elements in the interconnecting lines. In practice no problem will be likely to arise which involves so many unbalanced conditions. However conditions are frequently encountered in which certain sections of an otherwise balanced system may be heavily unbalanced. Even inequality of interconnecting lines must be dealt with as in the case of busses and flexible leads to furnaces. With the general equations at hand, substitution of the balanced section of the problem brings about considerable simplification, and absence of mutual elements which may not exist or which may be neglected causes many terms to vanish.

Numerical Example

The circuit given in figure 2 will be used to show how the loci are determined for an actual case. The internal voltages and impedances at the source will be assumed to be balanced. The source phases will be considered to be free of mutual elements. The connecting lines will be considered unequal with respect to reactance components. A passive resistance load, of which 2 phases are equal and the remaining phase is variable, is connected to the busses. The loci of the current in the variable load phase Z_{11} and the voltage across it will be determined as R_{11} varies from short circuit to open circuit. The constants of the circuit are as follows:

$Z_{aa} = Z_{bb} = Z_{cc} = (0.404 + j2.02)10^{-3}$ ohms

$Z_{ab} = Z_{bc} = Z_{ca} = 0$ $Z_{11} = R_{11} + j\,0$ ohms

$Z_{rr} = (0.080 + j1.131)10^{-3}$ ohms $Z_{22} = (17.65 + j0)10^{-3}$ ohms

$Z_{ss} = (0.080 + j0.980)10^{-3}$ ohms $Z_{33} = (17.65 + j0)10^{-3}$ ohms

$Z_{tt} = (0.080 + j0.754)10^{-3}$ ohms $E_{ba} = 230\angle 0°$ volts

$Z_{rs} = (+j0.452)10^{-3}$ ohms $E_{cb} = 230\angle -120°$ volts

$Z_{st} = (+j0.415)10^{-3}$ ohms $E_{ac} = 230\angle 120°$ volts

$Z_{tr} = (+j0.528)10^{-3}$ ohms $E_{21} = E_{32} = E_{13} = 0$

Substituting these values in equations 7, the group constants become

$A_1 = (0.564 + j3.23)10^{-3}$ $A_2 = (-0.080 - j0.641)10^{-3}$

$B_2 = (0.564 + j2.92)10^{-3}$ $B_3 = (-0.080 - j0.263)10^{-3}$

$C_3 = (0.564 + j2.85)10^{-3}$ $A_3 = (-0.080 - j0.566)10^{-3}$

$D_1 = R_{11} + j0$ $D_2 = D_3 = (17.65 + j0)10^{-3}$

$K_1 = 230 + j0$ $K_2 = -115 - j199.2$

$K_3 = -115 + j199.2$ $K_4 = 0$

Substituting in equation 9 the denominator becomes

$$\Delta = (-5{,}140 + j1{,}010)10^{-12} + R_{11}(-92.4 + j2{,}050)10^{-9}$$

Substituting in equation 13, the numerator of I_1 is

$$\Delta I_1 = (466)10^{-6} \quad \angle 88.6°$$

The phase current I_1 written as a function of the variable resistance R_{11} is then

$$I_1 = \frac{(466)10^{-6}\angle 88.6°}{(-5{,}140 + j1{,}010)10^{-12} + R_{11}(-92.4 + j2{,}050)10^{-9}}$$

Evaluating the invariant points and using for the third point the value $R_{11} = 10^{-3}$ ohms, the corresponding currents are:

R_{11} Ohms	I_1 Amperes
0	89,000 $\angle -80.3°$
10^{-3}	76,900 $\angle -61.1°$
∞	0

The locus of I_1 is drawn in figure 3.

With no mutual impedance or internal voltages in Z_{11} the voltage V_{21} across this phase is I_1R_{11} which is

$$V_{21} = \frac{R_{11}(466)10^{-6}\angle 88.6°}{(-5{,}140 + j1{,}010)10^{-12} + R_{11}(-92.4 + j2{,}050)10^{-9}}$$

The same 3 points are computed as before, giving

R_{11} Ohms	V_{21} Volts
0	0
10^{-3}	76.9 $\angle -61.1°$
∞	227 $\angle -4.0°$

The locus of the voltage V_{21} is plotted in figure 4.

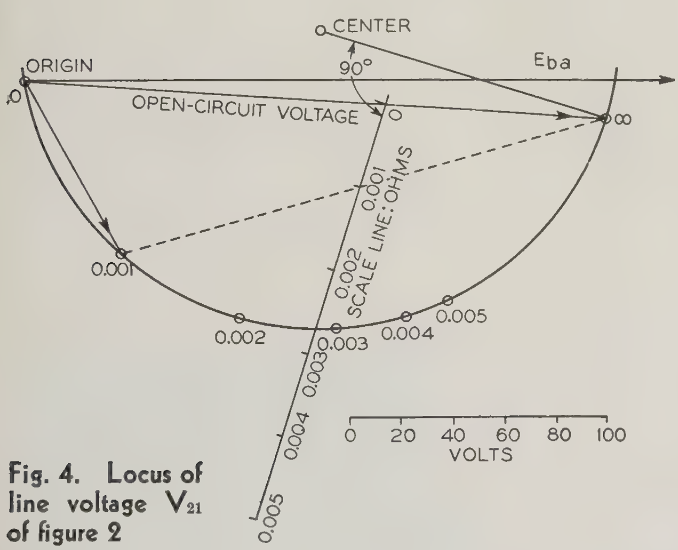


Fig. 4. Locus of line voltage V_{21} of figure 2

References

1. Current and Voltage Loci in 3-Phase Y-Y Circuits, A. C. Seletzky. Elec. Engg. (A.I.E.E. Trans.), v. 54, Sept. 1935, p. 970–6.

2. Circular Loci of Currents and Voltages in a General Network, A. C. Seletzky. To be published in the Franklin Institute *Jl.*

Some Applications of Instrument Transformers

Methods and apparatus are described in this paper for the measurement of amperage, voltage, and wattage, and for the calibration of meters and instrument transformers, by means of multirange precision transformers of special design. These methods are said to obviate the use of the multiplicity of primary standards commonly used in standardizing laboratories and to reduce the work of maintaining substandards in calibration. The methods and apparatus described have been used extensively in the laboratory and field by a large western power company; one of the principal benefits has been a noticeable improvement in the average accuracy of watt-hour meters on the system.

By
OTTO A. KNOPP
MEMBER A.I.E.E.

Pacific Gas and Elec. Co.,
San Francisco, Calif.

IN THE PAST 25 years considerable advance has been made in the methods of testing and calibrating meters and instrument transformers. During this time, the writer has disclosed, in the form of articles and patent papers, various testing and calibrating methods. The purpose of this paper is to consolidate the material pertaining to all those methods and apparatus which are founded on the same basic principle, manifested in the multirange precision instrument transformer, and also to add new material covering recent adaptations and applications.

An instrument transformer usually is understood to be a device for transforming an alternating voltage or current from a higher to a lower value; but in the methods described herein, the term "instrument transformer" has a broader meaning. It may be used not only for alternating current, but also for direct current and for stepping up as well as for stepping down. The first printed description of the d-c application appeared in 1914.[1] A more detailed description was published in 1915.[2]

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Feb. 25, 1936; released for publication March 28, 1936.

1. For all numbered references, see list at end of paper.

Both types of instrument transformers, for alternating as well as direct current, have 2 windings, a primary and a secondary, wound on a magnetic core. The current flowing through the secondary winding tends to neutralize the flux produced in the core by the primary current. Under this condition the ratio between primary and secondary currents approaches a value equal to the reciprocal of the turn ratio when the flux in the core approaches zero.

The d-c instrument transformer differs from its a-c counterpart only in that the secondary current is not produced directly by induction but is caused to flow automatically from a separate power source.

In the earlier construction[1,2] (see figure 1), the control of the secondary current was obtained by the action of a relay in the air gap of the core. In later developments made in Germany (see figure 2), the control is obtained by a small motor driven armature revolving in the air gap of the core. The induced alternating current in the armature, transformed into direct current by commutation, flows through the secondary winding neutralizing the flux produced by the primary current.

This apparent complication is outweighed by the following principal advantages:

1. The power for operating the measuring instrument is not drawn from the primary circuit. Therefore, when a transformer of the through type is inserted into a circuit, the current flow therein is not affected.
2. The ratio is independent of the secondary burden.
3. The reluctance of the magnetic circuit does not affect the ratio.

As a result of the conviction that with this d-c instrument transformer a true current balance could be established magnetically, efforts were made to use this principle for the proportioning of currents in calibrating and measuring work. However, because of the realization of the greater importance of proportioning alternating currents, and because of the greater simplicity of the a-c instrument transformer, efforts later were concentrated on overcoming some of the inherent inaccuracies of the a-c instrument transformers, in order to adapt them to this work. This was done by providing a multiplicity of taps and operating the transformer on a fixed ampere turn value, and usually with zero burden, or at least at a predetermined fixed burden.

Fig. 1. Portable d-c split-core current transformer with indicating and recording ammeter; 3,000 ampere capacity

Systems of winding were perfected so that the ratio and phase angle errors for all ranges did not vary more than 1 part in 10,000. In order to detect such changes in the ratio and phase angle, the transformers were wound so that they could be calibrated in themselves, that is, each was made an absolute standard. After 4 years of application in the laboratory, this method was described in an article published in 1916.[3]

Among the many ranges of different ratios, one provided for a ratio of 5 to 5 amperes for current transformers, or 110 to 110 volts for potential transformers. By taking the vector difference between primary and secondary, the ratio and phase angle errors of the transformer can be determined accurately by most simple methods and inexpensive equipment. In the equipment described in the article mentioned in reference 3, use was made of 2 watt-hour meters. In later years other equipment was built for deriving the results more quickly and more accurately. Description of this improved so-called "'one to one' standardizing method" will follow in a later section of this paper. A "detector wattmeter" used in this method was described in 1927[4] and also will be discussed in a later section of this paper.

By 1917 developments in the design of the instrument transformers had progressed sufficiently to permit the application of the apparatus for the proportioning of alternating currents, voltages, and wattages. The first so-called "calibrating transformer" and calibrating method[9,10] were described in an article published in 1920.[5] In this article it was shown that by applying (in case of a current transformer) a known current of, say, 5 amperes to the 5 ampere primary range of this transformer and maintaining the secondary current constant while changing the primary taps, one can determine or proportion the primary currents accurately in inverse ratio to the number of primary turns. It was pointed out that this principle is similar to that of the potentiometer where one applies a known voltage (standard cell) and maintains a constant so-called working current, which is determined by balancing a standard cell across a section of the working circuit, and thus, by tapping various known points of the working circuit, determines or establishes other values of potential.

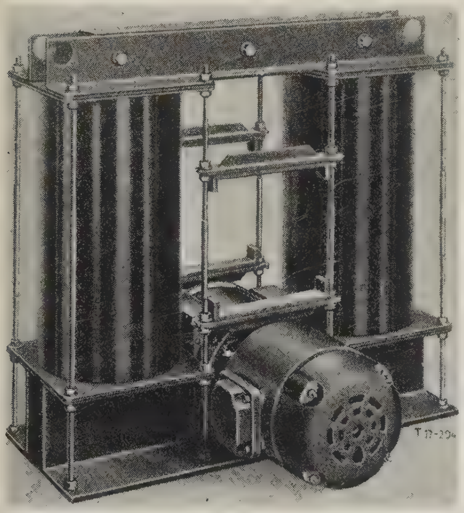


Fig. 2. Stationary d-c current transformer of the busbar type, with 10,000 and 20,000 ampere ranges

The similarity of the principles is shown diagrammatically in figure 3. In (*a*) the balance is detected by means of a galvanometer, zero current indicating correctness of values. In (*b*) a balance is shown when zero flux exists in the core around which the windings are wound. In (*c*) a balance is established automatically through the transformation process, in which the secondary current rises to a value that reduces the flux to the amount just sufficient to circulate the current through the secondary circuit. If the impedance of the secondary circuit were reduced to zero, the flux in the core also would approach zero. It is apparent, therefore, that in schemes (*b*) and (*c*), the various turns replace the resistances in scheme (*a*). Therefore, schemes (*b*) and (*c*) rightfully might be called "turn potentiometers."

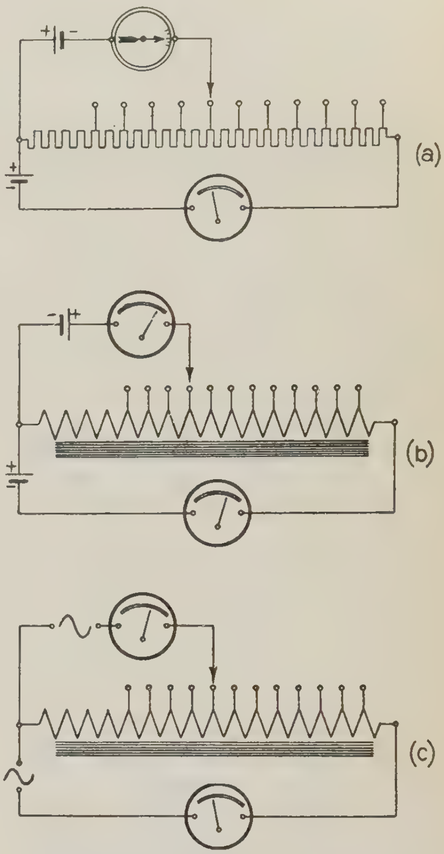


Fig. 3. Diagrams illustrating similarity of principle of calibrating transformer and potentiometer

(*a*)—Potentiometer
(*b*)—D-c magnetic current balance
(*c*)—A-c magnetic current balance or calibrating transformer

In weighing the relative values of the 2 systems, it can be reasoned that a turn is a more stable quantity than a resistance. The criterion for the value of the scheme is therefore the accuracy with which turn values can be established. This means accuracy not only of the number of turns, but also of the effective equality of all the turns of the primary relative to the secondary, or uniformity in coupling. This fact had been realized from the very beginning, and since 1920 considerable advance has been made in the art of turn adjustment[11] so as to widen the practical value of the turn potentiometer.

In this paper turn potentiometers are discussed as applied to alternating current, figure 3(*c*). These constitute ideal multirange measuring transformers for general laboratory work: calibrating of ammeters, wattmeters, watt-hour meters, and voltmeters, as well as current and potential transformers. Such multirange measuring transformers are absolute standards which can be calibrated in themselves by means of the "one to one" method. Details of these transformers are given in a later section under the heading "Precision Calibrating Transformer." By permanently connecting such a transformer of small size to the current coil of an ammeter, a very useful and accurate multirange ammeter was developed which is described in another section. Another precision transformer,[12] for large currents, which also can be calibrated in itself and which constitutes an absolute standard without a special 5 ampere range, is described in the section headed "Precision Multirange Current Transformer Standard for Large Currents." For precision measurement on power lines where a transformer cannot be inserted, a special split-core precision multirange transformer for high potential lines was developed embodying many of the foregoing features; this unit also is described in this paper. In another section the "Precision Multirange Potential Transformer" is described. Further useful applications of these multirange measuring transformers have resulted by employing a method of interconnecting 2 multirange transformers of the same ratings, one acting as a loading transformer for obtaining any desired predetermined current or potential from a given constant current, or potential value, and the other acting to transform these back, with inversed ratio, to a constant current

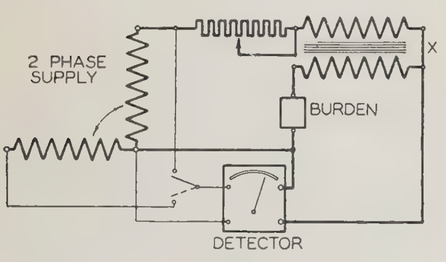


Fig. 4. "One to one" method for calibrating current transformers; X denotes transformer under test

or potential. This method is described under the heading "Uniload System."[13] The uniload system has found a number of very useful applications which are described in the latter part of the paper.

"One to One" Method for the Absolute Calibration of Instrument Transformers

Figure 4 shows the most convenient system for applying the "one to one" method to current transformers. By connecting the primary and secondary as shown, the difference between primary and secondary current is applied to the current coil of a so-called "detector wattmeter." The potential coil is excited with a potential that is in phase with the current when the ratio error is measured and in quadrature when the phase angle is measured.

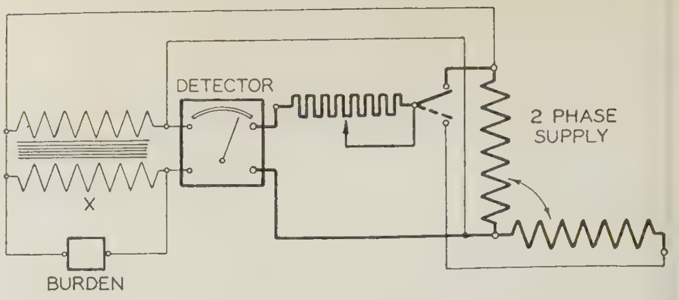


Fig. 5. "One to one" method for calibrating potential transformers; X denotes transformer under test

$$\text{Ratio error in per cent} = \frac{100 \times \text{detector wattage}}{\text{Secondary current} \times \text{wattmeter potential}}$$

$$\text{Phase angle error in minutes} = \frac{\text{Detector wattage} \times 3{,}438}{\text{Secondary current} \times \text{wattmeter potential}}$$

The constant 3,438 in the second equation is the factor with which the tangent of a small angle has to be multiplied to obtain the minutes in close approximation.

Figure 5 shows the system applied to potential transformers. The difference between primary and secondary potential is applied to the potential coil of the wattmeter; the current coil is excited in phase with the potential for ratio error determination, and in quadrature for phase angle error determination.

$$\text{Ratio error per cent} = \frac{100 \times \text{detector wattage}}{\text{secondary potential} \times \text{wattmeter current}}$$

$$\text{Phase angle error in minutes} = \frac{\text{detector wattage} \times 3{,}438}{\text{secondary potential} \times \text{wattmeter current}}$$

Detector Wattmeter

The detector wattmeter is used not only for the absolute calibration of multirange instrument transformers, but also for comparing other instrument transformers against a standard, using the Silsbee[14] and Brooks[15] deflection methods for current and potential transformers, respectively, shown in figures 6 and 7. This wattmeter will give accurate results, whereas the ordinary indicating wattmeter will not. The inaccuracy of the latter is caused by the mutual inductance between current and potential coils changing constantly as the pointer deflects over the scale and is zero only in one position, that is, when the 2 coil systems are at right angles to each other. When a wattmeter is used in the foregoing methods, it must be realized that the electromotive force in the current circuit is very small, particularly when current transformers of low ampere turn value are tested at zero burden. The electromotive force generated by induction in the current coil of the wattmeter from the potential or moving coil frequently may amount to an appreciable part of the first mentioned electromotive force, and, in exceptional cases, may exceed it, introducing large errors. These errors may be avoided by keeping the moving coil in a position of zero mutual inductance.

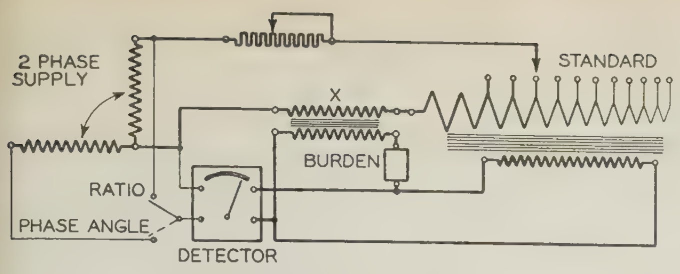


Fig. 6. Silsbee[14] deflection method for calibrating current transformers for ratio and phase angle with a multirange standard; X denotes transformer under test

This can be accomplished in a standard indicating wattmeter by using a torsion head attached to the zero adjuster and an additional pointer. The original indicating pointer is arranged so that in its zero position the moving coil is at right angles to the current coil system. By means of the torsion head the indicating pointer is kept at zero, and the torsion head pointer will indicate on a proportional scale the true wattage without the foregoing errors.

It is important to keep the resistance of both circuits of the wattmeter as low as possible and at the same time to provide high sensitivity. In practice, it was found possible in using standard high-grade wattmeters for making up the detector, to obtain sensitivity sufficient to determine the ratios within 0.02 per cent and phase angles within 30 seconds. With special portable suspension type of instruments, this can be increased materially. Figure 8 pictures a satisfactory type of detector wattmeter made from a standard low-power-factor wattmeter. The detector is best built with zero in the center of the scale and then scaled directly in per cent ratio error and in phase angle, so that deflection of the pointer to the right represents high secondary output or leading phase angle, and to the left, low output and lagging angle.

With an instrument transformer having zero error for ratio and phase angle, the detector will read zero, and when this transformer is used as a standard in the connections shown in figures 6 and 7, the detector will read directly the ratio and phase angle error of the transformer under test without any computation, if in all these cases the excitation current or potential for which the scale values were determined, are applied.

A watt scale in addition to the other scales makes the detector wattmeter very useful as an indicating wattmeter for general laboratory work.

Precision Calibrating Transformer

Design of the precision calibrating transformer, in principle, is applicable to both current and potential, but is most useful for calibrating work involving currents. The transformer designed for current work consists of a precision multirange current transformer having an inter-range accuracy of the order of 0.01 per cent and primary ratings for the various ranges that are proportional to the current values necessary for calibrating ammeters and wattmeters on the 10 or 15 calibrating points. Figure 9 shows the internal connections, and figure 10 a picture of such a transformer built to cover the calibrating range from 0.5 to 200 amperes.

Each of the primary taps or ranges as well as the secondary taps is connected to a dial switch for making quick changes in connections. For the particular transformer shown in figures 9 and 10, an 1,800 ampere turn value is used. Each section from the common to any one tap is wound with an even number of turns and in such a manner that the coupling with the secondary is exactly the same, as mentioned in the beginning of this paper.

In order to obtain the same secondary current, namely 5 amperes, when the rated primary currents are being sent through the taps with the rating 70–130–140 amperes and the taps with the rating 80–110–160 amperes, using the nearest even turn value as marked in the diagram instead of the theoretical fractional turn value, the following successful expedient has been used: Small series boosting transformers are employed in each of these taps which are just large enough to raise or lower the current in the particular range to bring the ampere turn value to exactly 1,800. If the situation is analyzed, it is found that when using the nearest even turn value, the first mentioned group (the 70–130–140 ampere taps) has the ampere turn value of 1,820; the second group (80–110–160 ampere taps) has an ampere turn value of 1,760. In the first group of taps, the current must be lowered somewhat more than 1 per cent; in the second group it must be boosted a little more than 2 per cent. In order to assure an accuracy of 0.01 per cent for the corrected range, the boosting transformers need be accurate only within 0.5 per cent. This very easily can be accomplished without any special adjustments.

The secondary of each boosting transformer is connected permanently with one terminal only. The other terminal is connected by a special contact finger at the time the corresponding primary range is

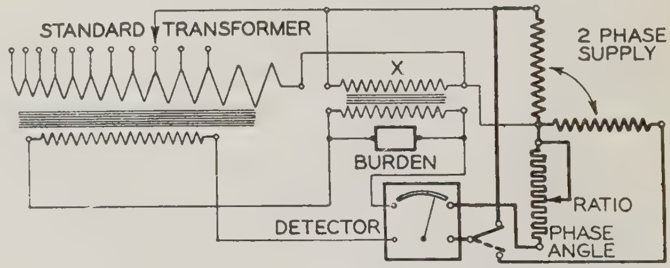


Fig. 7 (above). Brooks[15] deflection method for calibrating potential transformers for ratio and phase angle with a multirange standard; X denotes transformer under test

Fig. 8. Detector wattmeter

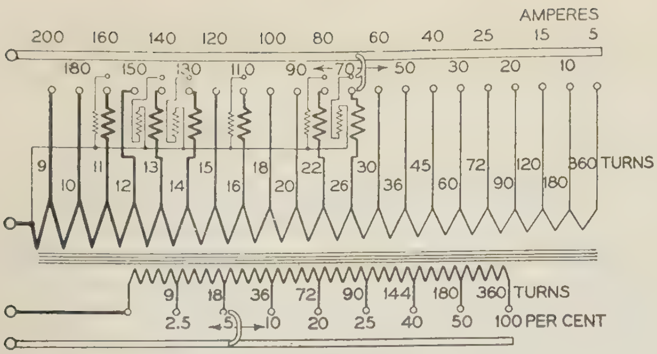


Fig. 9 (above). Internal connections of precision calibrating transformer

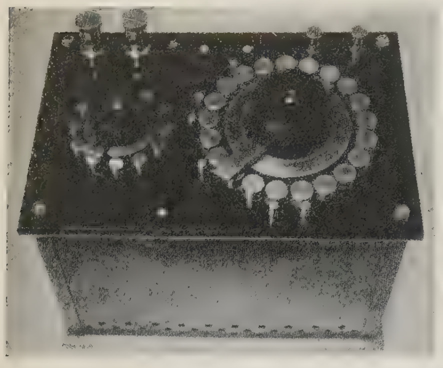


Fig. 10. Calibrating transformer

connected by the dial switch, as shown in the diagram.

The following simple procedure is used when calibrating, for instance, a 150-ampere iron-vane-ammeter: (Most instruments of this type cannot be calibrated accurately with direct current.) The main dial switch is set on the 5 ampere range and a current of 5 amperes, as measured with an accurate standard, is sent through the 5 ampere primary winding. The secondary dial switch is set on the 100 per cent tap and the secondary current of approximately 5 amperes is sent through a reliable 5 ampere ammeter with good repeating accuracy. By means of its zero adjuster, the reading may be brought exactly to the 5 ampere mark. After that, the 150 ampere ammeter can be switched in in place of the standard after which the main dial switch is turned to the 10, 20, 30, 40 . . . . 150 taps and the current is varied in each case so that the secondary ammeter reads the same each time.

A small capacity ammeter, *e. g.*, of 15 ampere rating, is calibrated in a similar manner except that the standard current of 5 amperes is sent through the 50 ampere range of the transformer instead of the 5 ampere range, and the secondary dial switch is set on the 10 per cent tap, thus giving each tap of the transformer a rating of $^1/_{10}$ of its normal value. In this simple manner any ammeter of 5 to 200 ampere full scale rating can be calibrated with great accuracy with only one known 5 ampere alternating current value and without a variety of equipment requiring in itself considerable maintenance and frequent calibration.

The same transformer can be used for calibrating wattmeters of 5 to 200 ampere current rating, merely by substituting for the standard primary and secondary ammeter, a standard primary wattmeter and secondary wattmeter, and applying to both, as well as to the wattmeter under test, a potential common to all and in phase with the current.

If in the foregoing method, watt-hour meters are substituted for the wattmeters, a very rapid and most accurate method is available by which a single observer may determine the characteristics of a watt-hour meter from the smallest load to heavy overloads with only one known reference load value and without the tedious maintaining of exact wattage readings, the constant counting of revolutions, and the careful timing. Figure 11 gives a diagram of connections. The first test is made using the reference standard at full load, counting the revolutions of the watt-hour meter under test and taking the revolutions registered by the auxiliary watt-hour meter. For the remainder of the test the reference standard can be dispensed with.

One point of the characteristic curve is fixed by the reading of the reference standard. The relative location of the other points of the curve now can be determined by letting the 2 meters run for a short time on each selected load point, making sure, after changing the primary dial switch to the corresponding position, that the current is regulated so that the auxiliary ammeter reads approximately the same as before and that the auxiliary watt-hour meter is started and stopped when a given point on the disk of the meter under test passes a fixed observation point.

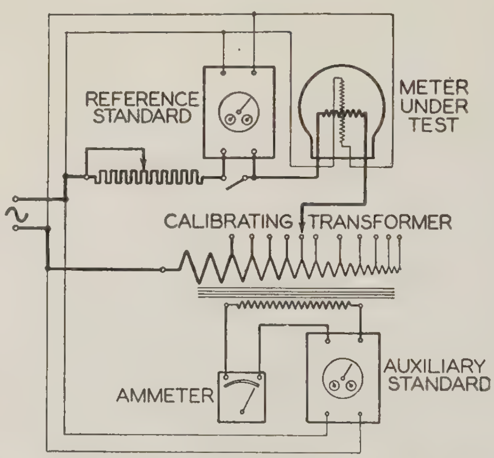
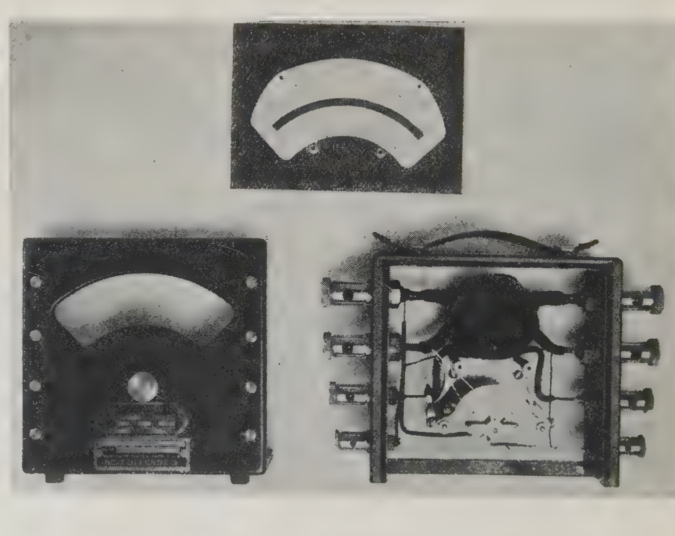
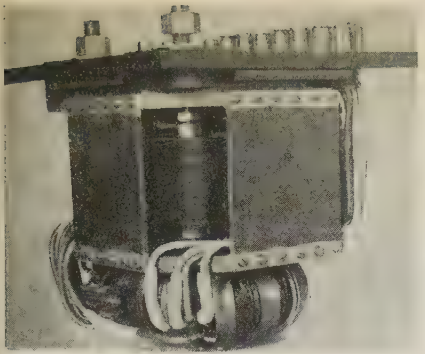
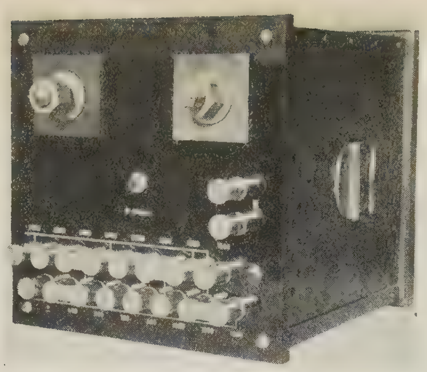


Fig. 11 (right). Method of determining the characteristic curve of a watt-hour meter by means of a calibrating transformer

Fig. 12 (below). Multi-range ammeter







Figs. 13 and 14. Precision multirange current transformer standard (left) without case and (right) with case

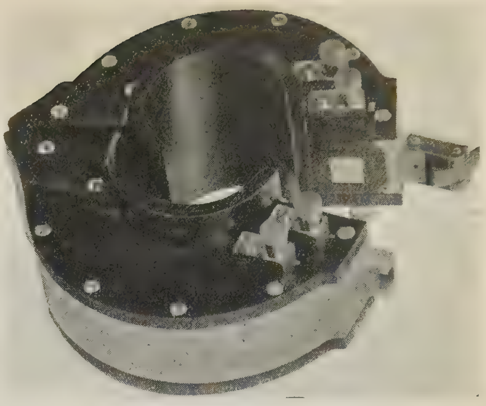


Fig. 15. Precision split-core multirange current transformer

In order to reduce calculations to a minimum, the auxiliary standard is best stopped at a time when the original reference reading is approximately reached, in which case the differences between the original reference reading and each of the successive readings divided by the reference reading give the accuracy variations of the meter under test relative to the reference point.

If by mechanical or electrical adjustment the reference reading of the auxiliary standard is made 10, the readings for each load point divided by 10 represent the correction factor of the meter under test. The various characteristic points therefore can be plotted directly as each reading is taken.

This calibrating transformer, having many commercially used current transformer ratios, such as 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40 to 1, can be calibrated in itself by the "one to one" method and used as a precision standard in a manner already described. If it is used with an ammeter that is especially calibrated with it in the secondary, the combination constitutes a precision multirange ammeter.

Multirange Ammeter

The last mentioned thought resulted in the development of 2 convenient portable multirange ammeters for general testing purpose, having the following ranges: 0.05, 0.1, 0.2, 0.5, 1, 2 amperes, and 2, 5, 10, 20, 50, 100, 200 amperes. The latter one is shown in figure 12. A small ring-core single-coil current transformer is mounted under the scale plate of the ammeter. A section of this winding is tapped across the current coil of the ammeter. This current coil has a very low impedance, having a special magnetic shield placed around it which increases the flux in the coil so that fewer ampere turns are required. At the same time, this shield has the purpose of reducing the frequency error of the ammeter. With increasing frequency, the transformer output into the current coil of the ammeter increases, but because of the shield the flux in the coil for a given current decreases. By proper design, it has been possible to produce an ammeter for power frequencies of 50 to 60 cycles which has negligible errors at triple frequency. Therefore, it will measure quite accurately on very distorted wave shapes. The current transformer can be wound without extreme care so that the inter-range accuracy does not vary more than 0.1 per cent.

Precision Current Transformer Standards for Large Currents

Precision current transformers for large currents are intended for use only as standards at zero burden; they have a single turn primary winding of special design which is used for all ranges. One transformer of this type is described here (see figures 13 and 14); it has the following ranges: 600, 800, 1,000, 1,200, 1,500, 1,800, 2,000, 2,400, 2,500, and 3,000 amperes. The 600-turn 5-ampere secondary is divided into 5 sections of 120 turns each, having taps at suitable places (one tap for each range) so that with the preceding primary current values, 5 amperes is obtained in the secondary.

The absolute calibration is accomplished with the "one to one" method and the detector wattmeter without a special 5 ampere calibrating winding. One section of 120 turns is disconnected from the other sections and is used as the secondary winding. The other 4 sections are connected in parallel and are used as the primary. Currents from zero to 25 amperes are used to calibrate the transformers up to 3,000 ampere turns. When using the regular 5 ampere detector wattmeter, the sensibility of the "one to one" method is increased fivefold without introducing errors due to the impedance in the detector, which easily was proved by adding impedance in the detector circuit without affecting the reading.

In order that the ratio and phase angle errors for the different ampere turn values obtained with the "one to one" method may be exactly the same as those obtained for the different taps, the following 2 rules are used in winding: (1) The resistance of the various windings from the common terminal to any one tap should be made proportional to the square of the number of turns. (2) The couplings of all the sections, or of the portions of winding between any successive taps, with the primary should be made exactly alike.

In respect to the first rule, it easily can be seen that with 25 amperes flowing through a 120 turn secondary (equivalent to 3,000 ampere turns) the internal burden will be the same as when 5 amperes

is flowing through a winding having 5 times the number of turns, producing the effect of 3,000 ampere turns, if this winding has 5^2 times the resistance (since the I^2R value is in one case $25^2 \times R$ and in the other case $5^2 \times 5^2R$, which are equal). For equal internal I^2R loss in the 2 cases, the flux in the core must be the same. Let the current in the secondary winding be I_1 and I_2, the voltage E_1 and E_2, the resistance R_1 and R_2, the turns n_1 and n_2, the frequency f_1 and f_2, and the flux ϕ_1 and ϕ_2. The internal burden then can be expressed as:

$$I_1{}^2R_1 = I_1E_1 \text{ and } I_2{}^2R_2 = I_2E_2$$

and since

$$E_1 = cf_1\phi_1n_1 \text{ and } e_2 = cf_2\phi_2n_2$$

it follows that

$$I_1{}^2R_1 = I_1cf_1\phi_1n_1 \text{ and } I_2{}^2R_2 = I_2cf_2\phi_2n_2$$

Since

$$I_1{}^2R_1 = I_2{}^2R_2$$

it follows that

$$I_1cf_1\phi_1n_1 = I_2cf_2\phi_2n_2$$

and since it is assumed that

$$I_1n_1 = I_2n_2$$

the final result is:

$$f_1\phi_1 = f_2\phi_2$$

Therefore, if the frequency is the same in both tests, as it usually is, the same flux is present in each test. This means that the exciting component of the primary current which causes the error is constant; hence the same error exists in both cases. All this is true if the second rule be fulfilled. In the particular transformer under discussion, this is accomplished (see figure 13) by winding all windings and each section thereof, including the single turn primary, in the form of a toroid uniformly distributed about a ring core.

Careful tests, using the sensitive "one to one" method and testing sections of equal numbers of turns against each other, assure exactly uniform coupling. The "one to one" method using 25 amperes gives a sensitivity of 1 part in 100,000.

Another feature of the construction is the loading transformer, which is an integral part of the whole unit whose secondary is the primary of the standard transformer. Thus a very compact equipment (see figure 14) has been developed, embodying in one unit an absolute multirange precision standard and a loading transformer, for the convenient calibration of commercial current transformers.

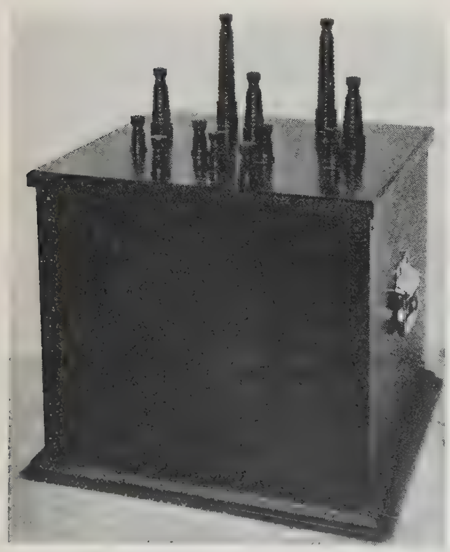


Fig. 16. Precision multirange potential transformer

Precision Split-Core Multirange Current Transformer

The problem of making so-called "over-all" tests on high voltage power circuits to check the accuracy of existing metering equipment without long service interruptions has confronted electrical utilities for many years. The available clamp-on or split-core transformers have been applicable only for current measurement on moderately high voltage circuits and not for power measurement on high voltage circuits, because of their large phase angle errors and the difficulty of insulating the core.

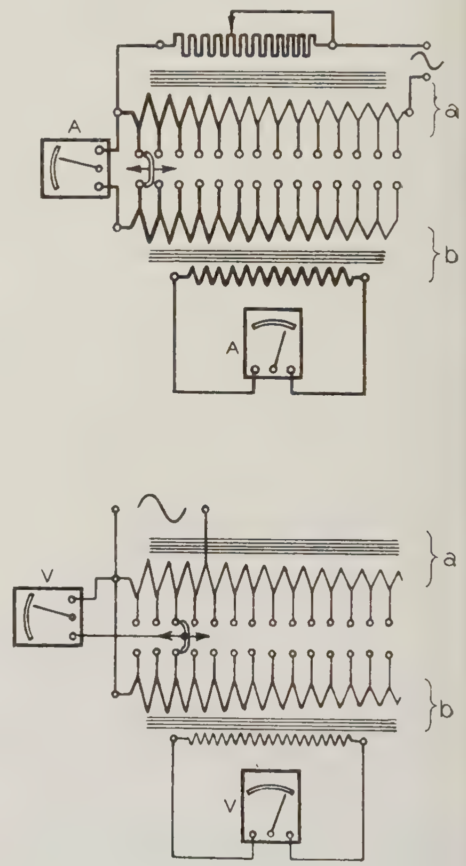


Fig. 17. "Uniload" system for current (upper diagram) and for potential (lower diagram)

a—Autotransformer
b—Measuring transformer
A—Ammeter
V—Voltmeter

In overcoming this deficiency, a split core transformer has been designed, shown in figure 15, which can be slipped safely over an 11,000 volt line, and accurate power measurements can be made by connecting a rotating standard test meter to the secondary. The slopes of the ratio and phase angle curves

are only 0.05 per cent and 4 minutes, respectively, between 10 per cent and 100 per cent load for the 800 and 1,000 ampere ranges.

By slipping into this core a properly shaped and wound coil which has numerous taps, this transformer can be used as a precision laboratory standard and can be calibrated in itself for ratio and phase angle, as explained in the early part of this paper. Some 30 ranges can be provided, ranging from 5 amperes to 1,000 amperes, by using a primary coil of 200 turns, and the accuracy mentioned is obtained.

The core surfaces which contact each other are ground and are self-aligning as a result of a special design of the hinge and clamping device, assuring a magnetic core reluctance almost as low as that of the uncut core.

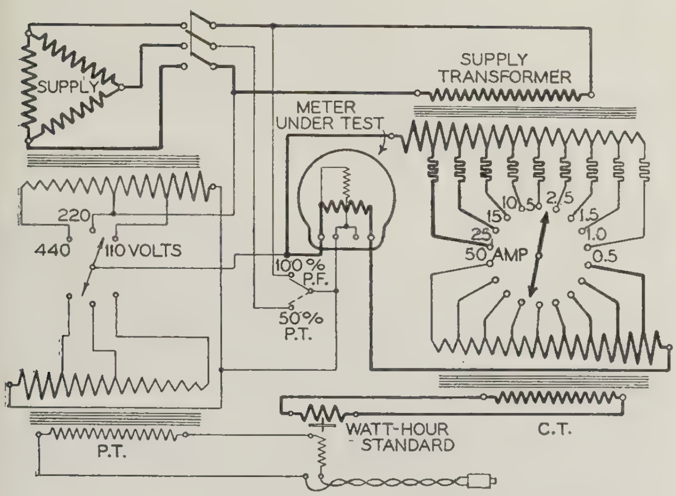


Fig. 18. Diagram of watt-hour meter calibrating equipment

P. T.—Multirange precision potential transformer
C. T.—Multirange precision current transformer
P. F.—Power factor

Precision Multirange Potential Transformer

Accurate potential transformers have been built which have the same ratio and phase angle errors for all ranges and which can be calibrated in themselves, 110 to 110 volts, as described earlier in this paper. This accuracy is obtained by adhering to the same design features embodied in the precision multirange current transformer, namely: (1) the uniform coupling of each section, between taps, with the secondary; and (2) the dimensioning of the resistances from the common terminal to any one tap so that they are proportional to the square of the number of turns.

Figure 16 presents a typical potential transformer having the following ranges: 110, 220, 440, 2,200, 4,400, 6,600, 11,000, 13,200 to 110 volts.

"Uniload" System

The "uniload" system is shown diagrammatically in figure 17. It readily can be seen that from a definite predetermined current or potential applied to the multirange supply transformers, any selected multiple of this current or potential can be obtained within the range of the equipment, for testing and calibrating of ammeters, voltmeters, wattmeters, watt-hour meters, and current and potential transformers; this selected current or potential can be reduced in each case to a definite single current or potential in the secondary of the precision multirange measuring transformer. A single switching operation will suffice to obtain the desired current or potential for the apparatus under test and to connect at the same time the corresponding range of the measuring transformer.

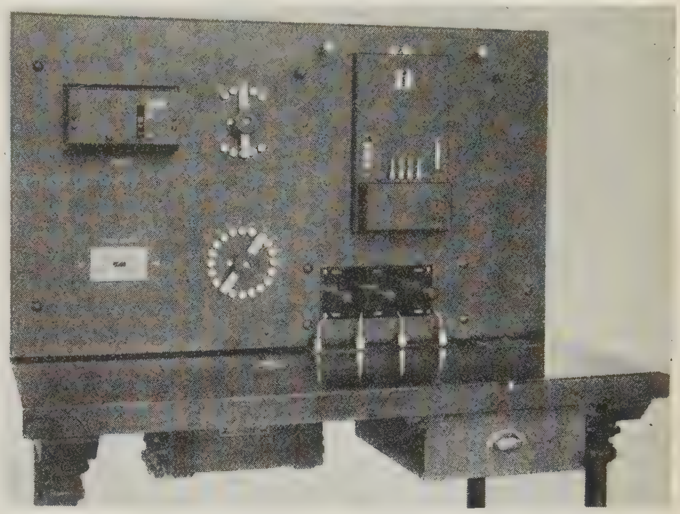


Fig. 19. Watt-hour meter calibrating equipment

Watt-Hour Meter Calibrating Equipment

The "uniload" system has been applied very successfully for the calibrating of watt-hour meters of all types and sizes. Figure 18 shows the diagram of connections of one type of watt-hour meter calibrating equipment, and figure 19 a picture. A great many of these equipments have been in service for more than 10 years and have proved the great value of the following features, resulting in an unusual stability of the calibration of the standard:

1. The standard never can receive more than the nominal 5 amperes or 120 volts irrespective of which switch is closed by the tester, as the "uniload" system is used for both current and potential.

2. Calculation for meter accuracy is practically eliminated, since in most cases by suitable count of revolutions the error of the meter is read directly off the dial of the standard.

3. No corrections for the error of the standard at different points of its characteristic need be applied since the standard operates only on one load point (100 per cent). The error at this point, including the error of the current and potential transformers is taken care of in the adjustment of the standard at this 100 per cent load point by comparing it against an accurate wattmeter inserted in place of the meter under test, while switches are set for 5 amperes and 120 volts.

Equipments of this kind have been built which cover in one unit all the commercial ranges, some 90 in all, from 0.5 to 500 amperes at 110, 220, and

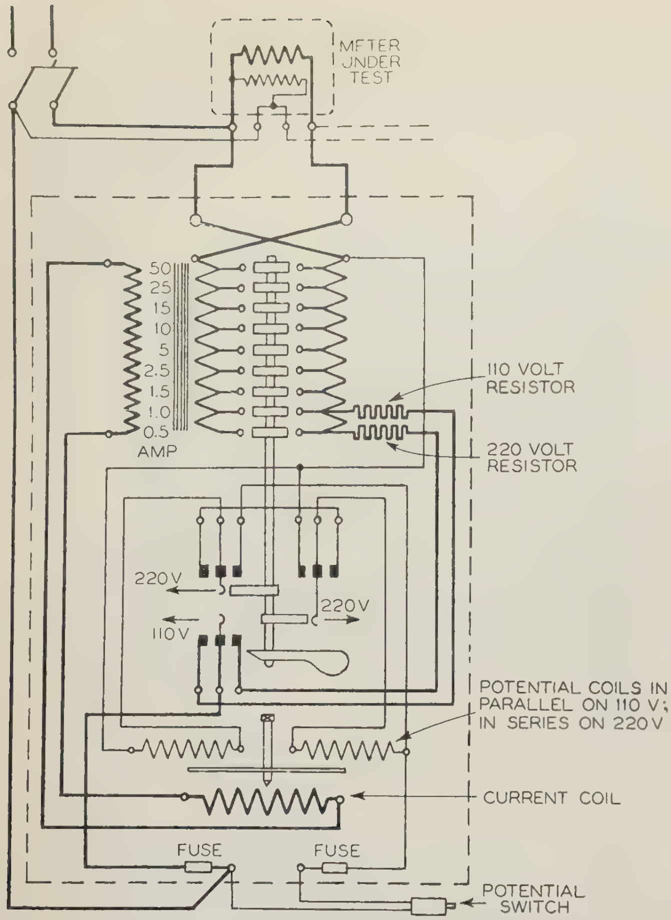


Fig. 20. Diagram of portable rotating standard for calibrating watt-hour meters in the field

440 volts. Without using undue care in the construction of this equipment, the calibration of all the ranges can be made accurate within 0.1 per cent.

Portable Rotating Standard

The foregoing watt-hour meter testing equipment has been built also into portable form to obtain the advantages mentioned for field testing. Figure 20 shows a diagram of this portable standard which is self-explanatory, and figure 21 a picture.

Current Transformer Calibrating Equipment

Figure 22 shows the application of the "uniload" system to the calibration of current transformers, and figure 23 a picture of a complete equipment. Here the secondary of the precision multirange current transformer instead of being connected to a measuring instrument is connected in series with the secondary of the transformer under test and to the current coil of a detector wattmeter. The throwing of a single switch selects the proper current tap of the supply transformer and at the same time the proper range of the multirange precision current transformer standard. A single rheostat R_c permits the current to be regulated from, say, 10 to 100 per cent of full load for each range whether it is of 5 or 500 ampere capacity. Ratio and phase angle errors of the transformer under test can be determined and the multirange standard can be calibrated in itself as described in early sections of this paper. A rheostat R_p is provided to regulate the potential applied to the detector to an exact 100 or 200 volts. In order to reduce the number of range switches to a minimum, 4 secondary turn taps are used, which, with 5 amperes flowing, give 1,200, 1,500, 1,600, and 2,000 ampere turns. Four corresponding turn taps in the primary winding permit the calibration of the multirange precision current transformer by means of the "one to one" method resulting in 4 calibration curves, A, B, C, D, respectively. The following 21 current ranges can be obtained with 9 switches (the figure represents the amperage and the letter the accuracy curve applying): 5D–10C, 12.5D–15A, 20C, 25D–30A, 40C, 50D–60A, 75B, 80C–100A, 125B–150B, 160C–200A, 250B–300A, 400C, 500D.

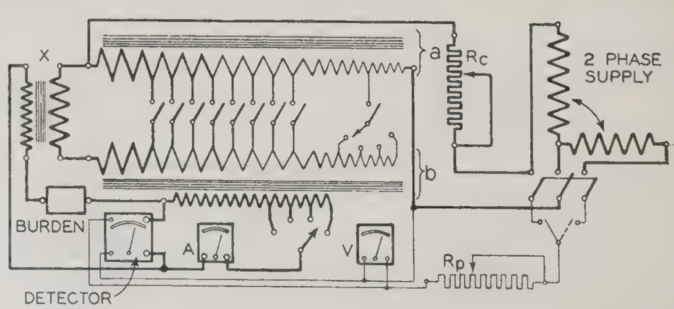
Potential Transformer Calibrating Equipment

Figure 24 represents the application of the "uniload" system to the calibration of potential transformers. The secondary windings of the precision multirange potential transformer and transformer under test are connected in opposition and the difference in potential is applied to the detector wattmeter. The operation of this system is otherwise similar to that described in the preceding section. The equipment pictured in figure 25 is built for calibrating transformers of the following ratios: 110, 220, 440, 1,100, 2,200, 4,400, 6,600, 11,000, 13,200 volts to 110 volts. Only low potential is brought to the panel. The high voltage connection is made

Fig. 21. Portable rotating standard

Fig. 22 (below). Diagram of current transformer calibrating equipment

a—Supply transformer
b—Measuring transformer
R_c—Current control rheostat
R_p—Potential control rheostat
A—Ammeter
V—Voltmeter
X—Transformer under test



with transformers disconnected, with the cabinet door open, and the supply circuit cannot be energized until the doors are closed.

The methods and apparatus described in this paper for standardization, calibration, and measurements have been used entensively in the laboratory and the field by a large electric utility, having nearly a million watt-hour meters connected to its system. The benefits have been manifold:

1. The average meter accuracy has improved noticeably in later years after a universal application of the principles all over the system.

2. The disagreement between the calibration of substandards in different parts of this large system has almost disappeared.

3. The cost of maintaining all substandards and meters in calibration has been reduced to a very marked degree.

4. There has been a beneficial psychological effect upon the tester. He has acquired a greater confidence in the results of his work since he is able to verify at any time, the accuracy of all the ranges of his testing equipment, by a check on the one-to-one range.

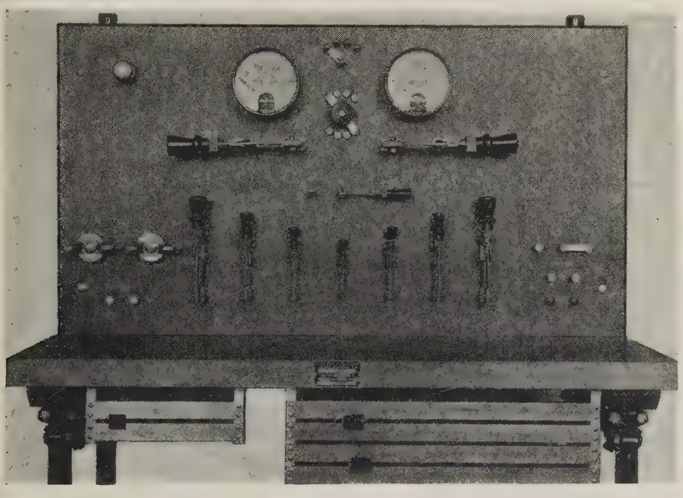


Fig. 23. Current transformer calibrating equipment

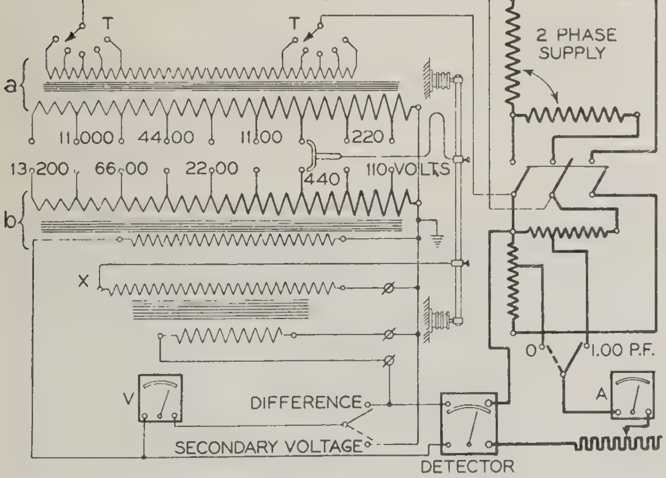


Fig. 24. Diagram of potential transformer calibrating equipment

a—Potential supply transformer
b—Measuring transformer
R_c—Current control rheostat
X—Transformer under test
A—Ammeter
V—Voltmeter
P. F.—Power factor
T—Regulating taps

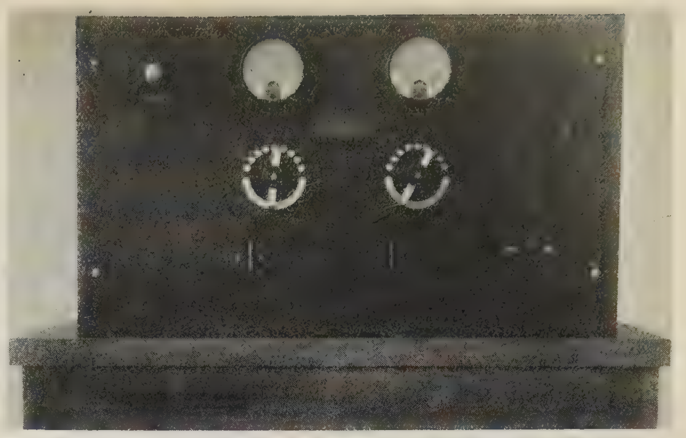


Fig. 25. Potential transformer calibrating equipment

It was thought that publishing these simple methods, even if only in brief form, might help other engineers confronted with the same problem to obtain similar improvements and economies on their systems and later lead to further developments along the lines indicated.

Future Developments

The application of the principles embodied in the described methods can be extended further along the following lines:

1. Measurements of small amperages, voltages, and thereby wattages, by stepping up to values of 5 amperes and 110 volts.

2. Development of standards for 5 amperes and for 110 volts.

3. Further development of the multirange potential transformer for higher potential, possibly by cascading and calibrating each step by means of the "one to one" method.

4. Elimination of the effects of capacitance currents at the higher voltages upon the ratio and phase angle of multirange potential transformers.

5. Development of auxiliary apparatus in order to use multirange transformers to better advantage.

References

1. Metering Direct Current Power With Current Transformers, O. A. Knopp. *Jl. of Electricity, Power and Gas*, v. 32, April 18, 1914, p. 334–5.

2. Measuring the Current in D-C Circuits, O. A. Knopp. *Elec. World*, v. 66, Oct. 2, 1915, p. 751–2.

3. The Commercial Standardization of Instrument Transformers, O. A. Knopp. *Elec. World*, v. 67, Jan. 8, 1916, p. 92–3.

4. Torsion Head Overcomes Wattmeter Error, O. A. Knopp. *Elec. World*, v. 89, Feb. 19, 1927, p. 407.

5. New Current Balance for Calibration Work, O. A. Knopp. *Elec. World*, v. 75, May 1, 1920, p. 991–6.

6. U. S. Patent No. 1,015,192, filing date Feb. 7, 1911.

7. German Patent No. 275,015, Klasse 21c, Gruppe 27.

8. U. S. Patent No. 1,171,213, filing date July 2, 1914.

9. U. S. Patent No. 1,372,821, filing date Nov. 20, 1916.

10. U. S. Patent No. 1,399,968, filing date March 2, 1917.

11. U. S. Patent No. 1,550,906, filing date May 1, 1922.

12. U. S. Patent No. 1,819,714, filing date Aug. 28, 1926.

13. U. S. Patent No. 1,587,841, filing date June 8, 1926.

14. A Method for Testing Current Transformers. Bureau of Standards *Scientific Paper No. 309*.

15. Testing Potential Transformers. Bureau of Standards *Scientific Paper No. 217*.

Cable Vibration—

Methods of Measurement

Test methods and equipment developed for the laboratory production in cables of vibrations (standing waves) similar to those experienced in transmission line service, and for determining the relative susceptibility of different cables to conditions tending to produce vibrations, are described. The special magnetic drive used to produce and sustain cable vibration has no friction loss except air friction, and hence is well suited to the measurement of the small amounts of power required to vibrate the cables.

By
JOSEPH S. CARROLL
ASSOCIATE A.I.E.E.
Stanford University, Calif.

JOHN A. KOONTZ, Jr.
MEMBER A.I.E.E.
Pacific Gas and Electric Co., San Francisco, Calif.

INCREASED use of conductors of large diameter on high voltage lines, and the desirability of obtaining a smooth surface to minimize the corona losses, have made vibration a factor of major importance in high voltage transmission line design.

The cable manufacturers, in attempting to obtain large diameter and light weight to meet corona and price requirements, have used a variety of methods. The transmission engineer now is faced with the problem of determining which design of cable is best suited to withstand vibration stresses. Under normal operating conditions in the field, one must wait for several years to obtain results. Hence, if some laboratory method of studying vibration could be developed it would be of great assistance. If cables could be vibrated (standing waves produced) under normal tension conditions and at loop lengths and amplitudes corresponding to those found in the field, a measure of the power required to cause such vibrations under such conditions should give a close approximation of the amount of power that the cable would pick up from the wind under actual operating conditions. Other things being equal, the cable requiring the greater amount of power to vibrate at a given frequency and amplitude would be the one best suited to withstand vibration in service, as the cable having the highest loss would vibrate at the least amplitude.

Tests on transmission lines have shown that practically all conductor vibration experienced in the field occurs in the vertical plane, and that the standing waves produced are sine wave in form.[1,2]

If a wire carrying an alternating current be placed in a magnetic field and the frequency and magnitude of the current properly controlled, we then produce the force required to vibrate the wire in a manner similar to the conditions frequently found in the conductors of a transmission line. Tests have been conducted abroad where cables were vibrated by passing an alternating current through the conductor, the conductor being placed between the poles of a magnet. However, because of the poor efficiency of the drive caused by losses in the conductor when carrying heavy currents,[3] this method was not well suited to the writers' problem.

The vibration motor finally developed for testing was what might be termed a modified radio loudspeaker consisting of a rectangular coil placed over the center tongue of the magnet frame. (See figure 1.) The coil was suspended rigidly below the cable and centrally located in a strong magnetic field. A coil so arranged will first be attracted and then repelled if an alternating current of the right instantaneous polarity is impressed. If the frequency of the alternating current corresponds to a resonant period of the cable, the cable will vibrate at an amplitude dependent upon the losses in the cable system and the magnitude of the applied force. The magnitude of the force can be controlled by limiting either the current flow or the magnetic field strength. In general, the magnetic field strength was held constant and the coil current varied.

Vacuum Tube Drive

In the early attempts to measure the power required to vibrate a cable, difficulty was experienced in obtaining results. The cables require such a small amount of power,[4] and this power must be supplied over a range of frequencies of from 3 to 30 cycles or

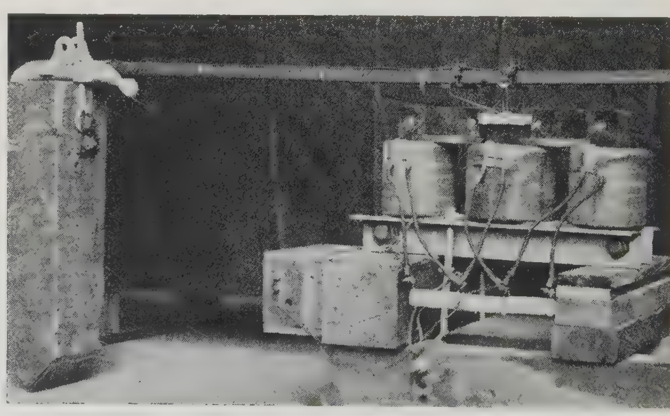


Fig. 1. Close-up view of experimental setup showing magnetic vibration motor near the end of a test span

A paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Feb. 24, 1936; released for publication March 12, 1936.

1. For all numbered references, see list at end of paper.

more per second, that vacuum tubes were considered to be best suited for this work. At the beginning of the tests the cables were vibrated and the losses measured, using vacuum tubes as the generators of the desired frequencies. If a second coil, similar to the drive coil, is used, any movement of this second coil in a magnetic field will generate a voltage of the desired frequency, which voltage if impressed on the grids of the vacuum tube would be amplified. The main drive coil is then connected to the output circuit of a vacuum tube amplifier, the grids of this amplifier being excited by impulses from the second coil or pickup coil.

A special push-pull circuit was worked out to improve the efficiency of this drive. The grid circuits of the vacuum tubes were connected to 2 separate coils mounted together in a single frame and located in the same magnetic field. For the purpose of this description, this double coil will be termed the pickup coil, and the other coil the drive coil. The pickup coil is free to move in the air gaps of an electromagnet identical with that of the electromagnet used for the driving coil. Any slight movement of this pickup coil will generate a voltage which will be impressed on the grids of the vacuum tubes. This voltage will be amplified and impressed on the drive coil in such a manner as to increase the original vibration. In practically all cases it was found that if the pickup coil was placed in a loop adjacent to the drive coil, and the leads to the pickup coil reversed, the conductor would be forced to vibrate at the required frequency.

The amplitude was controlled by varying the current in the field coil of the magnet of the grid pickup circuit. The ordinary radio push-pull circuits were not practical for this work because of the low frequencies required. As many as three tubes on each side were operated in parallel to obtain the required output. The diagram (figure 2) shows the circuit used and indicates a motor-generator set as a source of plate voltage. Any good d-c source would be satisfactory for this purpose.

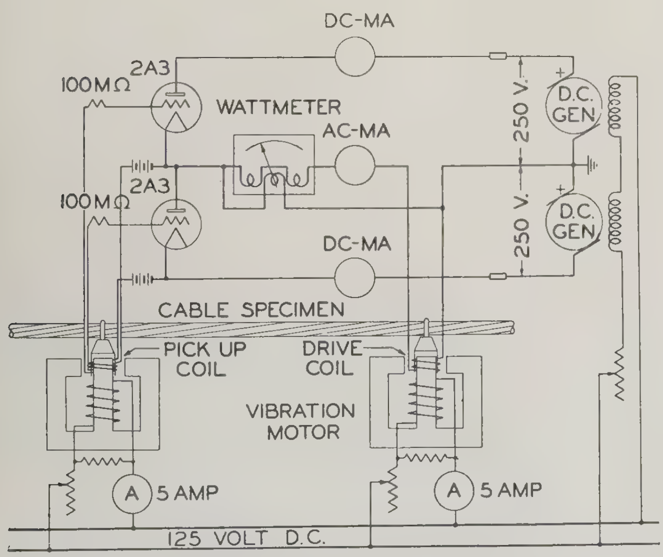


Fig. 2. Vibration motor circuit for vacuum tube drive

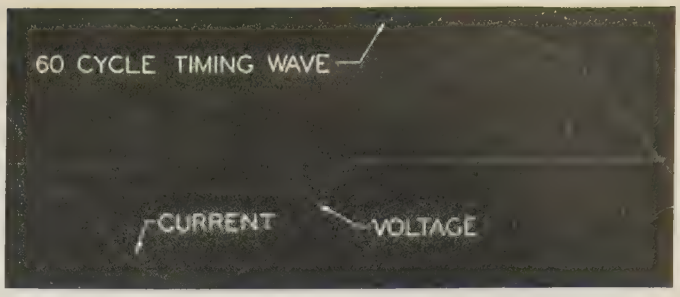


Fig. 3. Oscillographic record of current and voltage induced in the coil attached to the cable

A.C.S.R. expanded conductor, 1.4 inch (3.556 centimeter) diameter, 8,500 pounds tension, 167.4 foot length; 9 loops; amplitude of vibration, 5.15 centimeters; current, 0.25 amperes; voltage, 97.0

For power measurements it soon was found that the vacuum tube circuit would not deliver a sine wave of voltage to the drive coil. In effect, the vibration motor acted as a loud-speaker and, "figuratively speaking," rather faithfully produced all the tones and overtones that were impressed on it, with the result that the higher harmonics present in the vacuum tube voltage wave caused the cable to be driven at a complex rather than a simple wave.

Generator Drive

Inasmuch as losses in the cable were found to be quite different for different vibration frequencies, it was important that the cable be vibrated at a single frequency if the losses in different cables were to be compared successfully. A low frequency a-c generator having a good sine wave form was found to be a suitable source of power.

As a proof that the vibration motor was driving in the proper manner to give a sine wave motion to the cable, a special sine wave testing generator was operated and, from oscillographic records it was found that with a sine wave of voltage impressed on the motor coil, and vibrating the attached cable, the current in the coil was also sine wave in form. In other words, the counter electromotive force generated in the coil was sine wave in form, indicating a sine wave movement of the coil. (See figure 3.) The coil was rigidly attached to the cable, hence the cable was also moving with a sine wave motion.

The most difficult problem with a motor-generator as a source of power is speed control. The frequency of the vibrating conductor is very constant, whereas most methods used in driving a generator are subject to small variations in speed. In this problem of power measurements, it was necessary not only to hold the speed of the generator exactly at the desired speed, but also to hold the proper phase angle relation between the generator voltage and the counter electromotive force generated by the drive coil. This drive coil in the magnetic field has all the properties of the synchronous motor, it having a very definite V curve characteristic. Therefore, for best results the coil should be operated at unity power factor. Theoretically, this unity power factor condition could be held by use of a wattmeter and ammeter in the

circuit, but for the work in hand this was not satisfactory. Fortunately a 2 phase machine as indicated in figure 4 was being used. Phase 1 was used as a source of voltage for the coil driving the conductor. This current was passed through a second wattmeter. The potential circuit of wattmeter 2 was connected to phase 2. With this arrangement, wattmeter 1 measured the power input to the drive coil, while

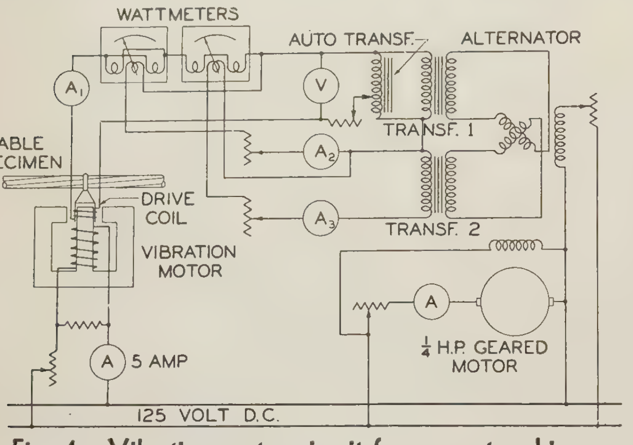


Fig. 4. Vibration motor circuit for generator drive

wattmeter 2 measured the reactive component of this power. By controlling the drive motor, the reactive component could be held at zero, providing unity power factor for the drive coil; also, with a center scale reactive component meter, it was possible to detect any tendency of the generator to lag or lead the cable, and make the necessary speed adjustment.

For small power measurements, a 2 phase alternator was used, driven through a reduction gear by a $^1/_4$ horsepower motor. The gear reduction was variable so that the speed of the driving motor could be held between 65 and 100 per cent of normal. One essential of a suitable drive is that of ample WR^2 in the generator. In the case under test this was obtained by a flywheel. With this arrangement, a frequency range of from 3 to 20 cycles per second was available. At these particular frequencies the wave form from this generator was a sine wave.

A good wave form was quite essential as it was desired to compare the power amplitude characteristics of various cables at definite frequencies, these frequencies covering the range shown by field tests to be present.

Weston 20 per cent power factor dynamometer wattmeters, with $^1/_{10}$ ampere current coils instead of the usual 5 ampere coils, were used for measuring the power. These instruments had a sensitivity 50 times that of the 5 ampere meter. The $^1/_{10}$ ampere current coil meter, when used with the 75 volt potential connection has a full scale reading of 1.5 watts; one division on the 150 division scale representing 0.01 watt. The $37^1/_2$ volt potential connection would give twice this sensitivity, 0.005 watt per division or 0.001 watt per $^2/_{10}$ division. The particular meter used had a special connection brought out directly from the moving coil of the potential circuit so that for voltages less than $37^1/_2$ an external resistance could be used, still further increasing the sensitivity of the meter and permitting power inputs of less than 0.001 watt to the driving coil to be measured with reasonable accuracy. Since the accuracy of such a dynamometer meter is very good with direct current when the average of 2 readings are taken with reversed polarity, there cannot be any question with respect to its accuracy when used on low frequency. Since the damping on these meters is very good, the vibration of the pointer at the low frequencies was not at all serious, even down to 3 cycles per second.

Typical amplitude loss curves for a 61 strand cable are shown in figure 5. The tests were made with the cable supported near the ends over standard line suspension clamps, these clamps being rigidly supported. The losses for this condition are high, most of the losses being in the end connections. However, this is fairly representative of conditions in the field, and probably will help to explain why the longer spans are more apt to give trouble from vibration.

For convenience of operation it soon was found that the driver could be placed near one end of the span, and the amplitude measuring equipment placed at the center of the span. Then, by taking measurements with the span vibrating in an odd number of loops, it was not necessary to move any of the equipment; the measuring device would always be located at an antenode.

Conclusions

1. The vacuum tube method of drive would be well suited for making life tests on cable. The equipment would require a minimum of attention and very small amounts of power. The frequency can easily be changed with minor adjustments.

2. The present motor drive is well adapted for making power measurements if a suitable variable speed sine wave power source is available. The efficiency of the motor drive can be made high over the operating range—above 90 per cent neglecting

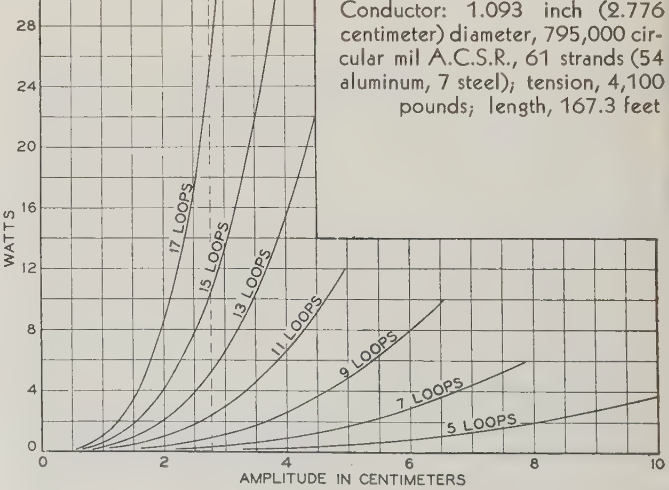


Fig. 5. Typical amplitude-loss curves showing relation of amplitude to driving power

the d-c field. The losses are mainly caused by the coil resistance; hence the necessary corrections can be made.

References

1. Notes on the Vibration of Transmission Line Conductors, Theodore Varney. A.I.E.E. Trans., v. 45, 1926, p. 791–9.

2. Resistance of Cable to Transverse Vibration, Especially Hollow Overhead Cable, K. Bechtold and H. Folkerts. *Elektrotechnik und Maschinenbau*, v. 47, July 14, 1929, p. 593–604.

3. *Siemens Review*, v. 8, 1932.

4. Vibration Transmission Line Conductors, Theodore Varney. A.I.E.E. Trans., v. 47, July 1928, p. 799–807.

5. Vibration in Electrical Conductors, Including Allied Researches. (An extensive bibliography prepared under the auspices of the A.I.E.E. power transmission and distribution committee for limited distribution; now out of print.)

Stray Load Loss Tests on Induction Machines—II

By
THEODORE H. MORGAN
MEMBER A.I.E.E.

VICTOR SIEGFRIED
ASSOCIATE A.I.E.E.

Both o
Worcester Polytechnic
Institute, Worcester, Mass.

A new test method is described for the determination of stray load loss in induction machines. By application to given machines, this method is shown to have an exceptionally high degree of accuracy. Values of stray load loss for the same machines are obtained by other recognized methods and results compared. Results support the validity of a previously described[1] "belted load-back method."

STRAY load loss now is recognized as a factor which must receive due consideration in the accurate determination of the efficiency of induction machines. Several types of tests for arriving at the value of this loss are in general use, but none as yet has achieved the goal of direct measurement under actual conditions of operation. The nearest approach to this ideal appears to be through the use of the loading-back method as applied to 2 machines operating from the same source of power. The principal advantage of this load-back test is that the sum of all losses is obtained by a direct measurement and not determined as the difference between total power input and output. Such a test method, illustrated by data on 2 machines, is described in a paper by Morgan and Narbutovskih.[1]

The objectives of the tests herein described are to confirm the validity of the belted load-back method and to compare it with other means of obtaining the stray load loss. To accomplish this result, a new test has been devised that isolates the total losses due to load and permits their direct measurement as a separate quantity. In this method, these load losses, which include only copper and stray load losses, are measured as the net input to 2 induction machines loaded back on each other through d-c machines directly coupled to them. The no-load losses of the induction machines are supplied continuously throughout the test as part of the input to the d-c machine furnishing the driving power, and are thus eliminated from measurements and calculations. Separation of the total load losses into copper and stray load loss components then depends only upon the accurate determination of resistance and slip.

Complete results of this d-c load-back test are given, as are results of tests for stray load loss made on the same machines by the belted load-back method[1] and by the d-c excitation method.[2] A comparison of efficiency determined from the separation of losses also is made with that from an input-output test. Close agreement between the losses determined by the new test and those of the belted load-back test is evidence that the latter is a fully satisfactory commercial test possessing inherent accuracy. The load-back test through d-c machines, although permitting even slightly higher accuracy, does have the disadvantage of being more complicated and requiring more equipment and a special technique.

The machines selected for the comparative tests for stray load loss were 2 identical squirrel cage induction motors rated at 10 horsepower, 550 volts, 10.3 amperes, 3 phase, 60 cycles, 1,750 rpm. The stators have 48 teeth and semi-closed slots, and the rotors 57 teeth and coffin shaped slots.

Summary

This investigation is concerned primarily with methods for the experimental determination of stray load loss in induction machines. While it is difficult

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 16, 1936; released for publication March 31, 1936.

1. For all numbered references, see list at end of paper.

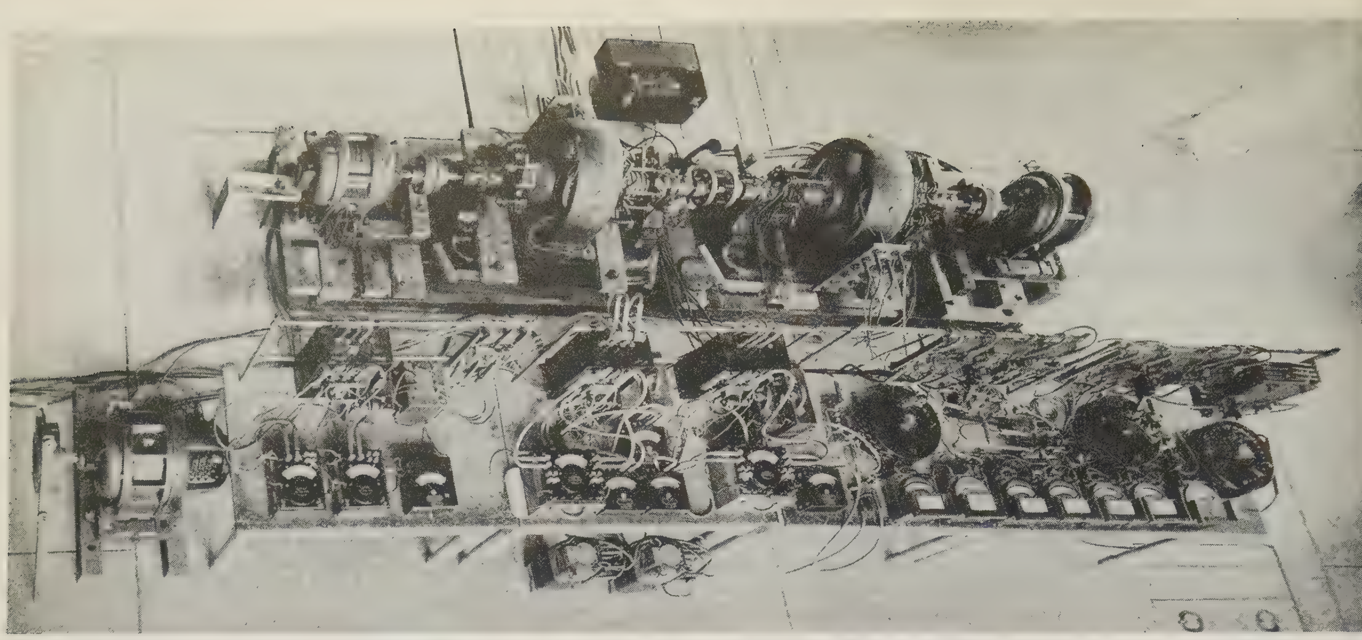


Fig. 1. Arrangement of machines and apparatus for d-c load-back test

to draw general conclusions from particular results, certain facts have been definitely established:

1. An accurate load-back test for stray load loss is developed in which the machine tested operates under actual load conditions. It is essentially a laboratory method in which stray load loss is measured more directly and to a higher degree of accuracy than by previous methods.

2. Stray load loss was determined by this load-back test, and 2 independent sets of measurements gave identical values of the loss. This loss value may be used as a basis of comparison for results of tests by other methods.

3. The belted load-back test is demonstrated to be inherently accurate for the determination of stray load loss. It has the additional advantage of being easy to perform.

4. The d-c excitation method, in the case of the machines investigated, gave a value for stray load loss which is sufficiently accurate for practical purposes. Future tests will show if this is generally the case.

5. The input-output method requires special technique in order to secure accurate values of stray load loss.

6. Stray load loss, while low in the machines tested, is in the order of magnitude of other losses and must receive consideration for accurate determination of losses and efficiency.

New D-C Load-Back Test Method

The general arrangement of the machines and apparatus for the d-c load-back test is shown in figure 1. The induction motors were direct connected through flexible couplings to 2 identical d-c machines of similar rating, all 4 machines being carefully aligned to eliminate coupling loss. An additional coupling also was provided between the d-c machines, and all couplings were arranged so that the machines could readily be disconnected. The wiring diagram of the machines and instruments for making this load-back test through d-c machines is shown in figure 2. Power was supplied to induction motors from a commercial source of good wave shape, with voltage regulators installed for maintaining constant balanced voltages. The d-c machines were separately excited, and power was supplied to their armatures from an independently operated generator.

Step A. The desired measurement is accomplished by an orderly sequence of machine operation, the initial step of which is to drive all machines slightly above synchronous speed with all 3 couplings locked. The induction machines are excited from the a-c supply at rated voltage, but the input power of the line wattmeter is made to read zero by adjustment of the d-c motor power. Also, during this operation, the d-c machines are loaded back upon each other to a predetermined value. Under these conditions, the d-c motor supplies the d-c generator output and the losses of all 4 machines, including the no-load iron and copper losses of the induction machines. The current in the a-c line is only the sum of the reactive magnetizing components of the no-load currents for the induction machines.

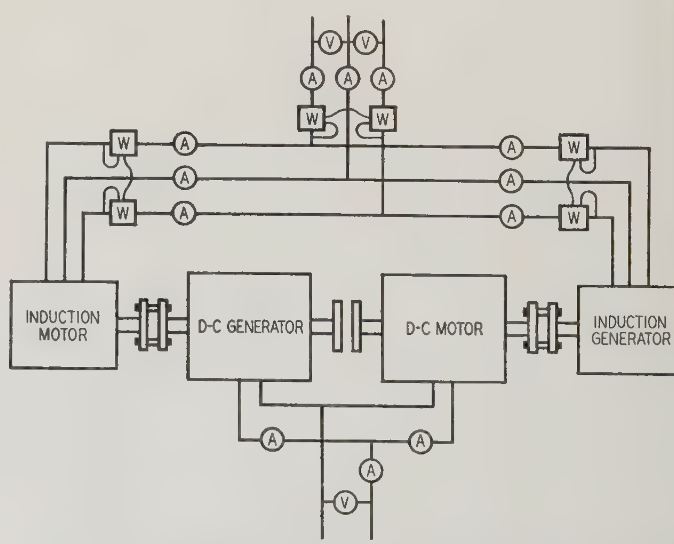


Fig. 2. Diagram of connections for load-back test through d-c machines

Step B. The second step in the test is to open the center coupling and thus to separate the machines into 2 mechanically independent motor-generator sets. The d-c machine armature currents again are adjusted to exactly the same values as previously used in step *A*. This is accomplished through a slight change in speed, causing one induction machine to operate as a generator and the other as a motor. The main difference in operation between this and the previous step is that now the induction machines are loaded. The total losses resulting from load on the 2 induction machines are now supplied as a separate quantity through the a-c line and measured by the line wattmeter. The only correction that must be made from the initial no-load adjustment of power is that required for the changes in rotation losses of all machines. These changes are small and tend to compensate each other in the 2 motor-generator sets.

Step C. A third step gives a separate measurement of the load losses by operating the combination of machines in such a manner that the additional input due to induction machine load is supplied by the d-c line and thus measured on the d-c input meters. This actually is a different method of obtaining the required quantity quite independently from step *B*, although similar in principle. The machines are loaded as in step *B*, the d-c generator armature current being maintained at the same value used in step *A*, but the d-c motor power is increased by an amount which brings the power input of the a-c line once again to zero. Thus the induction machine load losses are supplied by an increase in the total d-c power input. In arriving at the induction machine load losses in this case, corrections must be made for the change in the d-c motor load losses.

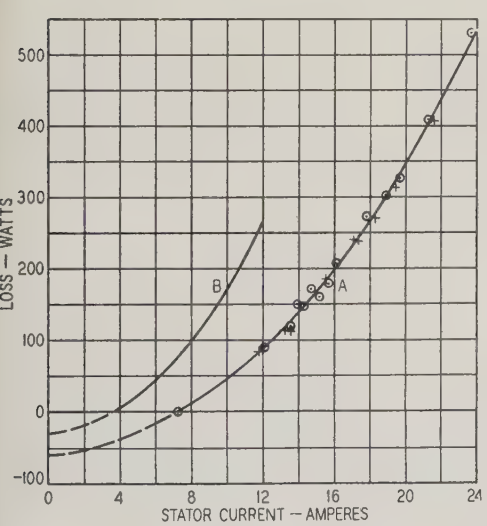


Fig. 3. Stray load losses as measured by load-back test through d-c machines

A—Stray load loss of both machines

B—Stray load loss of each machine

O—Loss supplied by alternating current

X—Loss supplied by direct current

A complete set of data for the stray load loss of each induction machine was obtained for this operating method as a separate and independent test.

Step D. Finally, step *A* is repeated, and any change in the no-load input is taken into consideration by averaging the readings of steps *A* and *D*. A change of more than a few watts is a distinct warning of some instability in the losses of the d-c machines.

Measurements

The integrity of this test, as carried out by steps *A* and *B*, depends primarily upon the wattmeter which previously had been calibrated, as were all instruments, against primary standards. Current coils of the wattmeters are placed in the line, and resistances are used for potential multipliers. Potential

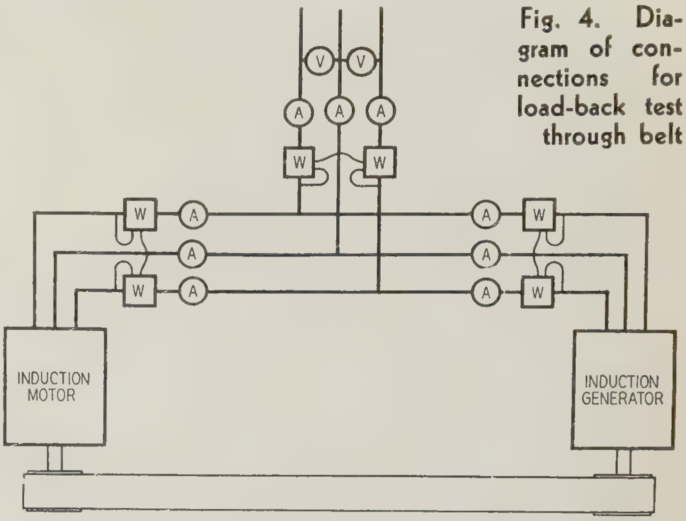


Fig. 4. Diagram of connections for load-back test through belt

circuit losses are supplied as shown by figure 2, and corrections for them would be necessary were it not that the initial adjustment for zero power on the a-c wattmeter in step *A* also includes this instrument loss with other no-load losses. The fact that this line wattmeter always is operating at very low power factor does not affect its accuracy, since only relative values of power are desired, and any power factor error will be compensated automatically in the initial zero power adjustment. This wattmeter normally is read when the machine instruments are not in the circuit. Any error in reading motor input and generator output is corrected by proportional adjustment of these values to give a difference equal to the net input of the system.

As a preliminary to the tests, the d-c machines were thoroughly calibrated for their conventional and stray load losses by a load-back test. The most variable factor in these losses was found to be the brush friction on the commutator. Control of this factor was accomplished through lubrication of the commutator with a thin film of paraffin, and instability of this loss was detected easily by fluctuations of d-c machine power and by a decided change in the commutator noise. The necessity for having complete information on the losses of the d-c machines arises from the fact that in step *C* of the load-back test, where the induction machine load losses are carried as additional input to the d-c motor, there is an increase in the power input and hence in the losses of that machine. The change in the losses must be taken into account in the calculation of the residual power assigned to induction machine

stray load loss. This fact does not seriously affect the accuracy of the test although it does increase the labor of calculation.

In the d-c load-back test, the stray load loss is obtained by a subtraction of copper losses from the change in losses between unloaded and loaded conditions. Standard test procedure requires d-c resistance of the stator to be used in determining stator copper loss, and this is measured by a d-c bridge placed at the junction of the machine leads with the a-c line. Rotor copper loss is taken as the product of slip and air-gap power. Slip is measured to a high degree of accuracy with the aid of the Edgerton stroboscope, and air-gap power is determined from the terminal power minus the stator copper and iron losses.

Results of D-C Load-Back Test

The results of stray load loss measurement by the d-c load-back method appear in figure 3, in which the sum of the stray load losses in the 2 machines is plotted as a function of the arithmetical sum of the stator currents. The experimental points from a-c measurements as determined from step *B* of the test procedure are shown by circles; those from d-c measurements from step *C* are indicated by crosses. These methods were found to be in such close agreement that they gave identical results, and only one curve could be drawn for the 2 sets of points. Thus, a single curve *A* is shown for both methods of measuring the stray load loss. A graphical separation[1] gives the stray load loss for each machine shown by curve *B*. Extension of curve *B* to zero current gives the amount by which it must be raised to make it pass through the origin.

Belted Load-Back Test

One of the purposes of this investigation being to establish further the validity of the belted load-back test,[1] a comparison was made of stray load loss determined for the same machines by the 2 load-back methods. The electrical connections to the induction machines (figure 4) are the same as for the d-c load-back test, and a-c measurements are of the same character. The loading of the machines is accomplished through a belt, the degree of loading being dependent upon the pulley ratio. This test method is only slightly less accurate than the d-c load-back method and is similar in nature, but it is quite different in principle from the test method in which the machine rotors are direct-connected and the stators excited from a-c sources of different frequency.[3,4]

The first measurement required is obtained by driving the belted combination from one machine with the other unexcited. The input under these conditions is that required for all no-load losses except the iron loss of the driven machine. When the second machine is excited, both become loaded, with the motor rotating below and the generator above synchronous speed. The input to the system from the a-c line thus is the sum of all losses of the combination, and the only loss not handled in the usual manner is that due to the belt slip. The no-load loss of the belt being included with those of the set, it remains only to determine the change in belt loss with load. This may be defined as the belt power times the ratio of belt slip *rpm* to motor *rpm*. Belt power is obtained by deducting the motor losses from the motor input, and the belt slip is determined from machine speeds measured by stroboscope and that expected because of the known pulley ratio. This pulley ratio is best determined by driving the belted combination by hand until both machines have turned an integral number of revolutions.

The final results of the belted load-back test, in the form of individual machine stray load loss, are shown in figure 5 by curve *C*, which is compared with curve *B* from the d-c load-back test (figure 3). Both curves are plotted so that they pass through the origin. The fact that curve *C* shows close agreement with curve *B* demonstrates the accuracy of the belted load-back test.

Segregated Losses at Full Load

Loss	Watts	Per Cent of Full Load Input
Iron	258	2.91
Friction and windage	294	3.32
Stator copper	476	5.37
Rotor copper	193	2.18
Stray load	185	2.08

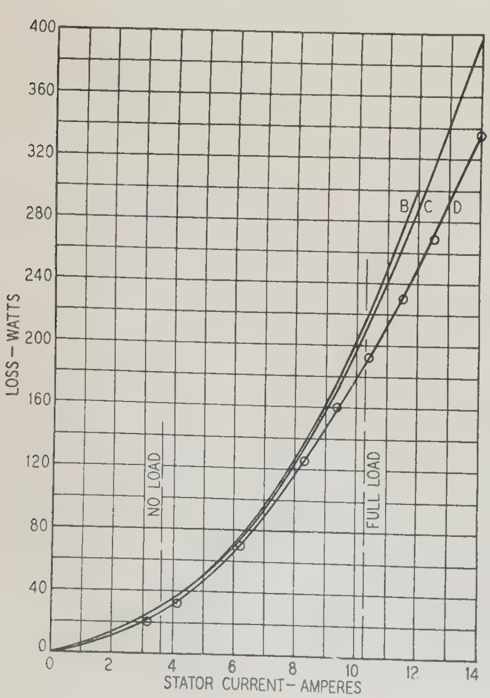


Fig. 5. Comparison of stray load losses determined by different methods

B—Loss by load-back through d-c machines

C—Loss by load-back through belt

D—Loss by d-c exitation of stator

D-C Excitation Method

Values of stray load loss also were obtained by driving the rotor at synchronous speed with d-c excitation of the stator winding.[2,4] A 1 horsepower d-c motor was accurately calibrated for its losses and was used to drive the rotor of the induction machine. Curve *D* of figure 5 is the result of this test, and it also

is plotted to pass through the origin. If the loss values are to be in agreement with the usual definition of stray load loss, they must be reduced in amount by the loss occurring when the stator is excited with current equivalent to the a-c no-load current.

Input to the rated motor is measured before and after application of direct current to the stator winding, and the increase in power supplied includes induction machine stray load loss and rotor loss. Standstill torque on the rotor with corresponding polyphase alternating current in the stator winding is a measure of this rotor loss.

Subtraction of stator copper loss from carefully measured line input to the induction motor during the standstill torque test gives a value slightly larger than the rotor torque loss. The difference is the loss attributable to the fundamental frequency leakage flux in the stator. The stator leakage flux loss is a component of the stray load loss under normal operating conditions, and was found to be approximately 15 per cent of the measured stray load loss.

Comparison of Results

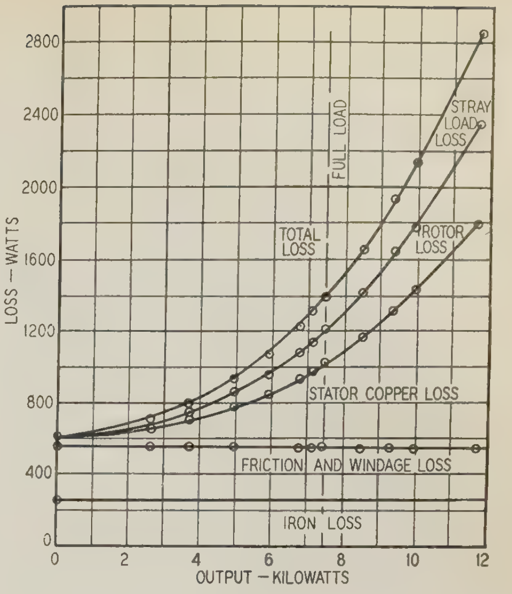
The curves of figure 5 give stray load loss as a function of stator current, and include that small portion of this loss due to current at no load. Curve *B* is the result of 2 highly accurate measurements in the d-c load-back test. The belted load-back test results, as shown by curve *C*, fall only slightly below the correct values. Both curves, when analyzed for relationship to the stator current, are exponential curves for which the exponent is not constant but increases from approximately 1.9 at low currents to 2.05 at high currents. Curve *D*, the loss by d-c excitation test, has a fixed exponent of 1.92.

Segregation of Losses

In these tests, the iron loss is considered as a stator circuit loss, although for simplicity in calculations the A.I.E.E. Test Code[4] specifies that it be treated as a rotor circuit loss. However, any error in rotor copper loss due to incorrect treatment of iron loss is negligible. The iron loss occurs largely at fundamental frequency and is supplied directly by the stator circuit. This was verified by driving the rotor of one machine at near-synchronous speed by a small d-c rated motor coupled to it. Measurements of the a-c line input were made with the machine operating at exceedingly small slip, first negative and then positive. An average value of input after subtraction of copper loss gives the iron loss at fundamental frequency. This was found to be 80 per cent of the total iron loss. The remaining 20 per cent, which occurs as a result of rotation, also was determined separately from measurements of input to the d-c motor.

The combined iron and friction and windage losses were measured in the usual manner by a-c no-load operation. Friction and windage loss then was determined by driving the induction machine by a small rated motor and the division made. Iron loss was

Fig. 6. Segregation of motor losses



obtained again by driving the machine rotor above synchronous speed, with the stator excited, to the point where the a-c line input was only that required for stator copper loss. The increase in power to the driving motor gives the total iron loss. Separation of iron loss from friction and windage loss is much more reliable when made by the rated motor method than by a-c no-load operation.

The relative proportions of all losses in the machines tested are shown in figure 6. The total loss is separated into its 5 components: (1) iron loss, including the no-load stray loss; (2) friction and windage loss; (3) stator copper loss as found from d-c resistance measurements; (4) rotor copper loss as determined from slip measurements; and (5) the stray load loss as determined by the load-back tests. Values for these losses at full load are given in the accompanying table. The full-load efficiency computed from losses as shown is 84.14 per cent. An input-output test of each induction motor was made by driving as generators the calibrated d-c machines that were employed in the d-c load-back test. The resulting full-load efficiency was 83.9 per cent.

Loading back one machine on another usually is considered to require identical machines in order to make a division of the stray load loss between them. However, this requirement no longer holds if one of the machines in a load-back combination already has been tested for stray load loss. The load-back method thus can be made available for tests of single machines.

References

1. Stray Load Loss Test on Induction Machines, T. H. Morgan and Paul M. Narbutovskih. Elec. Engg. (A.I.E.E. Trans.), v. 53, 1934, p. 286–90.

2. Measurement of Stray Load Loss in Polyphase Induction Motors, C. J. Koch. A.I.E.E. Trans., v. 51, 1932, p. 756–63.

3. Efficiency Tests of Induction Machines, C. C. Leader and F. D. Phillips. Elec. Engg. (A.I.E.E. Trans.), v. 53, 1934, p. 1628–32.

4. Test Code for Polyphase Induction Machines. A.I.E.E. Pamphlet, 1934.

5. Power Losses in Induction Machines, Paul M. Narbutovskih. Elec. Engg. (A.I.E.E. Trans.), v. 53, 1934, p. 1466–71.

Effect of Electric Shock on the Heart

A joint investigation of the effects of electric shock on the heart, extending over a period of several years, has yielded many significant results. It has been found that electric shock may derange heart action causing ventricular fibrillation without damage to heart tissue, but resulting in death within a few minutes. This heart effect establishes the maximum current that may be withstood safely for short durations. Threshold fibrillating currents were determined for different conditions of pathway, frequency, and duration, using numerous anesthetized animals of different species, comparable in size with man. The discovery was made that the heart is susceptible for only about 20 per cent of its cycle. Successful recoveries from ventricular fibrillation were obtained with large animals of several species by high intensity shocks of short duration.

By
L. P. FERRIS — Bell Tel. Labs. Inc., New York, N. Y.
FELLOW A.I.E.E.

B. G. KING — Columbia University, New York, N. Y.

P. W SPENCE — Bell Tel. Labs. Inc., New York, N. Y.
MEMBER A.I.E.E.

H. B. WILLIAMS — Columbia University New York, N. Y.
Member Am. Physiological Soc.

EXPERIMENTAL investigations of electric shock antedate all commercial uses of electricity, tests of the effects of frictional electricity on birds, beetles, and other living things having been made during the first half of the eighteenth century.[1–4] However, the records of such work, from the times of the early experimenters to date, leave much to be desired in the way of definite information that might be translated to man, particularly in defining the limits of dangerous shock. As a basis for the development of protective measures and practices, such knowledge is obviously important and this joint investigation at the College of Physicians and Surgeons of Columbia University was initiate in 1927 in the hope of obtaining some of the neede data.

The earliest experiments of this investigation wer made on guinea pigs to determine the effect of differ ent voltages by comparisons of the percentage fatal ties. From these early tests and further considera tion of the problem, it became evident that th physiological effects of electric shock are related t the magnitude of the current rather than to the volt age, owing particularly to large variations in contac impedances. Fatalities may occur from any of sev eral causes or combinations of causes, and the earl work of this investigation emphasized the necessit of differentiating the mechanisms of death, particu larly in investigating their limiting conditions Therefore, the work here reported has been directe along the lines of correlating the magnitude of cur rent with specific physiological effects. Heart ac tion has been given almost exclusive consideration because it seems to set the limit of current to b guarded against if fatalities are to be avoided.

In seeking a value of current which, if exceeded would be dangerous to man, it is important to con sider for different practical conditions the differen effects brought about as the current is increased Currents, for example, far smaller than those giving rise to any kind of harmful effects, are capable o causing sensation by stimulation of the sensory nerve endings in the skin, which is manifested as a tingling or sensation of warmth. Three recent independen studies have been made to determine the lowest cur rent at which any sensation is perceptible, referre to as the threshold current of sensation. The Elek trizitätswerken des Kantons Zurich[5] reports a threshold of 0.9 milliampere for 50 cycle current with hands gripping cylinders 2 centimeters in di ameter. Thompson[6] of the Electrical Testing Labo ratories made observations on 70 persons with one hand gripping a 1 inch rod and the other immersed in salt solution, and obtained a threshold of 1.05 milli amperes for 60 cycle current. In connection with the present investigation such determinations were made in 1929. The electrodes consisted of 8 inch pliers grasped in one hand and $^3/_{16}$ inch stranded cable in the other. The average threshold of 44 men and 3 women for 60 cycle current was 0.9 milliampere There was no marked difference between moist and dry hands. These independent determinations are in good agreement and well establish the threshold of perception of 60 cycle alternating current at about 1 milliampere.

As the current is increased above the threshold o sensation, a point will be reached at which the sub ject becomes unable to control the muscles subjecte to stimulation and, therefore, may be unable to re lease himself. The Elektrizitätswerken des Kantons Zurich[5] defines this as the limit of safe current and found it to be 15 milliamperes at 50 cycles for a cur rent pathway between hands. In accidental electric

A paper recommended for publication by the A.I.E.E. committee on instrument and measurements, and scheduled for discussion at the A.I.E.E. summer con vention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 7 1936; released for publication March 30, 1936.

1. For all numbered references, see list at end of paper.

shock, the ability to release oneself would depend somewhat on the type of contact as well as the magnitude of current. For some types of contact, muscular contraction would tend to break the contact rather than to improve it, while in others it is often possible for the victim to free himself by using muscles little affected by the current. Any currents, however, that prevent voluntary control of the skeletal muscles are dangerous because their pathway through the body might include the respiratory muscles and stop breathing during the shock. If prolonged, asphyxial death would result, but the time required is a matter of minutes rather than seconds, so that opportunity may be afforded for action to release the victim. No serious or permanent aftereffects are likely to appear merely from the cessation of respiration, provided it is not continued beyond the point where the victim can be resuscitated by artificial respiration. This effect of involuntary suspension of breathing resulting from enforced muscular contraction, which would cease upon breaking the contact, is not to be confused with respiratory inhibition—an effect on respiratory nerves—resulting from much higher currents than

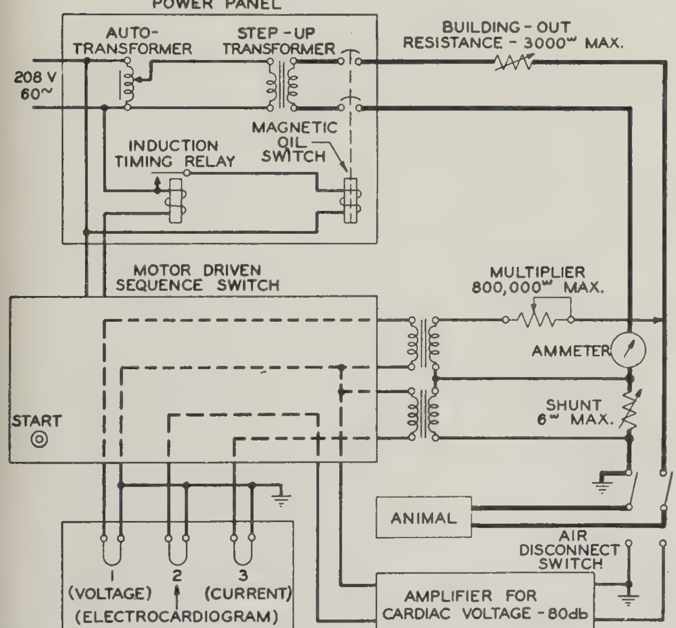


Fig. 1. Schematic diagram of power supply, control, and measuring apparatus for shocks longer than $^1/_2$ second

those necessary to enforce muscular contraction. Such respiratory inhibitions often last for extended periods after the shock has ceased.

Currents somewhat greater than those just necessary to stop respiration by action on the muscles may cause fatalities, even though the duration of such shocks is but a few seconds or less—far too short to be important from the standpoint of interruption of respiration and obviously too short to give any opportunity for rescue before the end of the shock. Death under such conditions is brought about by ventricular fibrillation, which is a disruption of normal heart action. This condition is an unco-ordinated asynchronous contraction of the ventricular muscle fibers in contrast to their normal co-ordinated and rhythmic contraction. It results from an abnormal stimulation rather than from damage to the heart. In the fibrillating condition, the heart seems to quiver rather than to beat; no heart sounds can be heard with a stethoscope; the pumping action of the heart ceases; failure of circulation results in an asphyxial death within a few minutes. The medical profession long has recognized that ventricular fibrillation once set up in man is unlikely to cease naturally before death. The value of current just under the threshold for ventricular fibrillation, therefore, may be taken as the maximum current to which man safely may be subjected, because regardless of rescue or aftertreatment, death is liable to result from greater current.

This experimental investigation, therefore, was directed chiefly toward determining the minimum current that would initiate ventricular fibrillation and the variation of this threshold current with several factors which enter into the practical application of the results in the development of protective devices and measures. From the standpoints of both physiology and engineering, it was important to determine the influence on this threshold of:

1. Species and size of animal.
2. Path of current through the body (determined by points of contact).
3. Frequency of the current.
4. Time of occurrence of short shocks in relation to the cardiac cycle.
5. Duration of shock.

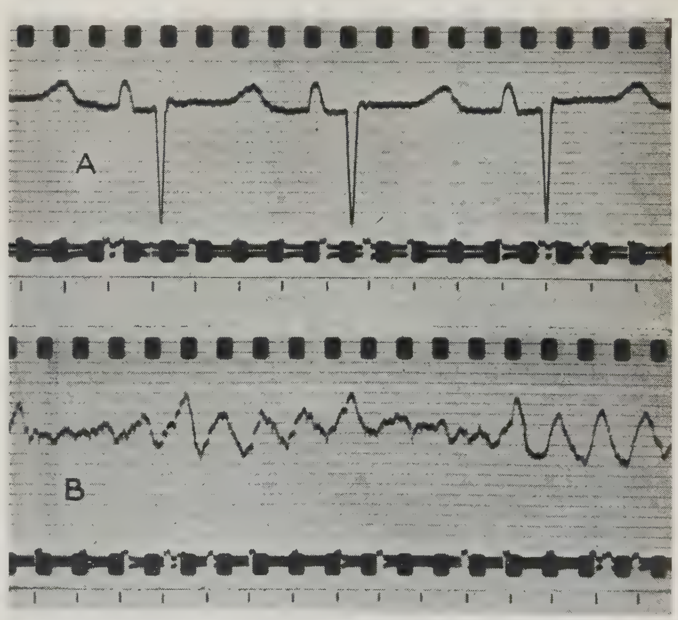


Fig. 2. Electrocardiograms of sheep

A—Preshock; normal
B—Postshock; ventricular fibrillation
Electrodes on right foreleg and left hind leg

It was considered important to determine also the effect of high currents, well above the threshold of fibrillation. Most of the experiments were upon species of animals comparable in body weight, heart rate, and heart weight to man. Thus the results should be more directly indicative of what might be expected in man than tests on smaller species. However, several species of smaller animals were included in order to establish the trend of the effects with variation in physiological and morphological factors. An experimental study of all possible combinations of variable factors mentioned would have required an excessively large number of specimens. This was avoided by holding certain factors to fixed values during most of the tests.

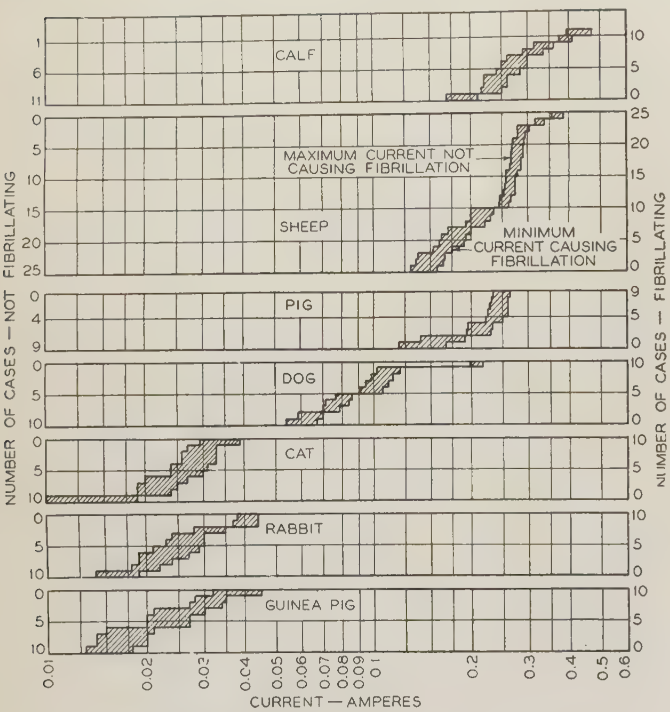


Fig. 3. Threshold currents causing ventricular fibrillation for different species

Shock duration 3 seconds; frequency 60 cycles; electrodes on right foreleg and left hind leg

General Testing Arrangements

The animals were kept under surgical anesthesia during the tests. For this purpose ethyl-methyl-butyl barbituric acid (Nembutal, Abbott), in solution, was used almost exclusively. It was administered by mouth, except for some of the earliest cases in which it was injected intraperitoneally. The sites for the electrodes were suitably prepared by washing to remove grease, and then saturated with salt solution. For the larger animals flexible metal electrodes, held in place by bandages wetted with salt solution, were used, and for the smaller animals clamps padded and wetted with salt solution.

For all a-c tests, power was obtained from mains at 208 volts, 60 cycles. The voltage was raised, in steps of 30 volts up to 3,000 volts, by means of a variable ratio autotransformer and a step-up transformer. Application of the shocks to the animals was controlled by a solenoid oil switch, except for shocks of less than one second in duration, where it was found desirable to co-ordinate the shock with the cardiac cycle. The duration of the longer shocks was governed by a timing relay, adjustable for durations of $^1/_2$ second to $^1/_2$ minute.

Electrocardiograms of each animal before and after shock, together with the shock current and voltage, were recorded on photographic film by means of a special 3-string electromagnetic oscillograph.[7] An electrocardiogram is a graphical record of the time variation of the voltage that always is associated with the action of the heart, taken between electrodes placed on chosen points of the body. An adjustable vacuum tube amplifier giving a maximum output of 100 milliamperes was necessary for recording the cardiac voltages, which are of the order of one millivolt. For identification and permanent record, the test number and statement of conditions, including constants of measuring apparatus, were photographed on the oscillograph film prior to every test.

A schematic drawing of the apparatus for applying the longer duration shocks is shown in figure 1. The "building-out" resistance, which appears between the power supply and the animal, served to maintain approximately constant currents for defi-

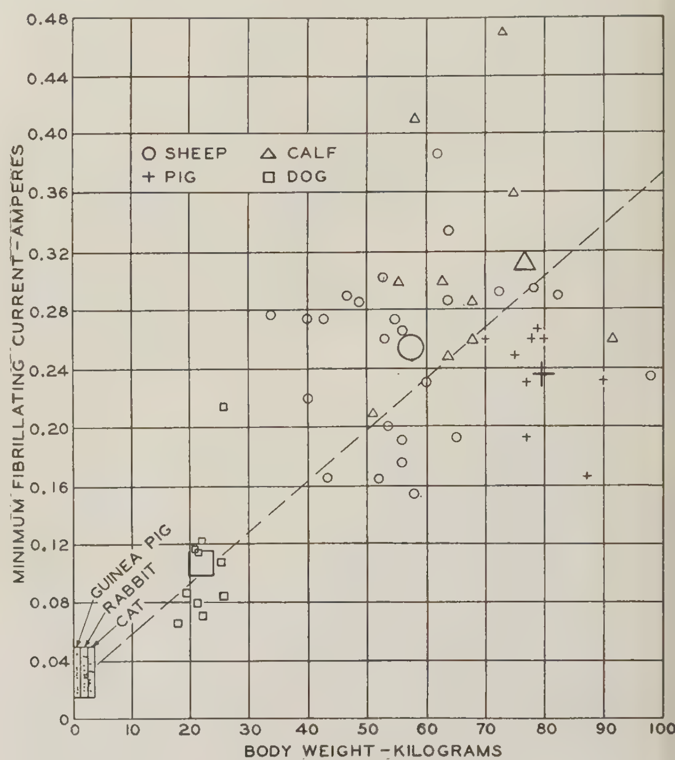


Fig. 4. Relation of minimum current causing ventricular fibrillation to body weight, for different species

Shock duration 3 seconds; frequency 60 cycles; electrodes on right foreleg and left hind leg. Averages are shown by larger symbols

nite applied voltages, despite differences in the electrode contacts. The "air disconnect switch" was used for transferring the connections to the animal from the electrocardiograph to the power supply and also served to protect the investigators while working on the animals. The motor-driven switch governed the sequence of events in applying the shocks and in photographing records on the oscillograph film. The ammeter allowed supplementary observations of the current when the shocks were as long as 3 seconds.

In studying the effect of shocks of durations less than the time of one heart beat, it became desirable to control their occurrence with reference to the cardiac cycle. For this purpose a marginal electromechanical tripping apparatus was developed which applied the shocks in predetermined parts of the cardiac cycle and controlled their duration. This apparatus is described in connection with the tests in which it was used.

For a threshold determination, successive shocks were applied to an animal, starting at a low current and increasing it for each successive shock, until ventricular fibrillation was initiated. The shocks were applied at intervals of 5 minutes or more, it having been found that the heart and respiratory system recovered from the effects of previous shocks within that time.

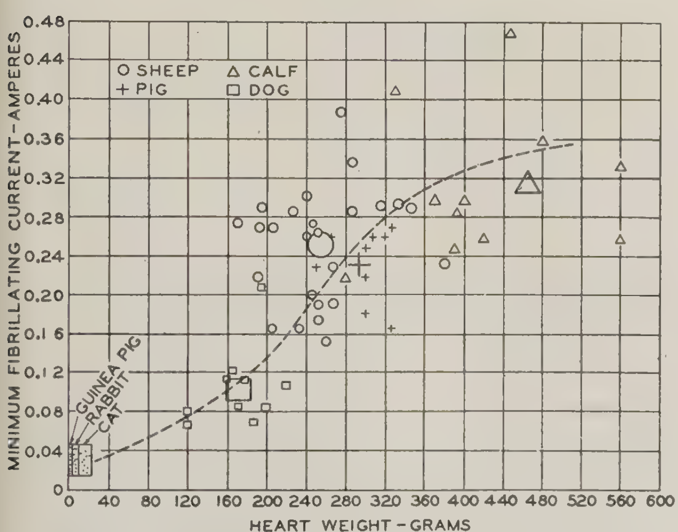


Fig. 5. Relation of minimum current causing ventricular fibrillation to heart weight, for different species

Shock duration 3 seconds; frequency 60 cycles; electrodes on right foreleg and left hind leg. Averages are shown by larger symbols

Threshold of Fibrillation

Thresholds were determined for 7 species of animals: the guinea pig, rabbit, cat, dog, sheep, pig, and calf. Standard reference conditions included the use of 60 cycle alternating current for a duration of 3 seconds with electrodes on the right foreleg and left hind leg. These conditions typify those of many accidental shocks to man and are very dangerous from the point of view of ventricular fibrillation because the heart is almost directly in the current path.

Three significant records were made for each shock. Two of these are illustrated in figure 2; the third, not shown, is an oscillographic record of shock current and voltage. Figure 2*A* shows a preshock electrocardiogram, a recurrent sequence of deflections, each complete cycle of which parallels the action of a heart beat. Figure 2*B* shows a postshock electrocardiogram. The postshock electrocardiogram indicates whether the heart continued its normal beat or fibrillated. Figure 2*B* does not have the characteristic prominences evident before shock but, instead, a recurrent wave having a fundamental frequency of about 10 cycles per second. The appearance of this typical fibrillating wave and the absence of heart sounds following shock were taken as conclusive evidence of ventricular fibrillation. In the 3 smaller species it was necessary to expose the hearts for visual observation during the shocks, because their hearts tend to recover naturally from fibrillation, often before an electrocardiogram could be obtained. As usual, these tests were performed with the animal under surgical anesthesia. In the larger species of animals the heart has little tendency to resume its co-ordinate beat once fibrillation is established, so that it was not necessary to open the chest.

Data from these tests are summarized in table I and are shown graphically in figures 3, 4, and 5. Figure 3 is a set of cumulative distribution charts, one for each species, in which data from individual animals are arranged in ascending order of minimum currents causing fibrillation and maximum currents not causing fibrillation. Curves of the actual threshold currents, portraying the boundaries between the nonfibrillating and fibrillating conditions, would lie somewhere within the cross-hatched areas. A greater number of sheep was used so as to secure better averages, since it had been decided to concentrate on this species for further testing. In figures 4 and 5, the minimum currents causing fibrillation for the individual animals have been plotted

Table I—Threshold 60 Cycle Currents Causing Ventricular Fibrillation in Different Species of Animals

Duration of Shocks 3 Seconds. Electrodes on Right Foreleg and Left Hind Leg

		Average Weights		Current, Amperes		
				Minimum Fibrillating		Maximum Non-fibrillating,
Species	Animals	Body, Kilograms	Heart, Grams	Average	Range	Average
Guinea pig	10	0.55	1.8	0.028	0.018–0.045	0.020
Rabbit	10	2.2	6.0	0.030	0.019–0.044	0.025
Cat	10	2.9	15.0	0.029	0.019–0.039	0.023
Dog	10	22	170	0.11	0.07 –0.22	0.092
Pig	9	79	300	0.24	0.17 –0.27	0.20
Sheep	25	56	270*	0.25	0.16 –0.39	0.24
Calf	10	70	420	0.31	0.21 –0.47	0.27

* Based upon average heart weight of other sheep of same body weights.

as functions of body weights and heart weights, respectively. The maximum current not causing fibrillation would have been as good an index of the variation of the threshold. The threshold current increases roughly with both the heart weight and body weight of the different species of animals, although if the 3 smaller species be considered alone this relationship does not hold, their threshold currents being practically the same despite widely different body weights.

These results serve to indicate the probable threshold current for man under similar conditions. The average weight of an adult man is approximately 70 kilograms and his heart weight, 330 grams. The average threshold current for a body weight of 70 kilograms is 0.26 ampere (from figure 4) and that corresponding to a heart weight of 330 grams is 0.29 ampere (from figure 5). Knowledge of such average currents is useful, but in the practical application of this information it is the lower limit of current causing ventricular fibrillation that must be taken into consideration. The thresholds differ widely for different individuals of the same species. The cumulative distribution of threshold currents from the determinations on sheep is shown in figure 6, including both the minimum fibrillating and maximum nonfibrillating currents, and indicates a minimum threshold of about 0.1 ampere—44 per cent of the average threshold.

Assuming a similar distribution for man, the results on the whole indicate that currents in excess of 0.1 ampere at 60 cycles from hand to foot would be dangerous for shock durations of 3 seconds or more.

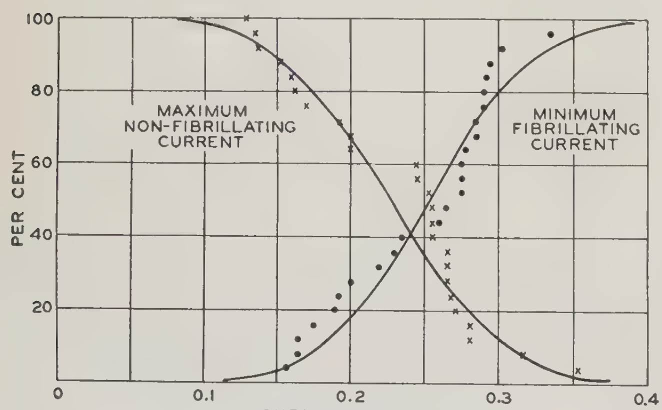


Fig. 6. Distribution characteristics of threshold currents for sheep

Shock duration 3 seconds; frequency 60 cycles; electrodes on right foreleg and left hind leg

Effect of Path of Current

It is well known that very small currents applied directly to the heart cause fibrillation; and inasmuch as the path of the current through the body affects the proportion of current reaching the heart, it was expected that the current path would have an important influence on the threshold current causing fibrillation. Tests, therefore, were made on 5 additional groups of sheep under the conditions previously described, except with different electrode positions to alter the current path. The averages for all pathways are given in table II and cumulative distribution charts in figure 7. The differences among the determinations for 4 of the pathways, namely, across the chest, chest to foreleg, head to hind leg, and foreleg to hind leg, do not appear great enough to be significant. The pathway between the forelegs indicates slightly higher thresholds. For the current path between the 2 hind legs the proportion reaching the region of the heart is evidently so small that currents up to at least 12 amperes (20 in 3 cases) produced no fibrillation. The apparatus could not be operated safely at higher currents. Tests with higher current do not seem necessary, because the damage to the tissues near the point of application of the electrodes indicates that persons receiving such a shock through the legs, while not likely to suffer from ventricular fibrillation, are liable to be injured seriously and might be maimed should they recover.

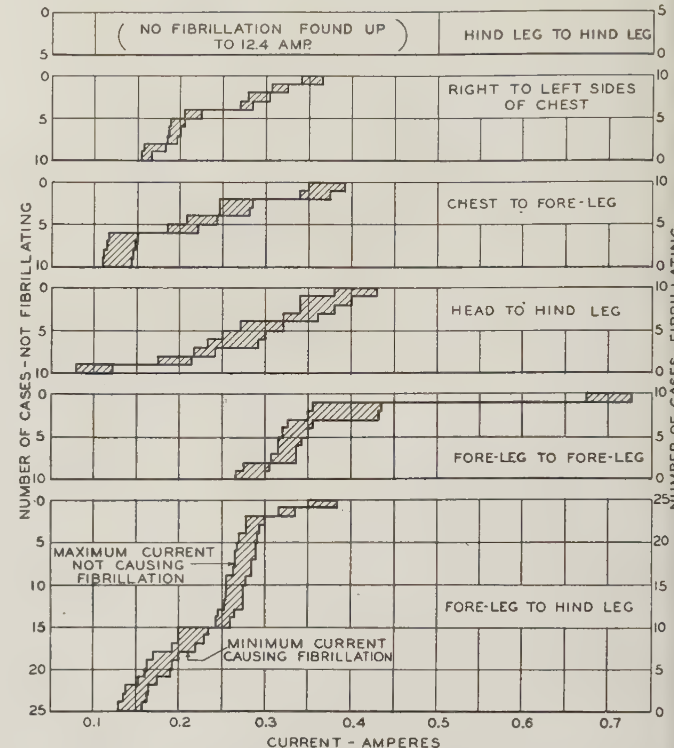


Fig. 7. Threshold currents causing ventricular fibrillation in sheep, for several different current pathways

Shock duration 3 seconds; frequency 60 cycles

Effect of Frequency

Since information useful in protection against electric shocks was the objective of this study, only the commercial range of power frequencies thus far has been investigated. In addition to the 60 cycle tests previously reported, others on sheep were made with 25 cycle current and with direct current, the general conditions previously described being main-

tained. Power for the 25 cycle tests was obtained from a motor generator and for the d-c tests from a battery. Oscillograms were not taken of the direct currents and voltages. These were read on meters.

The results of these tests are summarized in table III, and cumulative distribution charts of the data are shown in figure 8*A*. The thresholds for shocks at 25 cycles are significantly but not greatly higher than those at 60 cycles, while those for direct current are several times the a-c values. In figure 8*B*, the threshold currents have been plotted as a function of frequency. An additional point in the vicinity of 7 cycles per second would be desirable to fix definitely the curve between 0 and 25 cycles. An extension of this curve to higher frequencies would be of great physiological interest. Prevost and Battelli[8] reported 150 cycles to be the most dangerous frequency in causing ventricular fibrillation in dogs. They also observed that the voltage required to cause death at about 2,000 cycles was 10 times that required at 200 cycles.

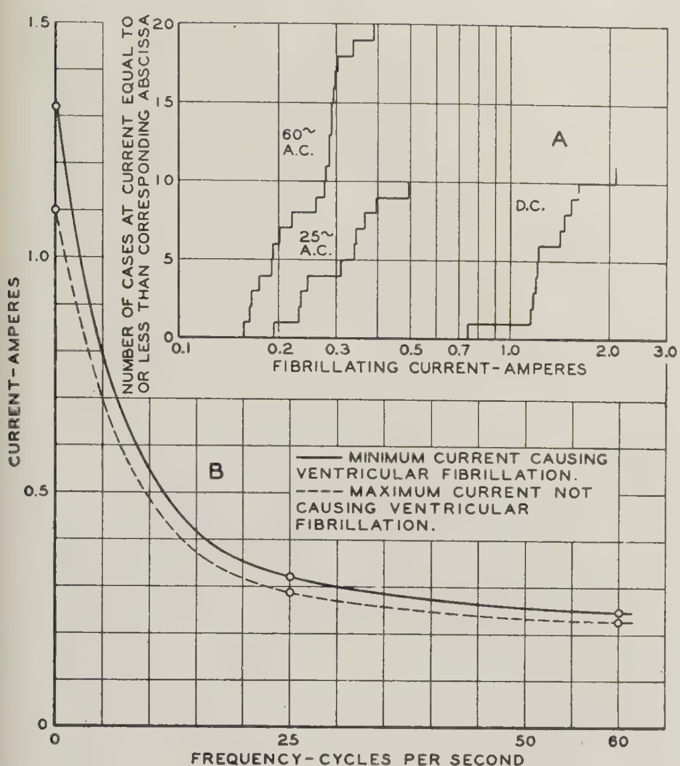


Fig. 8. Effect of frequency on threshold current causing ventricular fibrillation in sheep

Shock duration 3 seconds; electrodes on right foreleg and left hind leg

The d-c data suggest that the direction of the current may influence the threshold, slightly greater hazard resulting with the cathode on the foreleg. The data are insufficient, however, to support positive conclusions.

The electrocardiograms of ventricular fibrillation were alike whether caused by direct or alternating current. One spontaneous recovery from ventricular fibrillation appeared following a 25 cycle shock, the only such recovery observed in tests on about 500 sheep. It is not thought to be ascribable to the use of 25 cycles instead of 60 cycles in initiating the fibrillation.

Susceptibility of Heart in Different Phases of Cardiac Cycle

Physiologists have established that the heart is responsive to moderate electrical stimuli during the period of relaxation (diastole) whereas such stimuli during the period of contraction (systole) do not elicit further response. Figure 9 shows a normal electrocardiogram of a sheep, marked to indicate the relation of the phases of the cardiac cycle to the different so-called "waves" of the electrocardiogram, the latter being lettered in accordance with common medical practice. During the absolute refractory period, no stimulus, regardless of its magnitude, would be expected to cause further contraction.[9] The rapidity of change and magnitude of the cardiac voltage in the "*QRS* complex of waves" indicate a very rapid change in muscular activity from relaxation to contraction. When the whole ventricular tissue is active and the blood is being expelled from the cavities, no electrocardiographic response appears, since the uniformity of the muscular condition gives no marked potential difference. At the begin-

Table II—Threshold 60 Cycle Currents Causing Ventricular Fibrillation in Sheep for 3 Second Shocks Through Different Pathways

Electrode Positions	Animals	Current, Amperes — Minimum Fibrillating: Average	Minimum Fibrillating: Range	Maximum Non-fibrillating, Average
Right foreleg and left hind leg	20	0.25	0.16–0.39	0.24
Right foreleg and left hind leg	5*	0.26	0.23–0.28	0.23
Right and left forelegs	10	0.39	0.30–0.73	0.36
Head and left hind leg	10	0.30	0.12–0.43	0.26
Left foreleg and right side of chest	10	0.24	0.14–0.39	0.20
Right and left sides of chest	11	0.26	0.17–0.41	0.24
Right and left hind legs	5	No fibrillation found up to 12.4 amperes		

*No anesthetic administered; in all other tests the animals were anesthetized with nembutal.

Table III—Threshold Currents of Different Frequencies Causing Ventricular Fibrillation in Sheep

Duration of Shocks 3 Seconds. Electrodes on Right Foreleg and Left Hind Leg

Frequency, Cycles per Second	Animals	Observed Current, Amperes — Minimum Fibrillating: Average	Minimum Fibrillating: Range	Maximum Non-fibrillating, Average
0 — Foreleg positive	6	1.5	1.2 –2.1	1.2
0 — Foreleg negative	5	1.1	0.7 –1.5	1.0
0 — Entire Group	11	1.3	0.7 –2.1	1.1
25	10	0.32	0.19–0.49	0.29
60	25	0.25	0.16–0.39	0.24

ning of the *T* wave, the contraction starts to disintegrate. This wave is not as steep or of as great magnitude as the *QRS* complex and takes a longer time to complete, showing that relaxation is not completed so quickly. It is believed generally that groups of heart muscle fibers relax progressively and in some definite order so that parts of the heart muscle are completely relaxed while other fractions

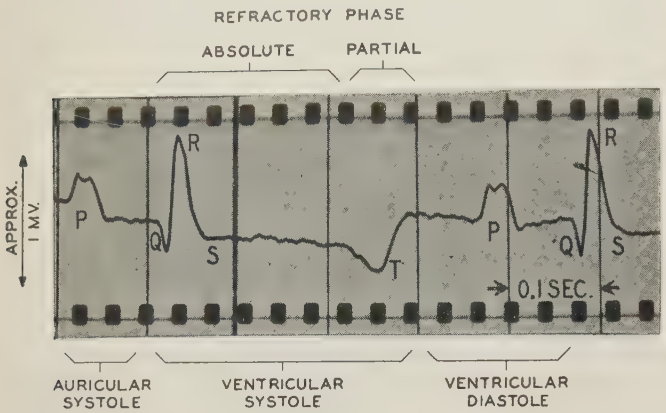


Fig. 9. Phases of electrocardiac cycle

are still contracted.[10] It is not realized generally that this state of diminishing contraction, designated as the "partial refractory phase," is coincident with the *T* wave.[11,12] In view of these facts and some erratic responses to shocks of 3 to 4 cycles of 60 cycle current, some important differences were expected in the response of the heart to very short shocks during different phases of the cardiac cycle.

To investigate this possibility, special apparatus was developed for applying short shocks at predetermined points in the cardiac cycle. Advantage was taken of the conspicuous differences among the electrocardiac impulses of a single heart beat, for initiating operation of circuits to control the application of the shock and the recording of the current, voltage, and electrocardiograms. Figure 10 is a schematic diagram of the apparatus used for this purpose. By proper timing and biasing, the marginal tripping circuit operated only on one of the steep portions of the *QRS* complex. The timing of the shock in relation to this reference point was regulated by means of an adjustable electrical delay circuit which started measuring time after the tripping apparatus had been actuated. The duration of the shock was controlled by a second adjustable delay circuit which measured time from the instant of the beginning of the shock and short-circuited the animal after the desired interval. Delays of from 0.2 to 1.2 seconds from the *QRS* complex and durations of from 0.03 to 0.55 second could be obtained.

Since the durations of all the different phases of the cardiac cycle do not change in the same proportion as the heart rate changes, it was necessary, for setting the timing circuit, to establish average intervals between the characteristic electrocardiac impulses. Figure 11 gives such average intervals, based upon a large number of sheep, as functions of the heart rate. Deviation for an individual sheep, from these averages, generally could be estimated from an examination of the first preshock electrocardiogram.

The oscillograph was started a few seconds before the shock was administered and operated continuously, recording a preshock electrocardiogram, oscillograms of current and voltage, and a postshock

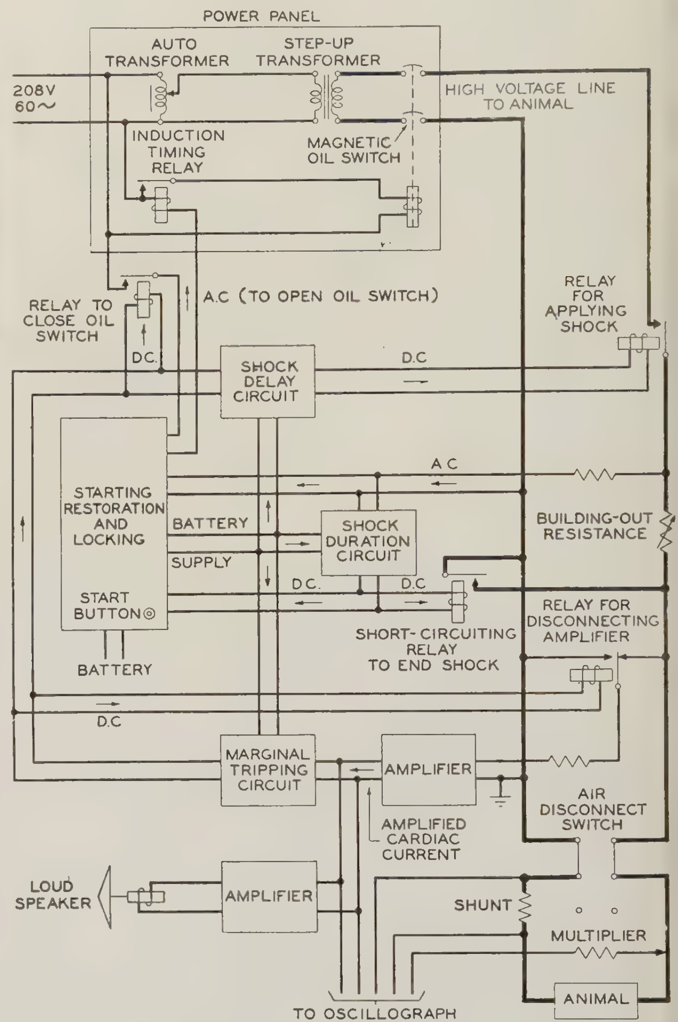


Fig. 10. Schematic diagram of apparatus for applying shocks at predetermined points in cardiac cycle

electrocardiogram of sufficient duration to show the effect on the heart. The exact time relation of the shock to the immediately preceding electrocardiogram thus could be determined, assuming only that the heart rate did not change during the 1 or 2 beats between the point of tripping and the beginning of the shock—an assumption wholly valid for normal hearts. Figure 12 shows typical records obtained in these tests. Time lines appear 10 times per second. The upper record of figure 12 shows a shock followed by a co-ordinate heart beat, while the lower record is of a shock resulting in ventricular fibrillation. The action currents from the body muscles persisting after the powerful contractions during a

shock, obscure to some extent all postshock electrocardiac records, making them appear to have high frequency irregularities. However, close inspection of the upper postshock electrocardiogram of figure 12 reveals the same typical sequence of prominent deflections that appear before shock, proving coordinate heart action.

For the earliest tests, successive shocks were applied to each of several sheep at intervals of not less than 5 minutes, with electrodes on the right foreleg and the left hind leg. The shocks generally were applied in the same part of the cardiac cycle, and the current was increased for successive shocks until the maximum output of the apparatus was reached, unless ventricular fibrillation resulted at a lower current. If fibrillation did not appear following any of these shocks, similar series of shocks were applied at other parts of the cardiac cycle. For other tests, the current was maintained approximately constant and the position of the shock in the cardiac cycle was varied until practically the whole cycle had been covered. Most of these shocks were of about 0.03 second duration, with a smaller group of about 0.1 second. Shock currents ranged between 1 and 17 amperes. The results of 913 shocks of 0.03 second duration on 132 sheep are plotted in figure 13 to show the positions of the midpoint of each shock in the cardiac cycle, approximate shock currents, and whether or not fibrillation occurred. Since the heart rates differed among the animals and from shock to shock, the relative positions of the shocks are shown in a single cardiac cycle (0.45 second duration for a heart rate of 132 beats per minute) corresponding to the average of all the preshock electrocardiograms taken in connection with these tests.

Table IV—Threshold Currents Causing Ventricular Fibrillation for Short Shocks in Partial Refractory Phase of Cardiac Cycle

60 Cycle Current Applied to Right Foreleg and Left Hind Leg of Sheep

Duration, Seconds	Average Weight, Kilograms	Animals	Current, Amperes: Minimum Causing Fibrillation, Average	Minimum Causing Fibrillation, Range	Maximum Not Causing Fibrillation, Average
0.03 (1933)	57	11	2.4	1.7– 3.0	2.0
0.03 (1934)	63	10	3.2	1.9– 5.3	2.7
0.03 (all)	60	21	2.8	1.7– 5.3	2.4
0.12 (1st series)	60	12	5.4	1.6–11.4	4.4
0.1 (2d series)	73	7	3.3	1.4–10.3	2.7
0.12 (3d series)	65	13	4.3	1.2–10.6	3.7
0.1–0.12 (all)	65	32	4.6	1.2–11.4	3.7
Statistical Division of 0.1 to 0.12 Second Shocks					
Mode *A*	58	8	9.7	7.3–11.4	8.0
Mode *B*	64	24	2.7	1.2– 4.8	2.3

Of 370 shocks of 0.12 second duration applied to 38 sheep, only 1 shock definitely outside the partial refractory phase resulted in ventricular fibrillation. This shock began at a point in the electrocardiogram between P and Q waves at which time the ventricles are completely relaxed and resting. Because of the high heart rate at the time of applying this shock, it covered 31 per cent of the complete cardiac cycle, much more than usual. A review of the history of this case revealed no physiological abnormalities nor any variation in methods of testing that might explain the unique result. Even with this one exception, however, the evidence is overwhelming that ventricular fibrillation results from such short shocks only when they occur during the period of diminishing contraction of the ventricles, corresponding to the T wave of the electrocardiogram. A preliminary report of these findings was made in 1934.[13,14] Since that time, considerable additional data have been obtained which confirm the original findings. The combined results of these tests show that, if it is possible to produce ventricular fibrillation with shocks of a duration of 0.1 second or less outside this sensitive phase of the cardiac cycle, the current would have to be in excess of 15 amperes, about the limit of these tests.

Effect of Duration

In the determination of thresholds for shocks of very short durations, a third or less of the duration of one heart beat, the time of occurrence of the shocks was regulated so as always to involve the partial refractory phase, corresponding to the appearance of the T wave. The thresholds were found in the same way as for 3-second shocks, by applying successive shocks at intervals of 5 minutes, each at increased current until ventricular fibrillation resulted. For a shock duration of 0.03 second, thresholds were determined for 21 sheep, 10 in the autumn of 1933, and 11 about a year later. The average results are given in table IV.

A second short duration (0.1 to 0.12 second) was chosen to insure complete coverage of the partial refractory phase of the cardiac cycle of most sheep, the apparatus being adjusted so as to initiate the shocks as nearly as possible at the beginning of the T wave. It was expected that complete coverage of the sensitive phase would result in a threshold current somewhat lower than for 0.03 second shocks, which seldom covered more than a third of this phase. This was based upon the fact that reduction

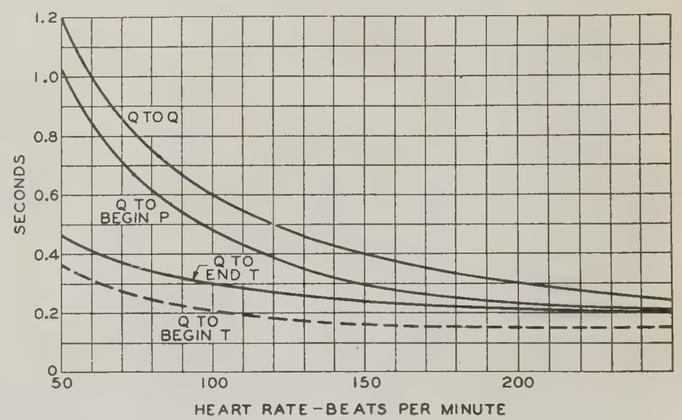


Fig. 11. Intervals of average electrocardiogram of sheep (see figure 9)

of the duration of shock from 3 seconds to 0.03 second had shown about a tenfold increase in average threshold current. This expectation was not realized in the first series of sheep tested with a shock duration of 0.12 second. Furthermore, the range covered by the individual thresholds seemed excessive. The thresholds then were determined for a second series of 7 sheep for a shock duration of 0.1 second, believed to approximate more closely the duration of the partial refractory phase. A third series of threshold determinations was made on 12 sheep for shock duration of 0.12 second as a check on the previous determinations and to provide more data for statistical analysis. The results are given in table IV.

In each series a majority of animals gave thresholds within the range for 0.03 second shocks, the thresholds for the smaller group of each series being distinctly higher. Taken as a whole, the individual observations of these 3 series group themselves so that from a statistical standpoint there is evidence of 2 "modes," and the results for these 2 modes also are

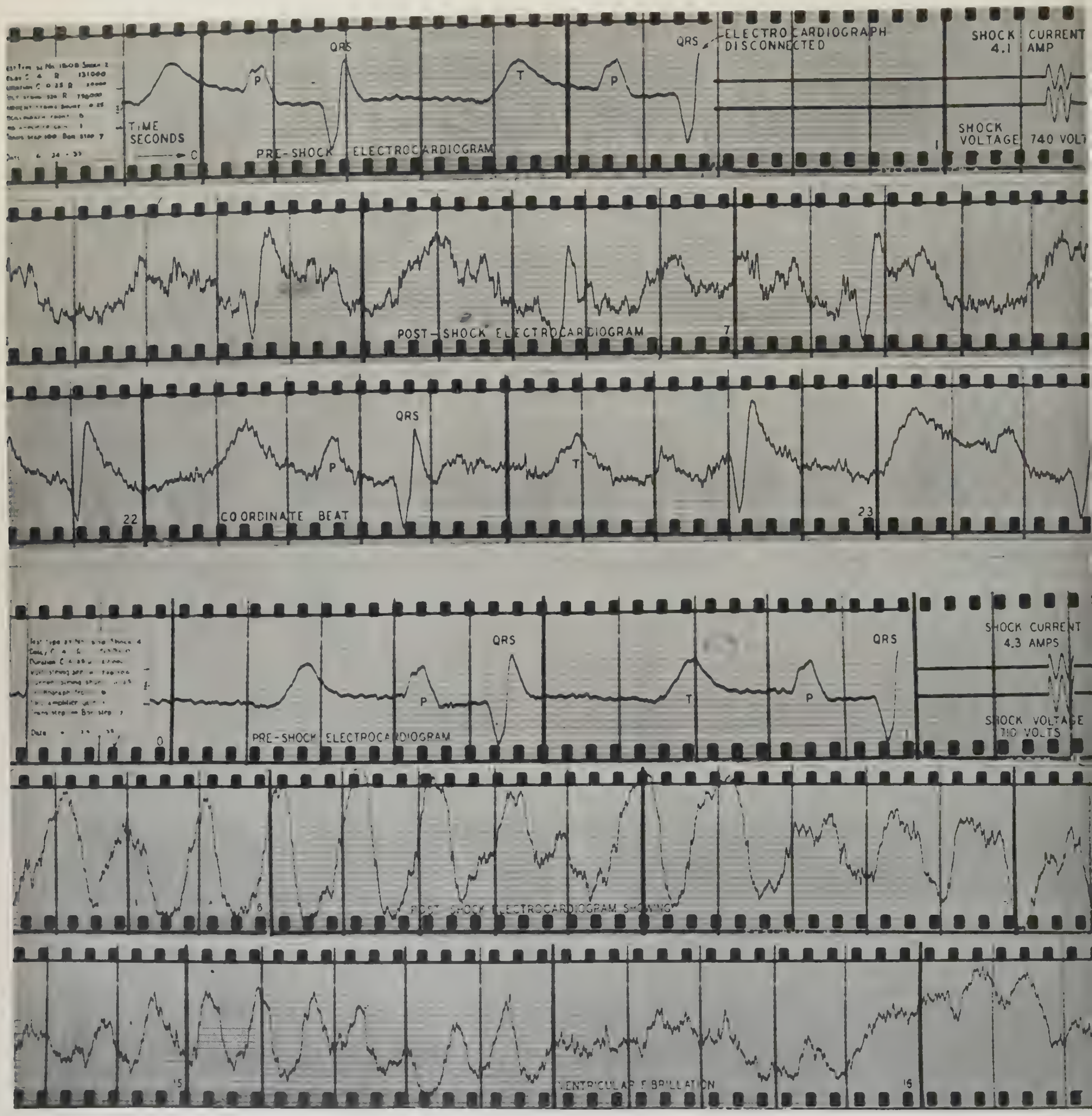


Fig. 12. Typical records of 0.03 second shocks to sheep

Top Group—Insensitive phase of cardiac cycle

Bottom Group—Sensitive phase of cardiac cycle

Electrodes on right foreleg and left hind leg

shown in table IV. Mode B closely approximates the average for 0.03 second shocks, while mode A is more than 3 times B. No consistent differences either in animals or test conditions have been found that would explain the occurrence of 2 modes. For the practical application of these results, the significant feature is that the larger group gave the lower threshold and that this is very close to the average for the 0.03 second shocks.

A third short duration of 0.47 second was chosen. This was slightly longer than the average period of the heart beat for the animals, so that the shocks covered a complete beat or more in a majority of cases. The regulating apparatus was adjusted so as to start these shocks at the beginning of ventricular systole as indicated by the QRS complex of the electrocardiogram. This fixed duration gave a variety of thresholds for 10 sheep. There seems to be evidence that this wide range in results was related to variations in the ratio of the duration of the shock to the period of the heart beat, the heart rates of the sheep varying considerably. This led to tests on an additional series of sheep in which the duration of each shock was adjusted to approximate closely the duration of the heart beat at the time of its application. The averages for each of these modifications of the same general type of tests are given in table V. Although these show good agreement, the deviations for the adjusted duration series are much smaller.

Table V—Threshold 60 Cycle Currents Causing Ventricular Fibrillation in Sheep, for Shocks of Duration Approximating One Heart Beat

Shocks Begun at Onset of Ventricular Contraction. Electrodes on Right Foreleg and Left Hind Leg

Duration	Average Weight, Kilograms	Animals	Current, Amperes: Minimum Causing Fibrillation, Average	Minimum Causing Fibrillation, Range	Maximum Not Causing Fibrillation, Average
0.47 sec	61	10	0.93	0.18–2.35	0.78
1 heart beat (0.36 to 0.55 sec)	64	10	1.08	0.55–1.5	0.91

In figure 14 the individual threshold currents for all durations of shock have been plotted as functions of the ratio of the duration of shock to the length of the heart beat. The curve shows what is believed to approximate the trend of the average results. It is apparent from this figure that duration of shock plays a most important part in determining the danger, and that about 10 times the current is required to cause ventricular fibrillation if the shock duration is reduced from 3 seconds to 0.1 second or less. As the duration of the shock approaches the period of the heart beat, the threshold current is very sensitive to changes in duration. It is believed that the current required to initiate fibrillation increases markedly as the duration is decreased below 0.03 second,

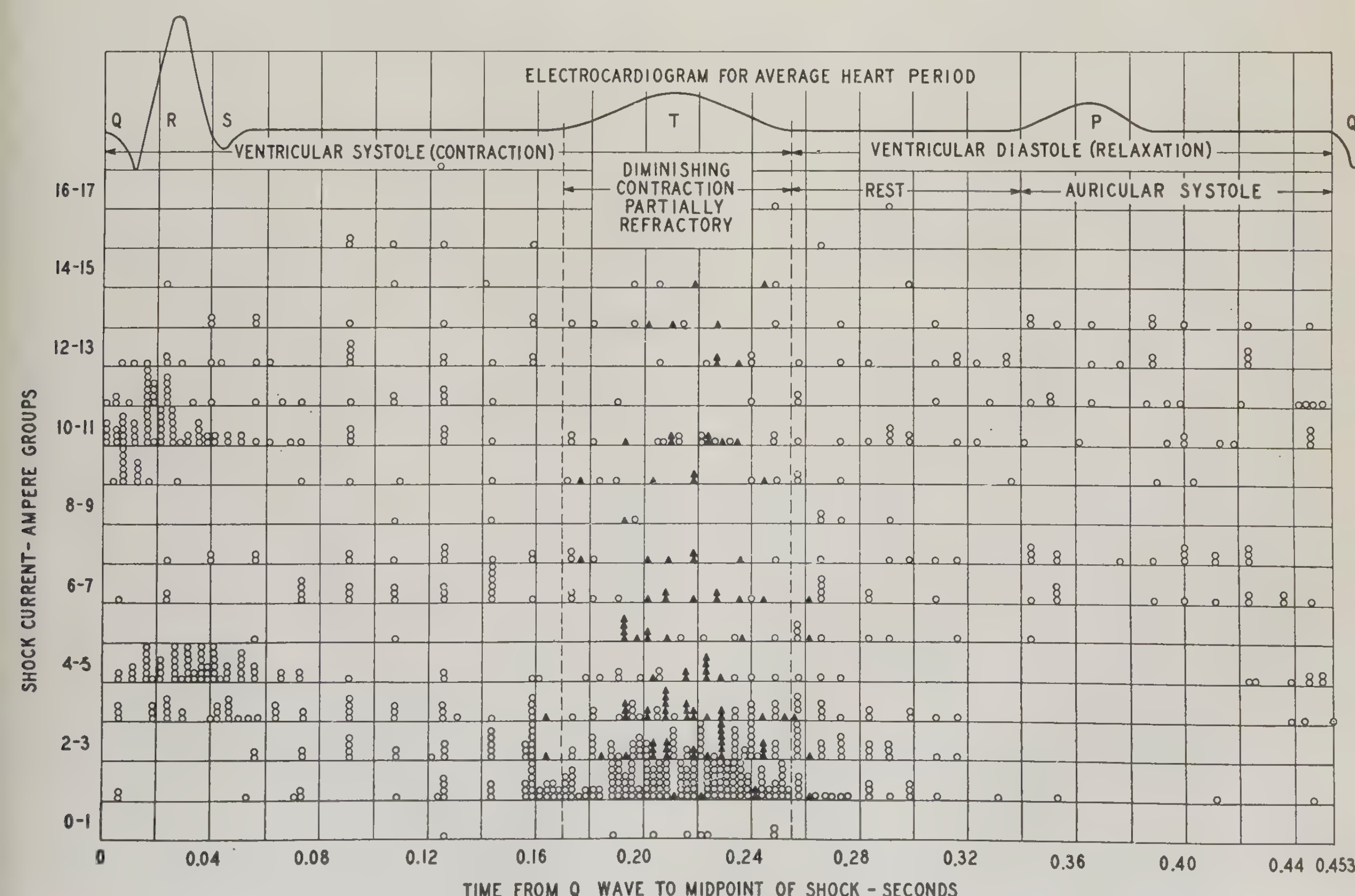


Fig. 13. Distribution in cardiac cycle and results of 913 shocks of 0.03 second duration to 132 sheep

Frequency 60 cycles; electrodes on right foreleg and left hind leg Solid triangles indicate fibrillation; open circles, co-ordinate beating

and additional observations covering one such duration seem highly desirable. Another important additional duration for study would seem to be about that of 2 heart beats.

It is not believed that increasing the duration of the shock beyond 3 seconds would reduce the average threshold current appreciably below 0.25 ampere. The establishment of ventricular fibrillation by electric shock is the result of the stimulating effect of the current rather than of injury to the heart tissue.

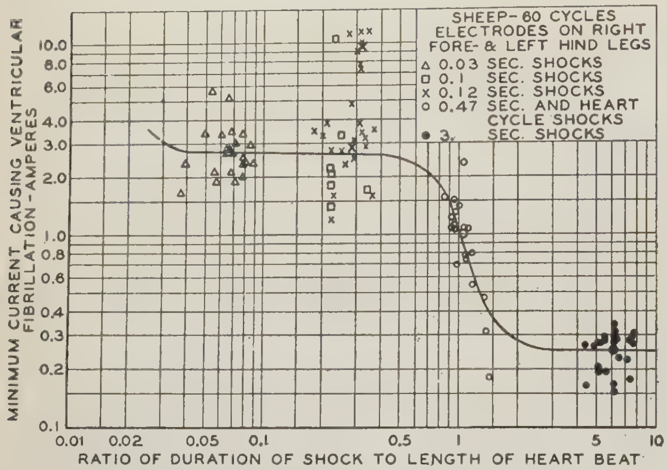


Fig. 14. Effect of duration of shock on threshold current

Thus, there should exist a current below which no such disturbance would be caused, even though the duration of the shock were extended indefinitely. From the flatness of the curve in figure 14, it would appear that this minimum threshold current had been reached. There is further support for this view in the results of some guinea pig experiments made in 1928 in which the incidence of ventricular fibrillation was equal for 10 and 2 second shocks, conditions otherwise being the same.

Ventricular fibrillation has been found to be the only serious cardiac effect of the currents applied in these tests; however, temporary disruptions of normal cardiac activity frequently have been observed. A most common effect of electric shock is a change in heart rate. Electrocardiograms after shock frequently indicate disturbances of conduction in the heart (auriculoventricular block and bundle branch block). Premature heart beats (extrasystoles) and fibrillation of the auricles also have been observed. The persistence of any of these conditions for more than a few minutes is rare. There is no evidence of any cardiac abnormalities or the presence of cardiac damage in electrocardiograms taken at intervals up to 2 months following the shocks. Opportunities for observations of this sort were afforded on more than 25 animals. Pathological examination of the hearts of several sheep and guinea pigs confirms this negative finding.

Table VI—Results of Like Shocks Repeatedly Applied to Sheep in Sensitive Phase of Cardiac Cycle

Duration of Shocks 0.03 Second. Frequency 60 Cycles. Electrodes on Right Foreleg and Left Hind Leg

Shock Current, Amperes	Animals	Shocks	Fibrillations	Per Cent Shocks Causing Fibrillation	Fibrillations on Shock Noted: 1st	2d	3d	4th or Over
1	6	62	0	0	0	0	0	0
4 (3.5–5)	11	16	11	69	7	3	1	0
12 (10–13)	6	17	6	35	2	1	0	3

Effect of Repeated Shocks in Different Phases of Cardiac Cycle

The threshold current causing fibrillation for short shocks in the sensitive phase of the heart cycle had been determined by applying successive shocks with increasing current to the same animal until fibrillation resulted. The same method had been used in attempting to find a threshold in the insensitive phases of the cardiac cycle. To obtain an independent check on these results and to ascertain whether the cumulative effect of successive shocks was important, repeated shocks of the same short duration were applied to several groups of sheep at constant current levels both above and below the previously determined thresholds.

In the sensitive phase of the cardiac cycle, shocks of 0.03 second duration were applied at 3 current levels: 1 ampere, distinctly below the average threshold of 2.5 amperes previously determined; 4 amperes; and 12 amperes. The results are summarized in table VI. Even in the most sensitive phase of the heart cycle an average of 10 1-ampere shocks never produced fibrillation, while the initial 4-ampere shocks usually caused it. The 12-ampere shocks seemed not quite so prone to produce fibrillation as the 4-ampere shocks. These results confirm the previous threshold determination in the sensitive phase of the heart cycle, showing that such threshold must lie between 1 and 4 amperes. Were the cumulative effects of successive shocks an important factor in determining whether fibrillation would result, it would be expected that the repetition of 1-ampere shocks as many as 10 times in a short interval would have produced fibrillation in some of the animals subjected to such a series of shocks, and that the 4-ampere shocks, which are not greatly above the previously determined average threshold, would not have been so prone to cause fibrillation on the initial shock. That these nearly identical repeated shocks showed otherwise is strong evidence that the cumulative effect of successive shocks did not depress the thresholds determined by the method of building up the current in successive shocks until the fibrillating point was reached.

As further evidence that successive shocks have little cumulative effect, a careful study of the electrocardiographic records of many animals taken just prior to each of a succession of shocks showed that heart action almost always returned to normal within 5 minutes of the application of current, provided fibrillation did not occur.

To test the nonoccurrence of fibrillation for cur-

rents less than about 12 amperes applied during the insensitive phases of the cardiac cycle, as shown by previous exploratory tests (see figure 13), 2 groups of sheep were given repeated 0.03 second shocks, one group at about 4 amperes and the other at about 11 amperes. All these shocks were regulated to occur at the beginning of systole, at which time the heart is in a state of increasing contraction rather than decreasing contraction, as in the partial refractory phase. In neither case is the heart muscle in a homogeneous state. The duration of the nonhomogeneous state is shorter for contraction than relaxation. The nonhomogeneity of the heart muscle in these 2 phases suggested that if fibrillation would result from shocks outside the partial refractory phase, the most likely position in the cardiac cycle would be coincident with the *QRS* wave of the electrocardiogram. The results are summarized in table VII, from which it is evident that repeated shocks at current levels well above the threshold for the partial refractory phase do not cause ventricular fibrillation when applied at the beginning of systole. The earlier exploratory tests had indicated the same results for currents above and below these current levels. Figure 13, which already has been referred to, includes these later tests as well as the original exploratory tests.

Table VII—Results of Like Shocks Repeatedly Applied to Sheep at Beginning of Systole

Duration of Shocks 0.03 Second. Frequency 60 Cycles. Electrodes on Right Foreleg and Left Hind Leg

Shock Current, Amperes				
Average	Range	Animals	Total Shocks	Fibrillations
4	3.8–4.5	11	98	0
11	9.5–12	12	116	0

Two other groups of sheep were subjected to repeated shocks at fixed currents of about 7 and 12 amperes; however the shocks were not applied at the same point in the cardiac cycle, but instead approximately at the beginning of late diastole, onset of systole, midsystole, early diastole, and finally, in the middle of the partial refractory phase. A total of 104 shocks falling outside of the partial refractory phase was applied without ventricular fibrillation, whereas 22 out of the 38 shocks applied within that phase resulted in ventricular fibrillation. The results are summarized in table VIII. Data from the shocks applied in the sensitive phase of the heart cycle are comparable with those given in table VI.

Other Observations on Short Duration Shocks

In determining thresholds and in applying repeated shocks at currents above the threshold in the sensitive phase of the heart cycle, an effort was made to time these shocks to occur as nearly as possible in the middle of this phase. Deviations of the time relations of individual hearts as compared with the averages for a large number, together with slight variations in the operation of the tripping apparatus, caused a distribution of the shocks over the whole of the partial refractory phase. All these shocks on sheep have been classified as to current values and positions of their midpoints relative to the cardiac cycle, divided for this purpose into 0.02 second intervals. The percentage of shocks causing fibrillation in each of these groups is shown in figure 15 as a function of the position of the shocks in the cardiac cycle, timed from the beginning of systole. Some of the shocks of which the midpoints fall outside the partial refractory phase, at least partially overlap this phase. The curves for the different currents indicate that not only is the partial refractory phase the only part of the cycle shown where short shocks cause ventricular fibrillation, but also that the middle of this sensitive period is the most susceptible.

From the results of this study thus far given it might be concluded that shocks would have to involve the partial refractory phase of the heart in order to cause ventricular fibrillation. Other tests, however, show that shocks less than half a heart beat in duration cause ventricular fibrillation even though timed so that had the heart continued its normal beat the shock would not have involved the partial refractory phase. The data upon which this

Table VIII—Results of Like Shocks Applied to Sheep in Successive Phases of Cardiac Cycle

Duration of Shocks 0.03 Second. Frequency 60 Cycles. Electrodes on Right Foreleg and Left Hind Leg

Position in Cardiac Cycle	Shock Current, Amperes: Average	Shock Current, Amperes: Range	Animals	Total Shocks	Fibrillations	Per Cent Shocks Causing Fibrillation	Fibrillations on Shock Noted: 1st	2d	3d	4th or Over
Late diastole (*P* to *Q*)**	7	6–8	10	26	0	0				
	12	10–14	10	19	0	0				
Onset systole (*QRS*)**	7	6–8	2	4	0	0				
	12	11–12	7	8	0	0				
Midsystole (*S* to *T*)**	7	5–8	9	14	0	0				
	12	11–14	11	16	0	0				
Early diastole (End of *T* to *P*)**	7	7	4	5	0	0				
	13	11–14	8	12	0	0				
Partial refractory period (*T* wave)**	7	5–8	12*	14	12	86	10	1	0	1
	13	10–14	11*	24	10	42	3	5	1	1
Totals			22	142	22					

* A single animal included in these totals survived 4 shocks at 13 amperes, then developed fibrillation from 1 shock at 6 amperes.
** These letters refer to corresponding waves in the electrocardiogram (see figure 9).

statement is based are summarized in table IX. It is well known to physiologists that an electrical stimulus during diastole can cause a premature heart beat, known as an "extrasystole." This premature beat will have a ventricular contraction and relaxation similar to those of a normal heart beat, though smaller in magnitude and duration. The relaxation corresponds to the partial refractory phase of the normal cycle. This suggests that extrasystoles were caused in these cases with consequent advancement in time of partial refractory phase, so that shocks half a heart beat in duration overlap, in part at least, this premature partial refractory phase, thereby bringing about fibrillation even though the shock would not have been long enough to reach the normal partial refractory phase. Pending further research, this hypothesis is advanced as a possible explanation.

Effect of High Currents

A comparison of the curves of figure 15 for different currents indicates that fibrillation is somewhat less probable at the highest currents, 4 or more times the threshold, than at currents only slightly greater than the threshold. This would not be inconsistent with the indications of this study that fibrillation is the result of the nonhomogeneous response of the heart fibers to the stimulating effect of the shock current, and, therefore, might be less liable to occur with a stimulus powerful enough to affect a great many fibers simultaneously, in spite of their differences in refractoriness. To test this evidence, repeated shocks were given to a group of 11 sheep at currents between 23 and 26 amperes, as high as could be obtained with the test apparatus. The electrodes were placed on the 2 sides of the chest to reduce the impedance and allow the maximum current to pass through the heart. Previous tests of the effect of different current pathways (table II and figure 7) had shown practically identical threshold currents for electrodes on the 2 sides of the chest and for electrodes on the right foreleg and left hind leg, the latter position prevailing in most tests. The shocks were of 0.03 second duration and were timed to occur in the sensitive phase of the cardiac cycle. Each of 10 sheep received 5 such shocks at 5 minute intervals without fibrillating. One of these 10 sheep a week previously had received a series of 3 such shocks, also without fibrillating. The eleventh sheep of this group fibrillated after the first shock of 23 amperes.

Subsequent to the high current shocks, each of the 10 surviving sheep was given additional similar shocks, except that the current was reduced to between 4 and 5 amperes. Five sheep developed ventricular fibrillation on the first shock, and 3 on the second shock. A single animal survived 5 shocks, and another animal 2, without fibrillating. A comparison of the results of the high current shocks with those at 5 amperes and with the results previously obtained with the smaller currents shown in figure 15 definitely establishes that the susceptibility of the heart to ventricular fibrillation becomes very much less when the current is increased to about 25 amperes, 10 times the average threshold value for shocks of short duration. This variation of susceptibility to fibrillation is shown graphically in figure 16, the data from the recent high current shocks being combined with those for the central portion of the partial refractory period of figure 15.

To determine whether such a decrease in susceptibility to fibrillation would occur also for shocks of about 3 seconds duration if the current were increased about 15 times the average threshold, 5 sheep were subjected to 4 ampere shocks of 3 seconds duration. In all cases ventricular fibrillation resulted on the initial shock, indicating either that at this duration no such decrease in susceptibility takes place with increase of current, or that 4 amperes is not a sufficiently high current to bring it about. The apparatus was not capable of giving much higher currents

Table IX—Development of Fibrillation in Sheep by Short Shocks Not Involving Normal Sensitive Phase of Cardiac Cycle

Current 4 Amperes, 60 Cycles. Electrodes on Right Foreleg and Left Hind Leg. Shocks Initiated During Ventricular Diastole

	Shock Durations in Seconds and as Percentages of Duration of Heart Beat											
	Final Fibrillating		1st Before Fibrillating		2nd Before Fibrillating		3d Before Fibrillating		4th Before Fibrillating		5th Before Fibrillating	
Test	Sec	%	Sec	%	Sec.	%	Sec	%	Sec	%	Sec	%
1,495	0.26	62	0.24	57	0.22	50	0.21	50	0.18	42	0.15	32
1,496	0.22	46	*									
1,497	0.21	47	0.19	42								
1,498	0.21	57	0.21	53	0.21	54	0.22	51	0.17	35		
1,499	0.20	45	0.20	38								
1,500	0.27	41	0.21	45								
1,501	0.19	44	0.19	39								
1,502	0.24	43	*									
1,503	0.20	43	0.17	35								
1,504	0.22	45	*									
1,505	0.20	44	0.18	40								
Geometric means		47		43								

* Fibrillation on first shock.

for this duration. It seems probable that 25 ampere shocks would be less liable to produce fibrillation, but such shocks for a duration as long as 3 seconds, particularly if brought about through accidental contact, would cause severe burning and probably other serious injury.

These results are in agreement with those of Prevost and Battelli,[15] who found a diminished tendency to ventricular fibrillation as the shock voltage was increased, in their experiments on dogs and smaller animals conducted at the University of Geneva and reported in 1899.

Recovery of Heart From Ventricular Fibrillation

Recovery of the heart from ventricular fibrillation by the application of a short intense shock was reported first by Prevost and Battelli[16] in 1899. Battelli[17] subsequently made extensive use of this method of recovery in his study of the survival of the brain during cerebral anemia in dogs. He caused circulatory failure by initiating ventricular fibrillation with an electric shock, and then re-established a co-ordinate heart beat, and hence a normal circulation, by application of what recently has been termed a "counter-shock."

Considerable search in technical literature has brought to light an interesting paper by Abildgaard[18] published in 1775 recounting a very early investigation of the effects of electric shock. Abildgaard reports an unsuccessful attempt to kill a young horse with the intention of subsequently trying to revive it. Failing in this, he reports success with chickens, the earliest record found of the use of electricity to counteract the effect of a previous electric shock:

"Therefore unwillingly leaving the young horse, I seized a hen, which at the first shock, directed from a single vessel (Leyden jar) on the head, I prostrated, so that the hen lay entirely dead without any feeling nor could it be aroused by any stimulant; indeed another shock, now having been given on the head in vain, I believed myself to have been mistaken in the hope of resuscitation; for the hen remained dead even though shocks on the head had been repeated. Little contented with this success, I tried the electric shock directed through the breast to the dorsal spine, nor in vain; for having been left on the ground it raised up suddenly and quietly walked on its feet."

* * * * * *

"I tried this same experiment with a poultry-cock, which, after many continuous shocks had been directed on the head, appeared altogether dead so that blood ran forth from the nose and mouth, but from the shock on the breast it quickly flew away and knocked the electric vessel on the earth and broke it."

Kouwenhoven, Langworthy, and Hooker[19] recently reported the recovery of dogs from fibrillation by application of a counter-shock. They also reported the successful use of certain drugs to improve the chance of such recovery.

In the present investigation, the determination of the threshold fibrillating current for different species, and under different conditions previously described, required the production of fibrillation as the end result of such tests and therefore furnished an extended opportunity for experiments on the recovery of the heart from this condition. Counter-shock has been employed almost exclusively, although the application of pharmacological methods has been experimented with in collaboration with Wanger and Blackberg[20] of Columbia University.

The smaller species of animals generally recover spontaneously from ventricular fibrillation. A great many guinea pigs, rabbits, and cats were used during 1928. In a few instances where ventricular fibrillation persisted after shock, a subsequent intense shock was applied. The results on these animals were uniformly good and normal heart action followed immediately upon the cessation of the counter-shocks. However, because of the proneness of these small

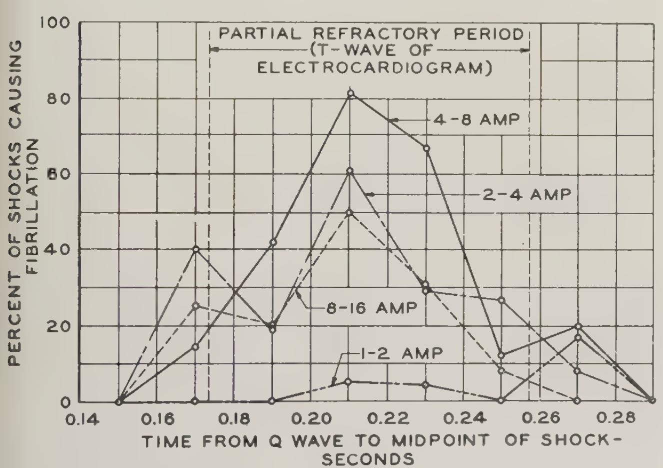


Fig. 15. Effect of position of shock in cardiac cycle on susceptibility of sheep hearts to ventricular fibrillation

Shock duration 0.03 second; frequency 60 cycles; electrodes on right foreleg and left hind leg

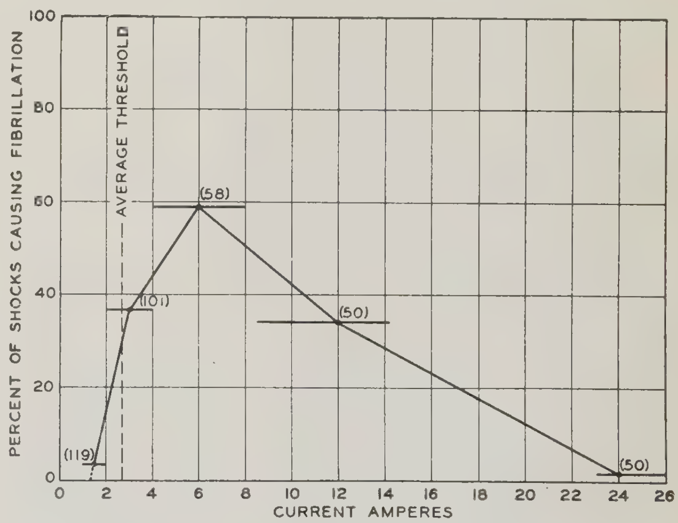


Fig. 16. Effect of current on susceptibility of sheep hearts to ventricular fibrillation

Shocks of 0.03 second, 60 cycles, applied in partial refractory period of cardiac cycle

Number of shocks and current spread indicated for each point on curve

animals to recover spontaneously, it was impossible to be sure that these recoveries were the result of the counter-shocks.

The opportunity to experiment on recovery from ventricular fibrillation with large animals arose with the use of sheep and other large species in 1932. Initially, counter-shocks were applied through the electrodes used for the fibrillating shock (on the forelegs). Application of full voltage (3,000 volts) with no "building-out" resistance in the circuit gave currents of from 4 to 7 amperes, but did not bring about recovery. Special electrodes then were provided for the counter-shocks, one fixed on the chest surface and the other under the skin just over the shoulder blade. Application of 3,000 volts to these electrodes gave currents up to 25 or 30 amperes, indicating a very low resistance pathway. This tripped the circuit breaker of the power supply (rated at only 10 kva) within about 0.1 second. The large area of the electrodes and the saturation of the one on the chest surface with salt solution prevented burning at the points of contact. Since the path of the current in this case included the spinal cord, the counter-shock conceivably might injure this important nerve highway. After a few tests of this kind all subsequent experiments with counter-shock were performed with electrodes outside the skin on both sides of the chest so as to include the heart between them. This did not result in as large currents, but was fully as effective and had the great advantage that the animal could be kept for future observation. Results of the application of counter-shock for recovery are summarized in table X, grouped in accordance with the experimental conditions for the shocks initiating fibrillation. Three types of results have been indicated. The conclusive recovery and failure to recover are self-explanatory. In certain cases temporary recoveries of co-ordinate heart beat were obtained, but these beats either decayed to a complete standstill or fibrillation returned. In most of these cases normal respiration was not recovered. In some of them ventilation of the lungs was not possible by pressure on the chest because of congestion of the trachea or a closure of the epiglottis, rendering passage of air impossible. The congestion of the trachea was a result of the use of nembutal in large quantities for maintaining deep anesthesia over an extended period. Closure of the epiglottis has not been explained, and whether it is a result of the anesthetic or the electric shock, or a combination of these is not known. This occurred most frequently with pigs. Operating on the trachea through the neck, or mechanically opening the trachea by insertion of a tube, if done in time, would relieve this condition and save the animals whose hearts had been recovered.

Figure 17 is an electrocardiographic record of heart action before and after a shock that caused fibrillation and at different stages after the application of a counter-shock that arrested the fibrillation and allowed the heart to resume its co-ordinate beating. The current and voltage of the fibrillating shock and the counter-shock are shown also to the same scale for comparison. The fibrillating shock of 6 amperes and 0.03 second duration occurred during the sensitive phase of the cardiac cycle. The counter-shock which followed $1^1/_2$ minutes later was of 26 amperes for 0.1 second duration. Times marked on the different sections are referred to the beginning of the record. It may be observed that the last electrocardiogram is practically identical with the preshock electrocardiogram. Many sheep have been observed

Table X—Results of Applying 60-Cycle A-C Counter-Shocks at 3,000 Volts to Recover Hearts From Ventricular Fibrillatio

	Fibrillating Shock				Conclusive Recovery			Temporary Recovery			Other Failure After Averag of Mor Than 2 Shock
Species	Duration, Seconds	Current, Amperes	Electrode Positions	Animals	1st Shock	2d Shock	3d Shock	1st Shock	2d Shock	3d Shock	
1. Counter-Shock Through Same Electrodes on Forelegs, Current 5 Amperes, Duration 1 Second											
Sheep	3	0.3–0.7	Forelegs	2	0	0	0	0	0	0	2
2. Counter-Shock Electrodes on Chest and Back, Current 25–30 Amperes, Duration 0.06 to 0.1 Second											
Sheep	3	0.3–0.7	Forelegs	10	1	1	0	1	2	0	5
Sheep	3	0.1–0.4	Chest and foreleg	11	8	1	0	0	0	1	1
Sheep	3	0.1–0.4	Head and hind leg	9	1	0	0	0	0	0	8
Calf	3	0.2 0.5	Foreleg and hind leg	2	0	2	0	0	0	0	0
Totals for group 2				32	14 Recoveries (44%)			4 Temporary Recoveries			14
3. Counter-Shock Electrodes on Right and Left Sides of Chest, Current 22–27 Amperes, Duration 0.06 to 0.1 Second											
Calf	3	0.2–0.5	Foreleg and hind leg	9	7	1	0	0	0	0	1
Pig	3	0.2–0.3	Foreleg and hind leg	9	3	2	0	1	0	0	3
Dog	3	0.1–0.2	Foreleg and hind leg	9	3	0	0	1	0	0	5
Sheep	3	0.2	Foreleg and hind leg	1	0	0	0	1	0	0	0
Sheep	0.07	20–25	Chest	2	2	0	0	0	0	0	0
Sheep	3	0.1–0.4	Chest	10	6	2	1	0	0	0	1
Sheep	3	0.7–2 (d-c)	Foreleg and hind leg	11	4	2	0	0	1	1	3
Sheep	0.03 ±	2 5	Foreleg and hind leg	62	30	10	1	2	3	0	16
Sheep	0.12 ±	1–10	Foreleg and hind leg	13	3	0	0	0	1	0	9
Totals for group 3				126	77 Recoveries (61%)			11 Temporary Recoveries			38

Range of elapsed time from fibrillating shock to counter-shocks: group 1—1 minute, 30 seconds to 7 minutes, 30 seconds; group 2—1 minute, 10 seconds to 4 minutes; group 3—55 seconds to 2 minutes, 30 seconds.

for periods of months after recovery from fibrillation, with no evidence of abnormalities. Several have given birth to normal lambs, and in many instances the recovered sheep have been used in subsequent tests and again recovered.

The possibilities of counter-shock have not been fully explored to determine the optimum conditions for its application, particularly as regards magnitude and character of current, its duration, and points of application. In regard to the latter, however, it would seem that some short path embracing the heart would be best. Any technique of recovery of the heart must be applied promptly so as not to permit deterioration of the brain which might result in impairing the competency of the victim if recovered. While the time limit would depend on many factors, it is a matter of minutes rather than seconds. The prompt application of artificial respiration ventilates the lungs and is believed also to maintain a small circulation of blood, sufficient to delay degeneration in the brain. This is of fundamental importance in the development of practical recovery methods. Wiggers recently has pointed out that maintenance of coronary circulation is essential for recovery from fibrillation.

Were suitable arrangements and methods developed for the practical application of counter-shocks, such shocks might be applied mistakenly to a victim whose heart was not in ventricular fibrillation. To determine whether in such an event ventricular fibrillation would be caused, 5 "counter-shocks" of about 25 amperes for about 0.06 second were applied to each of 9 sheep whose hearts were beating normally. Only 3 of the 45 shocks applied caused fibrillation, and in every such case recovery was obtained by the immediate application of another similar shock. This experiment was performed in 1932 prior to the development of apparatus for the controlled placement of short shocks, so that the shocks naturally fell at random. It was also prior to the determination that such high current shocks of short duration were not likely to cause fibrillation even when the shock occurred during the sensitive phase of the heart cycle. In the light of the subsequent experiments, it is evident that the liability of causing ventricular fibrillation by randomly placed short shocks at the high currents employed in counter-shock is small.

Summary of Results and Conclusions

1. Current rather than voltage is the proper criterion of shock intensity.

2. The stimulating effect of current through the heart can derange its action, causing ventricular fibrillation without damage to the cardiac tissues but resulting in death within a few minutes, unless the fibrillation is arrested.

3. A current just below the threshold for ventricular fibrillation is the maximum to which man safely may be subjected. Based upon numerous tests on animals of several species comparable in size with man, this maximum current is about 0.1 ampere, for a duration of one second or more and a pathway between an arm and a leg.

4. The threshold fibrillating current is affected by:

a. Species and size of animal. Among the different species the threshold current increases roughly with both body weight and heart weight.

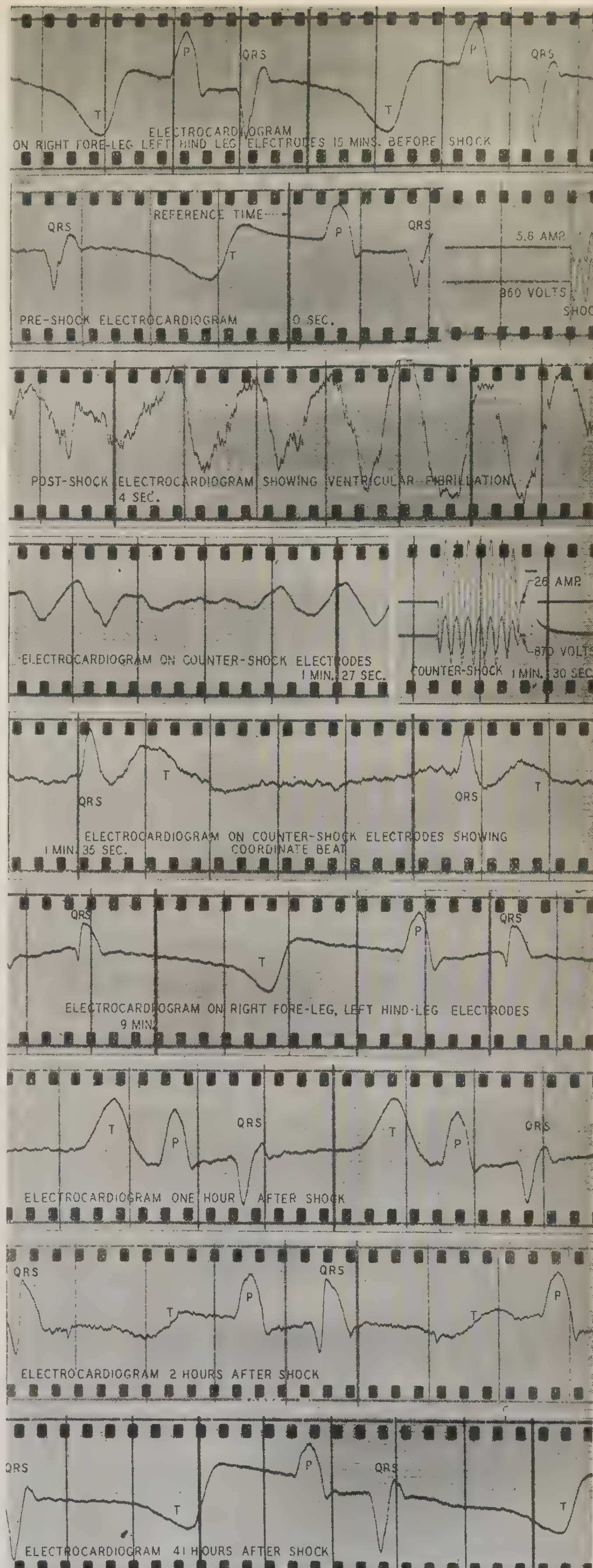


Fig. 17. Record of restoration of sheep heart from ventricular fibrillation by a counter-shock

b. Current pathway. Pathways from arm to leg, across the chest, chest to arm, and head to leg may be expected to give about the same threshold current. The pathway between the arms would be expected to give a somewhat higher threshold current. For the pathway from leg to leg, the proportion of current reaching the region of the heart is so small that fibrillation is not liabel to result, even at currents of 15 amperes or more, although such currents probably would burn the victim unless the contacts were good and the shock of short duration.

c. Frequency. For shocks of one second or more in duration, the 25 cycle threshold current is about 25 per cent higher than the 60 cycle value, and the d-c threshold current 5 times the 60 cycle value. For shock durations of a small fraction of a second this relation probably does not hold, all thresholds being expected to approach one another.

d. Time of occurrence of short shocks in relation to cardiac cycle. The heart is most sensitive to fibrillation for shocks occurring during the partial refractory phase of its cycle, which is about 20 per cent of the whole and which occurs simultaneously with the *T* wave of the electrocardiogram. With shocks of a duration of about 0.1 second or less, it is practically impossible to produce ventricular fibrillation, unless such shocks coincide in part at least with this sensitive phase of the cardiac cycle. The middle of the partial refractory phase is more sensitive than its beginning or end.

e. Duration of shock. The threshold current varies inversely with shock duration but not uniformly, being most sensitive to change as the duration approaches the duration of one heart beat. Within the sensitive phase of the heart cycle the threshold fibrillating current for shock durations of about 0.1 second or less is 10 or more times the threshold for durations of 1 second or more. Shocks $^1/_3$ or more of the heart cycle in duration may cause ventricular fibrillation, even though they would not extend into the sensitive phase of the cycle if the heart continued its normal beat after the initiation of the shock. The reason for this is probably the initiation of a premature heart beat which brings about a premature sensitive phase prior to the end of the shock.

5. Successive shocks have no cumulative effect on the susceptibility of the heart to fibrillation.

6. The susceptibility of the heart to fibrillation by short shocks increases with current up to several times the threshold, then diminishes, becoming very small at currents of the order of 25 amperes through the body in the vicinity of the heart. However, other serious injury may be expected from such currents when brought about by accidental contacts.

7. Fibrillation produced by an electric shock will in the majority of cases be arrested by a subsequent electric shock of high intensity and short duration through the heart, allowing the resumption of co-ordinate beating with no permanent damage.

With about 60 per cent success in recovering animals comparable in size with man from ventricular fibrillation by the application of a rather arbitrarily chosen counter-shock, further study is desirable to develop the optimum conditions and practical apparatus for utilizing this method in accident cases. The use of a simple electrocardiograph appears desirable since ventricular fibrillation cannot be recognized positively by stethoscopic examination.

Should a counter-shock be applied mistakenly to a co-ordinately beating heart, the liability of its causing fibrillation is small and, should this occur, another counter-shock probably will arrest the fibrillation and bring back co-ordinate heart action.

To be successful, a counter-shock must be administered promptly after the fibrillating shock, probably within a few minutes.

The use of a counter-shock does not in any way lessen the need for maintaining respiration, by artificial means if necessary. In fact, the administration of artificial respiration even in the interval before application of a counter-shock is highly advisable, not only for respiration itself, but because of the accompanying slight circulation which will assist in the nutrition of the heart and delay degeneration of the brain.

References Cited in Text

1. Nachricht von einigen Electrischen Versuchen, Daniel Gralath. Versuche und Abhandlungen der Naturforschenden Gesellschaft in Danzig, Part I, Danzig, 1747, p. 506.

2. Recerches sur les Causes Particulières des Phènomènes Èlectriques (a book, third edition), L'Abbe Jean Antoin Nollet. Chez les Frères Guerin, Paris, 1753, p. 366, *et seq.*

3. Experiments and Observations on Electricity (a book, fifth edition), Benjamin Franklin. F. Newbery, London, 1774, p. 38, 160–1, 333.

4. The Intellectual Rise in Electricity (a book), P. Benjamin. Wiley and Sons, New York, 1898, p. 507, 522, 529.

5. Beitrag zur Kenntnis der Vorgänge beim Stromdurchgang durch den Menschlichen Körper, *Elektrizitätswerken des Kantons Zurich. Bull.* des Schweizer Elektrotechnischen Vereins, v. 2, No. 13, 1929, p. 428.

6. Shock Threshold Fixes Appliance Insulation Resistance, Gordon Thompson. *Elec. Wld.*, v. 101, June 17, 1933, p. 793.

7. Oscillograph for Recording Transient Phenomena, W. A. Marrison. A.I.E.E. Trans., v. 48, No. 3, July, 1929, p. 939.

8. Influence du Nombre des Périodes sur les Effets Mortels des Courants Alternatifs, J. L. Prevost et F. Battelli. *Jl. de Physiol. et de Path. gén.*, v. 2, 1900, p. 755.

9. Recerches sur les Excitations Electriques du Coeur, E. Marey *Jl. de l'Anat. et de Physiol.*, v. 8, 1877, p. 60.

10. Bundle Branch Block and Arborization Block, Frank N. Wilson and George R. Herrmann. *Arch. Int. Med.*, v. 26, 1920, p. 153.

11. An Experimental Study of Incomplete Bundle Branch Block and of the Refractory Period of the Heart of the Dog, Frank N. Wilson and George R. Herrmann. *Heart*, v. 8, 1921, p. 229.

12. The T-Wave Deflection of the Électrocardiogram, Frank N. Wilson, A. Garrard MacLeod, and Paul S. Barber. *Trans.* Assn. of Am. Physicians, v. 46, 1931, p. 29.

13. Susceptibility of the Heart to Electric Shock in Different Phases of the Cardiac Cycle, H. B. Williams, B. G. King, L. P. Ferris, and P. W. Spence. *Proc.* Soc. Exper. Biol. and Med., v. 31, 1934, p. 873.

14. The Effect of Electric Shock on Heart Action With Special Reference to Varying Susceptibility in Different Parts of the Cardiac Cycle, B. G. King. Columbia University Thesis, Aberdeen Press, New York, 1934.

15. La Mort Par Les Courants Électriques—Courants Alternatifs À Haute Tension, J. L. Prevost et F. Battelli. *Jl. de Physiol. et de Path. gén.*, v. 1, 1899, p. 427.

16. Quelques Effets des Dècharges Electriques sur le Coeur des Mammifères, J. L. Prevost et F. Battelli. *Jl. de Physiol. et de Path. gén.*, v. 2, 1900, p. 40.

17. Le Rétablissement des Fonctions du Coeur et du Systeme Nerveux Central apres l'Anémie Totale, F. Battelli. *Jl. de Physiol. et de Path. gén.*, v. 2, 1900, p. 443.

18. Tentamina Electrica in Animalibus Instituta, C. P. Abildgaard. Societatis Medicae Havniensis *Collectanea*, v. 2, 1775, p. 157.

19. The Effect of Alternating Electrical Currents on the Heart, D. R. Hooker, W. B. Kouwenhoven, and O. R. Langworthy. *Am. Jl. Physiol.*, v. 103, 1933, p. 444.

20. Studies in Revivification—Organization of Resuscitation Measures, J. O. Wanger and S. N. Blackberg. *Am. Jl. Med. Sci.*, v. 183, No. 2, 1932, p. 241.

References Not Cited in Text

Accident, Danger, and Death by Electricity

21. Gutachten über die höchstzulässige Wechselstromspannung in Kontaktleitungen für Bahnen sowie über die zweckmässige Anordnung von Hochspannungsleitungen länge des Bahnkörpers, Gisbert Kapp, Silvanus P. Thompson, and H. F. Weber. Report to Brown, Boveri, & Co., Baden. Buchdruckerei Berichthaus, Zurich, 1897.

22. Death by Electricity, R. H. Cunningham. *Elec. Wld. and Eng.*, v. 34, 1899.

23. Les accidents causes par courantélectrique, M. A. D'Arsonval. *Bull.* Soc. Internat. des Élect., 1910, p. 355.

24. Death by Electric Currents and by Lightning, A. J. Jex-Blake, *Brit. Med. Jl.*, v. 1, 1913, p. 425, 492, 548, 601.

25. Contributo alla conoscenga degli infortuni per corrente ilettriche a basse tensione, Capello and Pallegrini. *Arch. di Antrop. crim.* (etc.), s. 4, 8, Torino, 1913, p. 201.

26. Le danger des courants électriques industriels et domestiques des faibles tensions, P. Chapius. *These de Paris*, 1919.

27. L'électrocution accidentelle par les courants d'éclairage domestique en med. légale, V. Balthazard. *Bull.* Acad. de Med., Paris, 87, 1922, p. 160.

28. Der Elektrische Unfall (a book), S. Jellinek. Leipzig and Vienna, 1927.

29. Electric Shock—Interpretation of Field Notes, Wills Maclachlan. *Jl. Ind. Hyg.*, v. 12, 1930, p. 291, and (Part II), *Jl. Ind. Hyg.*, v. 16, 1934, p. 52.

30. Hazard of Low Voltage Shock, H. B. Williams. *Jl.* Am. Medical Assn., v. 97, 1931, p. 156.

31. The Problem of Electric Shock, H. B. Williams. *Am. Jl. of Surgery*, v. 27, No. 1, 1935, p. 151.

32. Electrocution—Notes on the Mechanism of Electric Death, Harold H. U. Cross. *The Electrician*, May 26, 1933, p. 677.

Experimental Work and Studies

33. Einige neue Versuche über Herzebewegung, Ludwig und Hoffa. *Ztschr. f. rat. Med.*, v. 9, 104, 1850, p. 107.

34. Das Coordinationscentrum für den Herzkamerschlag, H. Kronecker. *Deutsche Med. Wchnschr.*, v. 10, 1884, p. 364.

35. On the Action of Electricity on the Human Heart, J. Dixon-Mann. *Med. Chron.*, Manchester, v. 2, 1885, p. 7.

36. Influence des Variations de la Force Électromotrice sur les Effets Physiologiques du Courant Continu, *Comp. Rend.* Soc. de Biol., Paris, 1891, 9 s., v. 3, p. 286.

37. La mort par l'électricite dans l'industrie; ses mècanismes physiologiques; moyens preservateurs, M. A. D'Arsonval. *Comp. Rend.* Soc. de Biol., 8 ser., v. 4, Paris, 1887, p. 95.

38. Experiments With Alternate Currents of Very High Frequency and Their Application to Methods of Artificial Illumination, N. Tesla. *Elec. Wld.*, July 11, 1891.

39. Sur les effets physiologiques compares des divers procédes d'électrisation; nouveaux modes d'application de l'énergie électrique, la voltaisation sinusoidal; les grandes fréquences et les hauts potentiels, M. A. D'Arsonval. *Bull.* de l'Acad. de Med., v. 27, Paris, 1892, p. 424.

40. Notes on the Effects of High Frequency Electrical Discharges Passed Through the Body, E. Thompson. Am. Electrotherap. Assn., v. 4, 1894, p. 261.

41. La Mort par les Courants Électriques, Courant Alternatif a Bas Voltage, J. L. Prevost et F. Battelli. *Jl. de Physiol. et de Path. gén.*, v. 1, 1899, p. 399.

42. La Mort par Les Dècharges Électriques, J. L. Prevost et F. Battelli. *Jl. de Physiol. et de Path. gén*, v. 1, 1899, p. 1085.

43. La Mort par les Courants Électriques—Courant Continu, J. L. Prevost et F. Battelli. *Jl. de Physiol. et de Path. gén.*, v. 2, 1900, p. 689.

44. Contribution a la connaissance de l'action des courants électriques continues a haute voltage sur les chevaux, S. Arloing. *Jl. de Physiol. et de Path. gén.*, v. 3, 1902, p. 967.

45. Electric Shocks at 500 Volts, A. P. Trotter. *Jl.* Inst. E.E., v. 31, 1902, p. 762.

46. Physiological Tolerance of Alternating Current Strength up to Frequencies of 100,000 Cycles per Second, Kennelly and Alexanderson. *Elec. Wld.*, v. 50, 1910, p. 154.

47. Sur les effets physiologiques des courants électriques, Weiss. Soc. Intern. des Élect. *Bull.*, v. 3, ser. 1, 1911, p. 417.

48. Expériences sur l'èlectrocution effectuèes du laboratoire central d'électricitè par la commission chargè d'èlaborer le texte de l'instruction sur les premiers soins a donner aux victimes de accidents èlectriques, M. Zacon, Soc. Intern. des Elec. *Bull.*, v. 3, ser. 1, 1911, p. 437.

49. Discussion on the Pathological Changes Produced in Subjects Rendered Unconscious by Electric Shock, J. A. MacWilliams. *Proc.* Roy. Soc. Med., v. 15, 1922, p. 49.

50. La Fibrillation Ventriculaire, L. Gallavardin. *Jl. de Med. de Lyon*, v. 8, No. 185, 1927, p. 453.

51. Electropathology, R. H. Jaffe. *Arch. Path.*, v. 5, 1928, p. 837.

52. Experimental Electric Shock, R. W. Ian Urquart. *Jl.* Inst. E.E., v. 31, 1927, p. 762 and (part II) *Jl. Ind. Hyg.*, v. 11, 1929, p. 155.

53. Effects of Electric Shock, W. B. Kouwenhoven and O. R. Langworthy. A.I.E.E. Trans., v. 49, 1930, p. 381.

54. Chemical Factors in Ventricular Fibrillation, D. R. Hooker. *Am. Jl. of Physiol.*, v. 92, 1930, p. 639.

55. Nerve Cell Injury in Cases of Human Electrocution, O. R. Langworthy. *Jl.* Am. Medical Assn., v. 95, No. 2, 1930, p. 107.

56. An Experimental Study of Abnormalities Produced in the Organism by Electricity, O. R. Langworthy and W. B. Kouwenhoven. *Jl. Ind. Hyg.*, v. 12, 1930, p. 31.

57. Electrocardiography of the Effects of Electrocution of the Heart, P. Duchosal. *Comp. Rend.* Soc. de Biol., v. 12, 1931, p. 639.

58. What Are the Effects of Electric Shock? O. R. Langworthy and W. B. Kouwenhoven. Elec. Engg., v. 50, No. 6, 1931, p. 306.

59. The Importance of the Points of Contact in Electrical Injuries, O. R. Langworthy and W. B. Kouwenhoven. *Jl. Ind. Hyg.*, v. 13, 1931, p. 145.

60. The Reaction of the Rabbit to Electric Current Directed Through the Heart, Harold Ettinger. *Am. Jl. Physiol.*, v. 105, No. 2, 1933, p. 457.

61. Experimentelle Untersuchungen über den Einfluss von niedergespanntem Gleich- und Wechelstrom auf Kreislauf und Atmung als Beitrag zur Frage des elektrischen Unfalls, G. Schlomka und G. Schrader. *Arch. f. Gewerbepathol. u. Gewerbehyg.*, v. 5, 1934, p. 615.

Resuscitation

62. Resuscitation, J. A. Brown. *Min. Elect. Eng.*, v. 3, 1923, p. 347.

63. Present Status of the Problem of Resuscitation in Case of Electric Shock, C. K. Drinker. *Jl. Ind. Hyg.*, v. 5, 1923, p. 255.

64. Counter-Shock as a Method of Resuscitation Following Electrocution, A. Campbell and L. Hill. *Jl. Ind. Hyg.*, v. 6, 1924, p. 267.

65. Artificial Respiration in Electric Shock and Gas Poisoning, C. K. Drinker. *Jl.* Am. Medical Assn., v. 83, 1924, p. 764.

66. Resuscitation After Electric Shock, Wills Maclachlan. *Jl.* Am. Medical Assn., v. 83, 1924, p. 756.

67. On Recovery of the Heart in Electric Shock, D. R. Hooker. *Am. Jl. of Physiol.*, v. 91, 1929, p. 305.

68. The Revival of the Heart From Ventricular Fibrillation by Successive Use of Potassium and Calcium Salts, Carl Wiggers. *Am. Jl. Physiol.*, 1930, p. 351.

69. Cinematographic and Electrocardiographic Observations of the Natural Processes in the Dog's Heart; Its Inhibition by Potassium and the Revival of Co-ordinated Beats by Calcium, Carl Wiggers. *Am. Heart Jl.*, v. 5, 1930, p. 351.

70. The Action of Antagonistic Salts, Carl Wiggers. *Am. Jl. of Physiol.*, v. 93, 1930, p. 197.

71. Recovery From Ventricular Fibrillation Initiated by Electric Shock Through Application of Short Period Shock at High Voltage, P. W. Spence, B. G. King, and S. N. Blackberg. Reported (not printed), Med. Soc. of the Col. of Phys. and Surg., Columbia Univ., New York, March 6, 1933.

72. Action of Quinine and Quinidine on Patients With Transient Ventricular Fibrillation, S. P. Schwartz and A. Jeger. *Am. Heart Jl.*, v. 9, 1934, p. 292.

ose returning through the Pacific Northwest from the A.I.E.E. summer convention at Pasadena, will find the Bonneville $32,500,000 deral power-navigation project (below) conveniently available from Portland, Oregon, via the famous Columbia River Highway

The Qualities of Incandescent Lamps

The application of scientific principles and engineering methods in the incandescent electric lamp field is outstandingly successful, and is one of the distinguishing features of American engineering. In this paper methods of appraising the qualities of electric lamps are discussed. The wide difference in quality between lamps that comply with American specifications and those that do not, and the economic consequences of the use of lamps that are without benefit of scientific methods are indicated.

By
PRESTON S. MILLAR
MEMBER A.I.E.E.
Electrical Testing Laboratories, New York, N. Y.

THROUGH this paper, electrical engineers are invited to turn their attention to the most important of the small utilization equipment served by electric power supply systems. Utilization equipment may well benefit from the same degree of engineering consideration and expertness that is expended upon generating and distributing equipment. The incandescent lamp has benefited more than most other small utilization equipment by sustained engineering treatment through the years of its history, perhaps because it was the earliest of electrical utilization devices, and because it is still the most generally employed. Where the engineer's point of view obtains and engineering procedure is followed, as with the more widely used American-made incandescent lamps, the results justify the engineering process. Where that viewpoint is lacking, as in the case of some of the less well-known domestic lamp products and of certain imported lamp products, the resultant inferiority of performance testifies to the lack of good engineering.

It may be recalled that the functional value of utilization equipment usually may be expressed in terms of safety, effectiveness, durability, convenience, and economy. When these criteria are applied in comparison of incandescent electric lamp products, certain aspects of value need not be discussed, because, being characteristic of all modern incandescent electric lamp products, they do not constitute variables. Thus, safety, a critical factor in some utilization equipment, is achieved so generally in electric lamps for ordinary lighting service that it may be dismissed without comment. Likewise, the color of the light needs no special consideration. The high temperature radiation departs but little from that of a black body, and is acceptable for general lighting purposes. These characteristics are so generally satisfactory that they leave little to be desired. It is with the qualities of effectiveness, durability, convenience, and economy that comparison is involved, and they are interdependent, because efficiency and economy may be enhanced at the expense of long life and convenience, and *vice versa*. It is characteristic of incandescent electric lamps that light output and efficiency may be increased at the expense of reduced life, increased annoyance due to outages, and increased inconvenience and expense of replacement. With respect to a 60 watt tungsten filament lamp the relation between efficiency and life may be expressed by the formula $t = Ke^b$, where t is the life in hours, K is a constant depending upon the inherent physical quality of the lamp, e is the efficiency in lumens per watt, and b for the lamps under discussion, is of the order of -6.8. Figure 1 shows the relation graphically. In the interest of simplicity this paper deals with multiple lamps of sizes generally used in residence service. Other types of lamps, such as street lighting series lamps and projector lamps, offer distinctive problems worthy of separate treatment. To include such treatment in this paper would extend the discussion unwarrantably.

Because of these characteristics it is necessary to determine the efficiency at which incandescent electric lamps shall be operated, and what life shall be obtained for optimum results. This decision is influenced by the following considerations:

1. Where electric energy is available at low cost the convenience that comes with long lamp life may be sought to a degree that is not economical where electric energy is more costly.
2. It is economical to seek higher efficiency of light production at the expense of lamp life when the price of lamps is low.
3. Economy requires a longer lamp life, even at the expense of efficiency of light production, when the price of lamps is high.
4. Longer life at the expense of efficiency of light production may be sought when for any reason frequent outages are of serious consequence or replacements are especially difficult.
5. A range of cost of electric energy and of ease of replacement of lamps must be taken into account in a practical consideration of efficiency and life.

The optimum life, taking into account only costs of lamps and of electrical energy, is found through the following formula that is correct for 1,000-hour, 60-watt lamps rated at 12.6 lumens per watt, and selling for 15 cents each: $t = 5{,}800\ p/rw$, where t is the optimum life, r is the energy cost in cents per kilowatt-hour, w is the mean watts throughout life, and p is the price of the lamp in cents. This neglects the objectionable features of outages and the inconvenience and expense of replacements, the importance of which it is difficult to appraise. Neglecting these less tangible aspects, and considering only the costs of lamps and of electric energy, the unit costs of light for good quality modern tungsten filament 60 watt lamps costing 15 cents

A paper recommended for publication by the A.I.E.E. committee on the production and application of light, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 9, 1936; released for publication April 2, 1936.

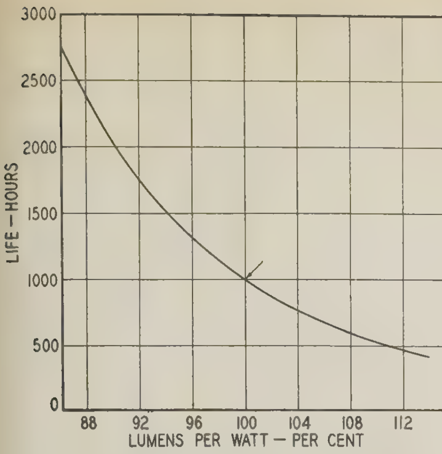
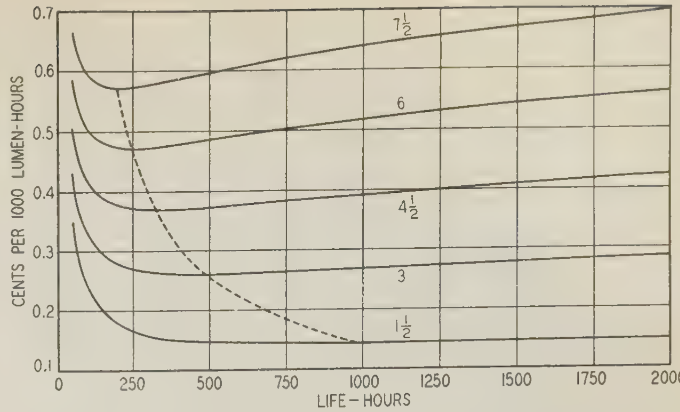


Fig. 1 (Left). Characteristic life-efficiency relationship for 60 watt lamps

Fig. 2 (Right). Unit costs of light for 60 watt lamps at 15 cents each, neglecting labor and inconvenience of outages and replacements



Numbers on curves indicate energy cost in cents per kilowatt-hour

each, with electric energy available at stated rates, are as indicated in figure 2. It will be observed that the lowest unit cost of light occurs at a considerably shorter life than the prevailing standard of 1,000 hours; however, when lamps are adjusted to yield a 1,000 hour life, the increase in cost of light is of the order of 6 per cent, if energy costs 4.5 cents per kilowatt-hour. This departure from the shorter hours of life corresponding with the lowest unit cost of lamps and current is a concession to the greater convenience and satisfaction that comes with freedom from outages and avoidance of replacements. It is the result of a consensus of judgment after taking into account the wide variation in conditions encountered in the use of lamps. This 1,000 hour convention for the 60 watt lamp is embodied in the federal specification for incandescent electric lamps (hereinafter referred to as "the specification") and is the commonly accepted standard in this country. Other types and sizes of lamps are adjusted to lives indicated by operating conditions, as 1,500 hours for sign lamps or 2 hours for photoflood lamps. With respect to life performance this specification stipulates minimum requirements for purchase of lamps by the government. The better makes of lamps quite generally exceed this minimum performance. In Canada and in Europe, where prices of lamps generally are higher, a longer average lamp life has been common heretofore, but by general agreement in this country the higher efficiency that comes with 1,000 hours of life (750 hours for 75-, 100-, and 150-watt inside-frosted lamps) is deemed to be desirable, even at the expense of somewhat more frequent replacements.

Method of Comparison

Appraisal of the qualities of incandescent electric lamps and comparison of their values are rendered difficult by the evasive character of their performance. Lamps of inherently inferior quality may be operated at relatively high efficiency, and their inferiority is not known until it displays itself in short life. Moreover, they may be operated at an efficiency lower than that of approved practice, thereby yielding a life longer than usual, and concealing their inferiority. Since the life of tungsten filament lamps varies inversely as about the seventh power of the efficiency, considerable deficiencies in quality may be concealed. For effectiveness in appraisal and comparison, it is desirable to observe the performance of lamps when they are operated at a common efficiency as required in the specification. The longevity and the rate of decline in light output under these conditions make practicable the appraisal and comparison of inherent values. The comparison, however, lacks somewhat in practical application, because various makes of lamps operate at various initial efficiencies other than those established as standard in the specification. In order to meet this condition and still to facilitate comparison and appraisal, the form of chart illustrated in figure 3 has been devised. In this chart the initial efficiency established in the specification is taken as 100 per cent, and the efficiencies of lamp products in terms of that basis are plotted on the scale of abscissas. The heavy-line descending concave curve, which corresponds with the curve of figure 1, indicates for other efficiencies the life of a lamp that operates for 1,000 hours at standard efficiency. Superior lamps fall above this curve and to the right of it on the chart. Inferior lamps are plotted to the left and below it.

Typical 60 watt size lamps complying with this requirement of the specification are sold for 15 cents each. They are supplied with electric energy at, say, 4.5 cents per kilowatt-hour. (This is slightly below the average domestic rate; the figures involving the assumption of the 4.5 cent rate therefore are conservative.) These lamps are characterized by a certain failure frequency. Lamps of a quality inferior to that required by the specification characteristically are issued by their manufacturers at lower efficiencies. Their lives at such efficiencies may be longer or shorter than 1,000 hours, with failure frequencies being dependent upon their average life, and upon divergence of individuals therefrom.

When lamps are issued at relatively low efficiencies, as is usual with inferior lamp products, the user obtains less light per lamp, and must use more lamps to obtain light similar to that of a high efficiency lamp. The only fair way to compare lamp products is to adjust them to a basis of equivalency of light production, for if the user is content with less illumination he may better obtain it with fewer efficient lamps than with a larger number of in-

efficient lamps under usual conditions. Therefore, in comparisons of lamp products with the aid of such a chart as that of figure 3, the criterion is equivalency in amount of light produced by lamps of each of the brands compared.

Assuming, then, as a criterion a lamp that complies with the specification, yielding 1,000 hours of life at 100 per cent efficiency, and supplied with electric energy costing 4.5 cents per kilowatt-hour, lamps of other efficiencies and lives may be plotted upon the rectangular co-ordinates. If the plotted point falls anywhere upon the 94.7 per cent abscissa, the use of that lamp to produce the same light as that produced by lamps complying with the specification will be attended by an additional cost of 15 cents to the user. It may be said that lamps of such inferior efficiency should be given away without charge if the user is not to suffer.

Lamps of efficiency lower than 94.7 per cent of that required by the specification involve the user in so much additional expenditure for electric energy to obtain the same amount of light that their use would be attended by unnecessary loss, even if they were given to the user without charge. The extent of such loss is indicated by the light-line ascending concave curves in figure 3. Thus, if no expense is to be incurred beyond that entailed in the use of specification quality lamps in obtaining a certain amount of light, the user who obtains lamps of 90 per cent efficiency and 1,150 hours' life should pay nothing for them. In addition, he should be presented with the sum of 15 cents per lamp to reimburse him for the additional expenditure he must make for electric energy at 4.5 cents per kilowatt-hour in order to obtain the same amount of light that he would obtain by the use of specification quality lamps.

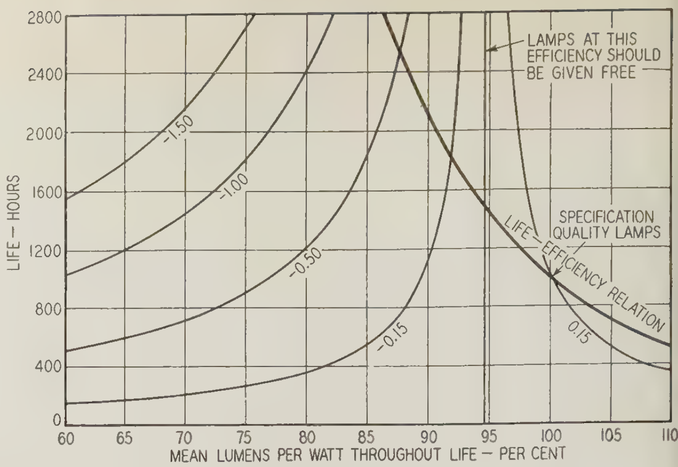


Fig. 3. Type of chart employed for comparison of practical values of lamp products

Number on each curve indicates the price in dollars that a consumer should pay (negative numbers, the amount he should *be paid*) for a 60 watt lamp of that brand

Table I—Average Life in Hours of Nonconforming American and Certain Imported Lamps, 1935

Brand	Rating, Watts	Number of Specimens Tested	Average Initial Efficiencies Lumens per Watt: Of Specimens	Average Initial Efficiencies Lumens per Watt: Required by Specifications[1]	Average Life, Hours[11]: At Marked Volts	Average Life, Hours[11]: At Specification Efficiencies
H	25	96	10.16	10.0	758	819
	40	10	10.87	10.7	840	942
	60	110	12.48	12.4	690	686
	100	10	15.80	14.9	406	622
L	25	100	9.11	10.0	889	444
	40	20	10.43	10.7	994	816
	60	108	11.06	12.4	1,797	761
	100	20	13.43	14.9	1,361	646
M	25	100	9.66	10.0	740	565
	40	10	9.88	10.7	1,707	966
	60	100	11.24	12.4	974	477
	100	10	13.30	14.9	889	394
T	25	110	10.28	10.0	637	770
	40	40	10.35	10.7	956	756
	60	119	11.90	12.4	1,132	845
	100	40	15.13	14.9	676	743
AA	25	100	8.33	10.0	1,582	426
	40	20	9.52	10.7	1,165	475
	60	110	10.30	12.4	2,295	626
	100	20	12.30	14.9	1,499	376
OO	25	100	9.01	10.0	1,045	487
	40	20	10.35	10.7	589	425
	60	110	10.94	12.4	1,615	663
	100	20	14.11	14.9	813	541
B2	25	99	9.64	10.0	886	673
	40	10	10.65	10.7	542	523
	60	100	11.47	12.4	1,045	559
	100	10	13.65	14.9	690	368
E2	25	10	8.11	10.0	2,636	582
	40	10	10.29	10.7	863	656
	60	10	11.23	12.4	874	438
F2	25	10	9.00	10.0	1,064	497
	60	10	11.27	12.4	1,632	837
	100	10	13.85	14.9	1,489	879
G2	25	10	10.28	10.0	609	744
	60	10	12.07	12.4	839	693
	100	10	14.73	14.9	644	593
H2	60	10	10.46	12.4	993	303
J2	60	10	10.24	12.4	2,462	645
K2	25	10	9.57	10.0	765	558
	60	10	12.16	12.4	590	516
L2	25	10	9.39	10.0	868	552
	60	10	11.88	12.4	816	605
M2	25	10	8.75	10.0	946	360
N2	25	100	9.82	10.0	540	435
	60	100	7.81	12.4	4,669	167
Imported[111]	25	119	8.40	10.0	1,280	326
	40	20	9.17	10.7	1,302	435
	60	120	10.30	12.4	677	181

[1] For lamps rated at 115 volts.

[11] There are slight apparent inconsistencies between these values. They are attributable to the fact that specification efficiencies are slightly different for lamps rated at 110, 115, and 120 volts. Lamps of these 3 voltages were tested in different proportions among the various brands. This simplified presentation of results, adjusted to specification efficiencies, is affected by the slight variable involved in the various voltage ratings.

[111] Information necessary for separation into makes and brands is not available.

Comparison of Lamp Products

With this understanding of the method employed in the comparison of lamp products, some results of tests made by Electrical Testing Laboratories will be presented to show the average initial efficiencies and the average hours of life of various makes of tungsten filament incandescent electric lamps of the usual sizes for domestic lighting. The data are made available through the courtesy of the General Electric Company, the Hygrade Sylvania Corporation, the lamp committee of the Association of Edison Illuminating Companies, and the Westinghouse Lamp Company. All tests involved were made by Electrical Testing Laboratories on lamps purchased in the open market during the year 1935 by that organization's inspectors.

Lamps made by the leading American manufacturers are adjusted so that they comply with the requirements of the specification, and either equal or exceed the performance stipulated. Therefore, they may be considered as representing (figure 3) lamps of 100 per cent efficiency and 1,000 hours of

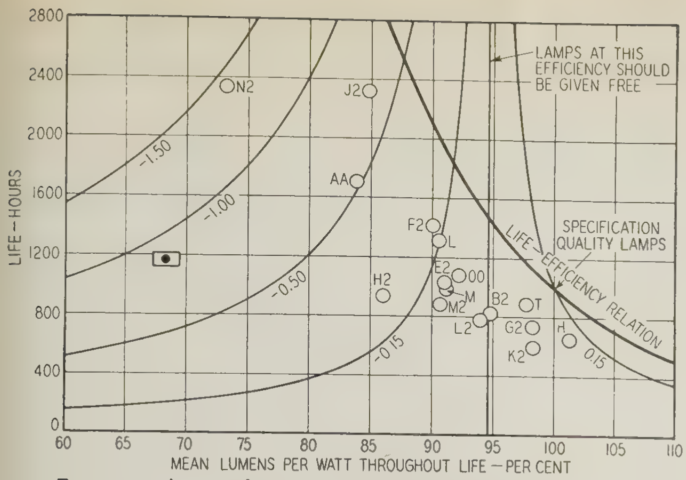


Fig. 4. A graphic comparison of the qualities of specification quality and some inferior brands of lamps

Number on each curve indicates the price in dollars that a consumer should pay (negative numbers, the amount he should *be paid*) for a 60 watt lamp of that brand

life. Extensive tests of Electrical Testing Laboratories have established this general conformity with specifications, and such lamps are referred to as "specification quality" lamps.

There are several brands of tungsten filament incandescent lamps that are not of the quality called for by the specification. The accompanying detailed data present in condensed form the average efficiencies and average hours of life of each of these brands of lamps as they have been investigated during the past year. In addition, a great many incandescent lamps reach this country from abroad, and data on specimens of some of these lamps are included. The author presents all data on lamps of the stated sizes which have been obtained for clients, and he is not responsible for the selection of makes or brands involved. Statements herein should not be applied to makes or brands that have not been tested.

It is reported that during the past year about 15 per cent of the total number of lamps sold in this country for general lighting purposes, including those of American and foreign manufacture, were of qualities that did not meet the specification requirements.

When the data of table I are plotted on a chart like figure 3, the result is figure 4. (The data embodied in table I for the purpose of this chart are consolidated by averaging the lives of lamps of all sizes and treating them as though they were all 60 watt lamps. If they were plotted size by size, the several charts would differ in detail slightly but the same general conclusions would be indicated.) As compared with 60-watt specification-quality lamps costing 15 cents each and supplied with energy at 4.5 cents per kilowatt-hour, the best of these nonconforming domestic and imported lamps are shown to be uneconomical for the user. Those of better efficiency than 94.7 per cent of that required by the specification are short in life; moreover, the user's light is produced less efficiently, and he has to replace his lamps more frequently than he would if he used specification quality lamps. Those lamps represented at the left of the diagram are long in life, but penalize the user seriously through their inefficiency, and through necessity for consuming much larger quantities of energy in order to obtain the same amount of light.

Through the 100 per cent efficiency-1,000 hour life intersection in figure 4 the light-line descending concave curve indicates other efficiencies and lives at which a 15 cent price for lamps would be justifiable, taking into account only costs of lamp and electric energy. On this basis, inferior lamps operated at relatively high efficiency are shown to less disadvantage, while those operated at very low efficiencies are shown to greater disadvantage. This is because the 1,000 hour standard has been determined by taking into account in a general way the annoyance of outages and the cost of replacements, but all the other representations exclude these intangible elements. For this reason the representations of efficiencies and lives shown in figure 4 have been adjusted in figure 5 to a basis of 1,000 hours of life. This minimizes the variable of lamp outages and replacements, all lamps compared having at least the same average lives.

For each lamp of brand $N2$ the user should be given the sum of $1.54 in addition to a free lamp. It appears, however, that brand $N2$ suffers needlessly because of the low efficiency (73 per cent of standard) at which the lamps are sold, which results in an average life of about 2,300 hours. If they were issued to yield an average life of 1,000 hours, their efficiency would be 83 per cent and the user's compensation, in addition to receiving the lamps free, should be 34 cents instead of $1.54 for each lamp. The best of these American nonconforming lamps that have been investigated are of such quality that if they are adjusted to yield a 1,000 hour life, under the conditions assumed for this study, they ought to be sold for no more than 4 cents each. The average of the various brands of certain imported lamps is seen to have a lesser inherent value than any of

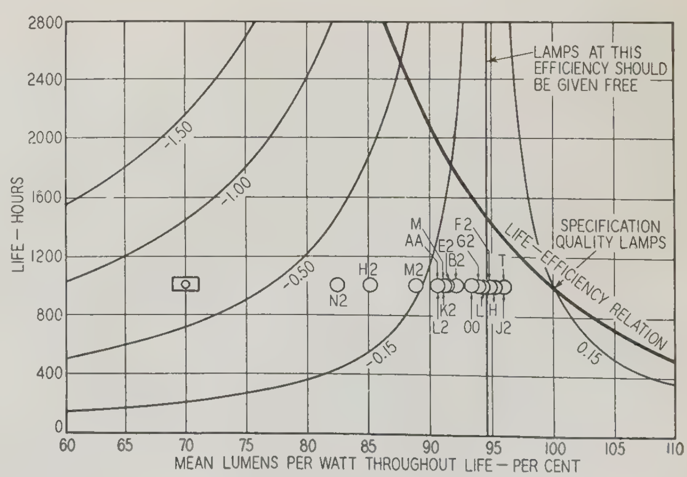


Fig. 5. A corrected comparison of the qualities of some brands of lamps

Number on each curve indicates the price in dollars that a consumer should pay (negative numbers, the amount he should *be paid*) for a 60 watt lamp of that brand

the domestic nonconforming lamps, partly because the imported lamps tested are of the vacuum type.

So far as average efficiencies and lives are concerned, such is the best available picture of the quality of the subspecification 15 per cent stratum of the incandescent lamps supplied for general lighting purposes in this country last year.

There are, however, aspects of the quality of incandescent lamps other than average efficiency and average longevity. The progressive lamp manufacturer, while endeavoring to improve his product otherwise, seeks to make it more uniform, more free from defects, and more consistent as to rating. The incandescent lamp is a delicate device, parts of which are operated close to the limit of their physical capabilities, and the elements of which are in delicate and easily disturbed balance. In the normal process of manufacture constant vigilance is required in order to protect the process from going awry in some particular which will affect adversely the performance of the lamps. The task is rendered more difficult by the great variety of services in which lamps are employed and the consequent large number of types and sizes of lamps, many of which present distinctive manufacturing problems. In addition, each step of the almost constant endeavor to improve products in various particulars introduces the possibility of defects not previously encountered. All incandescent lamp products, therefore, contain some individual lamps that are defective or irregular. The difference between superior and inferior products is in the proportion of such defective or badly rated individuals, and in the degree of consistency attained in the performance of the individual lamps of the group and of successively produced groups. In order to indicate the range throughout which these differences are encountered, a comparison is offered here between the qualities ascertained through extensive tests of lamps that substantially comply with the requirements of the specification, and of certain imported lamps that usually do not comply with the specification.

Physical Defects

For lamps of the types used in residence lighting, the inspection procedure applied to lamps purchased by members of the Association of Edison Illuminating Companies, recognizes approximately 100 distinctive defects and irregularities, and statistics are kept accordingly. Most of these defects are indicated in figure 6.

An increase in the usual small proportions of any of these defects indicates manufacturing deviation, and is a signal for attention by the manufacturers from whom these lamps are purchased. The sustained inspection of lamp products for all these defects is an important feature of measures designed to keep defects at a minimum. Even irregularities that do not impair performance, and have no significance for the user may increase in such proportions as to indicate manufacturing carelessness which, if uncorrected, may lead to other more serious difficulties. This is the reason for the influential character of the critical sustained inspection to which the principal American lamp products are subjected both by manufacturers and by certain purchasers. Two of the larger manufacturers of lamps even have gone to the extent of employing an independent organization to conduct routine inspections and tests of their products in order to insure impartial conclusions as to the success of their quality maintenance procedures.

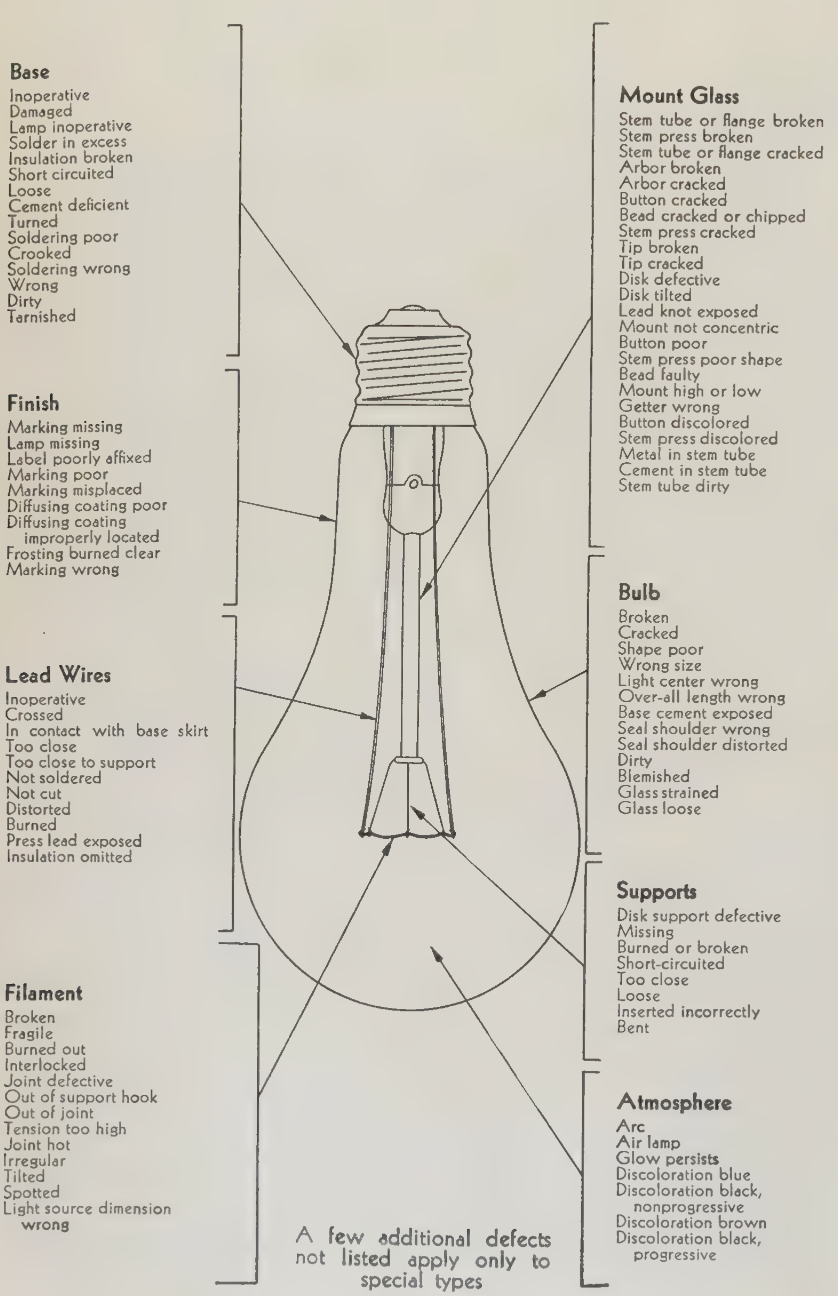


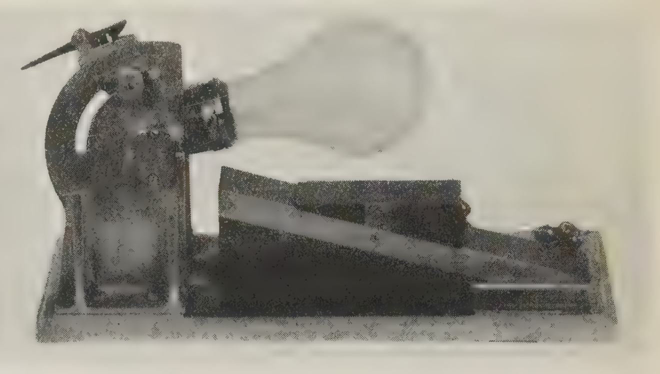
Fig. 6. Defects recognized in incandescent lamp inspection

Most incandescent electric lamps of all makes are relatively free from defects that occasion serious service difficulties. Occasionally a lamp may do damage through a short circuit or an arc that blows a fuse. Such lamps are assigned negative values, since they are worse than useless. Next with respect to serious consequences (other than economic) are lamps that are inoperative or fail soon after being placed in circuit. Lamps having broken filaments or interrupted circuits, or lamps that have leaked are perhaps to be assigned negative values because of the annoyance involved in replacing them. It is, of course, necessary that dimensional specifications be observed. In the case of a defect that renders the lamp useless because it will not fit into sockets or luminaires, a similar minus value is applied. Fortunately, serious defects of these types are rare, especially in the better products.

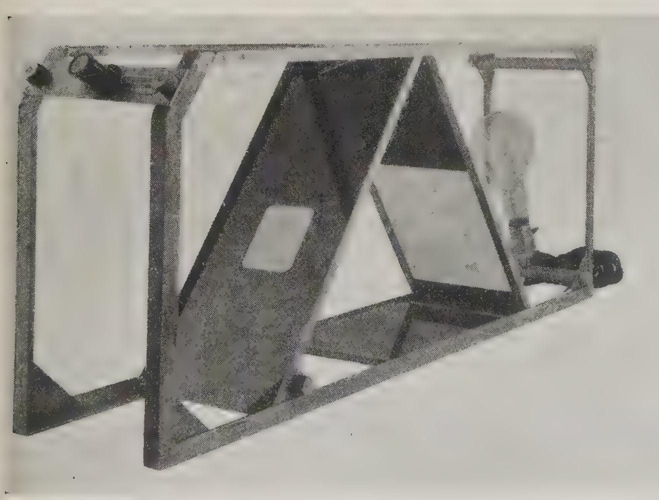
Lamp products are liable to a great variety of defects that shorten life and impair performance. The technique of detecting in new lamps symptoms which presage poor performance is highly developed. Some of the testing devices are shown in figure 7. Defective lamps may be assigned values ranging from 10 to 80 per cent, depending upon the seriousness of the defect encountered. The defects that shorten life are less prevalent in the good products than in the poorer ones.

In addition, there is a great variety of irregularities in lamp construction, some of which may affect appearance only. The more careful manufacturers seek to minimize such defects, even though they have little serious consequence in service. Figure 8 shows at the left the proportions of various classes of defects and irregularities encountered in lamps purchased in the open market, and as recorded under a rigorous inspection system for American lamps that comply with the specification and for certain imported lamps. At the right of figure 8 are estimates of the extent to which the presence of such defects diminishes the value of these 2 lamp products. Among American specification quality lamps, 5 per cent are recorded under this strict inspection as defective or irregular. Their presence is estimated to reduce the value of these products 1.1 per cent below that of a similar product free from such defects. Among the imported lamps 34 per cent are recorded under a similar inspection system as

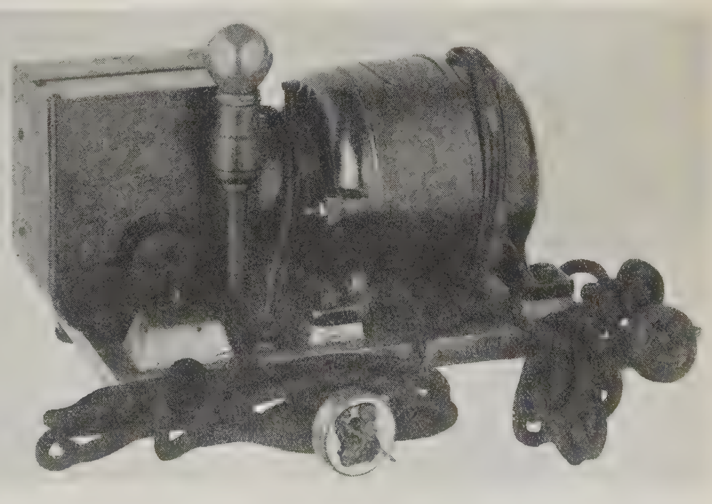
Base torsion gauge: applies a torsional stress of from 15 to 20 inch-pounds as a test of the adhesive strength of the cement between the base and the bulb



Fragility gauge: subjects lamps to a physical impact as a test of the mechanical strength of filament and glass parts



Polariscope: detects through the modification of polarized light the presence of strains in lamp bulbs



Spot tester: for detecting high resistance filament spots

Fig. 7. Four of the 27 testing devices used in lamp inspection

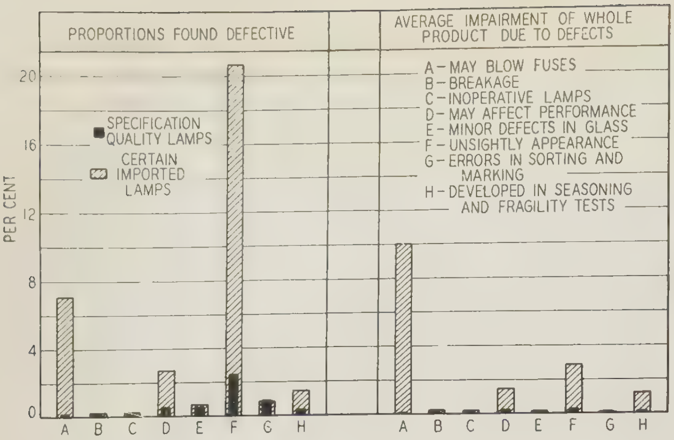


Fig. 8. A comparison of the proportions of various types of defective lamps

being defective or irregular, and it is estimated that these reduce the value of the products by 16 per cent.

A brief discussion of the subject of bulb strength may be in order at this point. The process of frosting the insides of bulbs tends to reduce materially their mechanical strength through impairment of the inner glass surface. American manufacturers of Mazda lamps have developed a process of fortifying this inner surface after frosting, which makes it more sturdy. Certain imported lamps are less satisfactory in this respect, as the following data show:

	American Specifications Quality Lamps	Certain Imported Lamps
Height from which a 30-gram steel ball must fall to break average *A*-21 bulb, centimeters	53	16

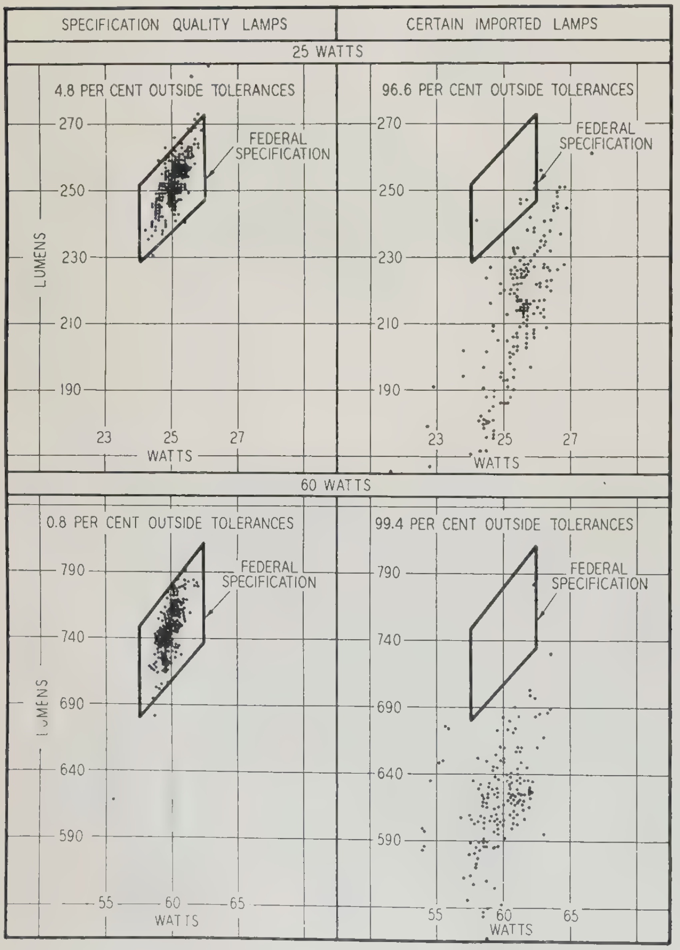


Fig. 9. Comparisons of the initial ratings of several brands of specification quality lamps and some imported lamps

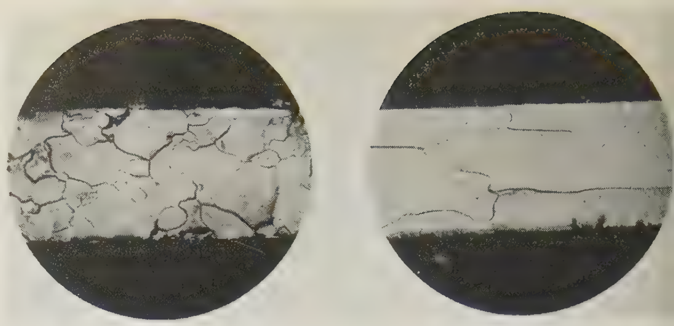


Fig. 10. Photomicrographs of plane longitudinal sections through tungsten filaments

Left—"Sag" filament Right—"Nonsag" filament

Ratings

The specification indicates efficiencies that are regarded as standard for modern practice, and sets up tolerances for watts and lumens per watt within which most of the lamps are expected to fall. Rating tests of representative lamps of specification quality American brands and of certain imported brands appear in figure 9 for lamps rated at 25 and 60 watts. The proportions of these American lamps that do not fall within the established tolerances are respectively 4.8 and 0.8 per cent, and the average for all sizes tested is 4.7 per cent. The imported lamps (which are mostly of the vacuum type, whereas the American lamps of the 60 watt size are gas filled) usually fail to comply with the rating requirements of the specifications. The proportions falling outside the established tolerances are respectively 96.6 and 99.4 per cent. In addition, there is a much lower degree of consistency in the rating of imported lamps. Of the total it may be said that about 5 per cent of the American specification quality lamps fail to comply with the federal specification as to watts and lumens per watt, but about 98 per cent of the imported lamps so fail.

Life Performance

The history of the development of drawn tungsten wire for incandescent lamps forms a gratifying chapter in American research and manufacturing procedure. In modern lamps, manufacturing requirements of great delicacy must be met. The fine filament wires (from about 0.0015 to 0.0020 inch in diameter for a 60 watt lamp) must be so uniform that when placed in the circuit they will attain a uniform temperature throughout the entire length, and it is important that throughout life the filament

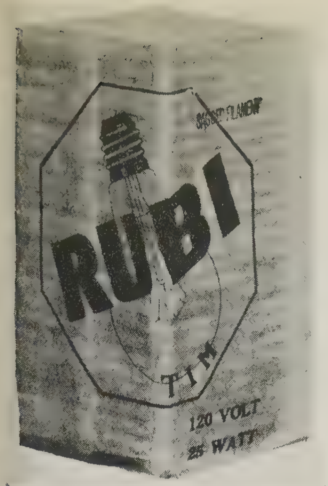


Fig. 11. An imported lamp carton bearing a rubber stamp imprint warning of an inferior type of lamp

coils shall undergo a minimum of deformation, notwithstanding the very high temperatures at which they are operated. Some years ago deformation of filament coils during life was avoided, but it was accomplished at the expense of desirable ruggedness of the filament. At that time it was necessary to choose between fragile filaments and others that offered a higher degree of ruggedness but became deformed in such a way as to reduce the filament temperature materially during life. Scientific developments more recently have made available filaments that do not deform materially and do not suffer from serious fragility toward the end of life. Figure 10 shows photomicrographs of 2 filaments, the one on the left being a "sag" filament composed of small crystals and the one on the right a "nonsag" filament having long interlocking crystals of a type that imparts strength and rigidity. The use of "sag" filaments usually is attended by serious decline in efficiency of light production. An important difference between the performance of specification quality American lamps and certain imported lamps is in this particular, since the filaments of most of the latter deform, with resultant decrease in light output during life. As a consequence, the mean lumens throughout life of certain imported lamps of the 60 watt size typically is 72 per cent of the initial value, but in specification quality American lamps it is 93 per cent. Figure 11 shows an imported lamp carton on which a rubber stamp imprint warns the discerning buyer of this feature of inferiority. In most cases there is no such warning.

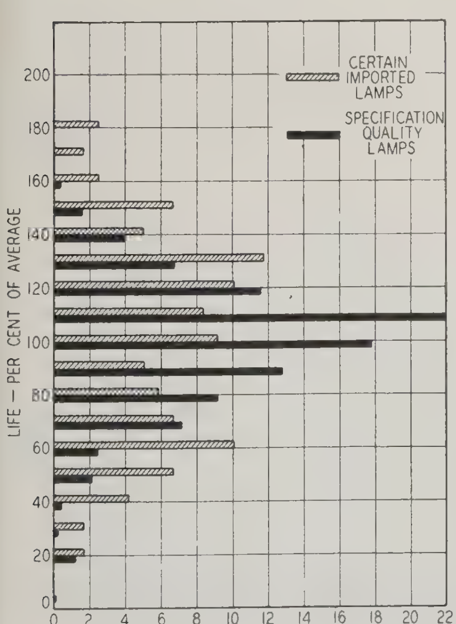


Fig. 12. Distribution of failures among imported lamps and lamps of specification quality

Specification quality American lamps, when tested in a laboratory under uniform operating conditions at the initial efficiencies prescribed in the specification conform quite closely on the average with the hours of life set up in the specification, generally exceeding and rarely failing to equal the values stipulated. In contrast with these are the results of equivalent tests on specimens of imported lamps set forth in table I. When these imported lamps are operated at rated voltage their average life is short in comparison with the specification requirement which, for the sizes indicated, is 1,000 hours. When they are adjusted to the higher efficiencies at which American specification quality lamps operate, their lives are even shorter. The comparison of these hours of life in the last column of table I with the 1,000 hours of life required by the specification affords one index of the relative values of these lamp products. In general, it may be said that the average deficiency in hours at specification efficiencies is zero for American specification quality lamps, but it is 74 per cent for certain imported lamps.

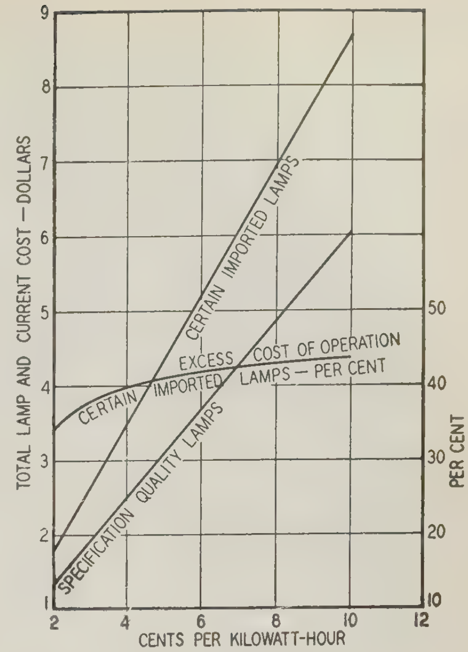


Fig. 13. Comparison of cost of operation for 1,000 hours of specification quality and imported lamps

The average hours of life for a lamp product is, of course, a very significant value. However, there are wide ranges in deviation of individual lamps from the average for a product. Uniformity of performance among individual lamps of a group is desirable, and a characteristic of reliable manufacturers is that their products are more nearly uniform than the products of other manufacturers. Figure 12 shows the variation of hours of life among specimens of 60 watt lamps tested. The solid bars

(*Continued on page 529*)

Fig. 14. A carton in which an imported lamp was purchased in the open market in 1935

Neutralizing Transformer to Protect Power Station Communication

In considering the subject of protecting communication circuits at electric power plants and substations by means of neutralizing transformers, this paper gives particular attention to the problems incident to the localized rise in ground potential caused by power system faults. Several specific installations are discussed and illustrated.

By
E. E .GEORGE
FELLOW A.I.E.E.
Tenn. Elec. Power Co., Chattanooga

R. K. HONAMAN
MEMBER A.I.E.E.
Bell Tel. Labs., Inc., New York, N. Y.

L. L. LOCKROW
Membership Application Pending
Bell Tel. Labs., Inc., New York, N. Y.

E. L. SCHWARTZ
ASSOCIATE A.I.E.E.
Bell Tel. Labs., Inc., New York, N. Y.

THE NEUTRALIZING transformer is a device for the protection of communication circuits against extraneous longitudinal voltages. It functions as its name implies, by neutralizing the voltages. This neutralization is accomplished in general by connecting a secondary winding of the transformer in series with each of the communication wires to be protected. By induction from another winding connected into a circuit subject to the same voltage, there is introduced into all the secondary windings a voltage substantially equal in magnitude and opposite in phase to the disturbing voltage. The result is the reduction of the voltage in the circuit to a value sufficiently low to prevent harmful effects.

Historical Review

Although the principle of neutralizing might be applied to any type of extraneous voltage, except that due to direct contact, its main application has been in connection with (1) voltages of fundamental frequency induced from either power or railway propulsion circuits and (2) voltages impressed upon communication circuits serving power stations as a result of an abnormal rise in station ground potential.

The basic idea was invented by Professor Chas. F. Scott in 1906 and, as far back as 1907, neutralizing transformers were applied to circuits exposed to the electrification of the New York, New Haven, and Hartford Railroad to eliminate the effects of fundamental frequency induction in telegraph circuits. Several other installations were subsequently made for protection against similar effects.[1,2]

In 1926 a neutralizing transformer was installed on circuits of the Hydro-Electric Power Commission of Ontario to protect a remote metering circuit serving a power station ("power stations" as used in this paper include substations) against abnormal ground potential rise.[3] In 1932 a trial installation of several transformers to protect a toll telephone line against 60 cycle induced potentials[5] was made in New York State under the auspices of the Joint Subcommittee on Development and Research of the Bell System and the Edison Electric Institute.[4] More recently several transformers have been installed in the territory of the Tennessee Electric Power Company[6] for the purpose of protecting circuits serving power stations against rise of ground potential.

Scope of This Paper

Although these applications are of interest from the standpoint of a general consideration of this device, it is not practicable within the scope of a single paper to deal with all of them. This paper, therefore, is confined to consideration of the use of the transformer in protecting circuits serving power stations, particularly against ground potential rise. This problem takes on additional interest in view of the use of commercial telephone circuits by power companies for a wide range of communication serv-

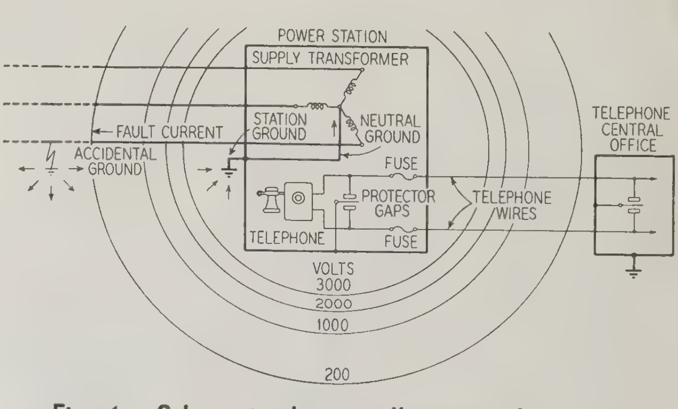


Fig. 1. Schematic diagram illustrating the rise in station ground potential caused by power line faults

A paper recommended for publication by the A.I.E.E. committee on protective devices, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 19, 1936; released for publication April 1, 1936.

The authors wish to acknowledge the assistance of the Southern Bell Telephone and Telegraph Company in connection with the testing and installation of the neutralizing transformers.

1. For all numbered references, see list at end of paper.

ices, including not only telephone, but telemetering, remote alarms, supervisory control, and pilot wire relaying. In all these services there is required a high degree of reliability and continuity.

GROUND POTENTIAL RISE

The important factors in the problem of protecting against ground potential rise are illustrated in figure 1. This shows a communication circuit from a telephone central office into a power station at which the power system neutral is grounded. This circuit is equipped at both ends with protector gaps connected between the line wires and the respective grounds to safeguard personnel and to prevent damage to terminal equipment that might be caused by lightning or by contact with high voltage circuits.

At the time of an accidental ground on the power line, fault current passing from the transformer bank at the station over the line to the fault and back through the station ground will produce a rise in the potential of the station ground with respect to a remote point. The resistances of station grounds vary from fractions of an ohm to several ohms, and with currents of several thousand amperes, potential rises of several thousand volts may readily appear for an appreciable time. Power stations located in cities, and those having extensive grounding systems, generally will have station grounds of lower resistance than small stations in rural districts. In actual cases, due to the configuration of the grounding structure and soil irregularities, the equipotential lines by no means are as regular as indicated. The potential, however, is roughly inversely proportional to the distance from the station ground, and the largest drop in potential usually occurs within a region of from 200 to 500 feet from the station. These potentials will operate the protector blocks on the communication circuit and may ground them

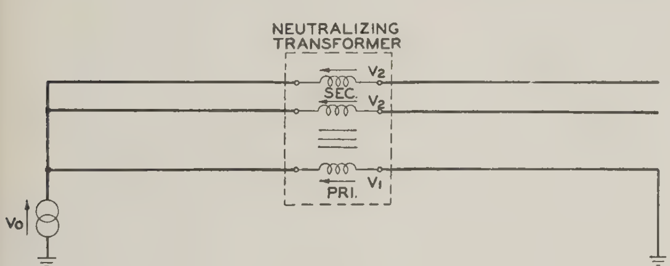


Fig. 2. Schematic diagram of a neutralizing transformer

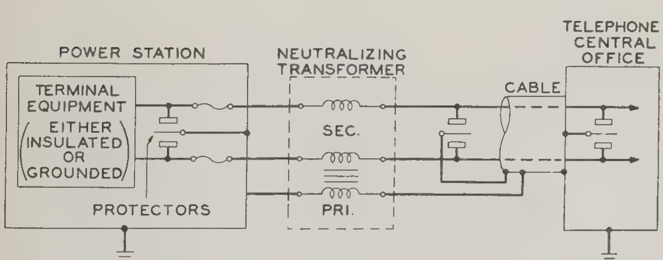


Fig. 3. Arrangement of a neutralizing transformer on a pair of wires leading into a power station

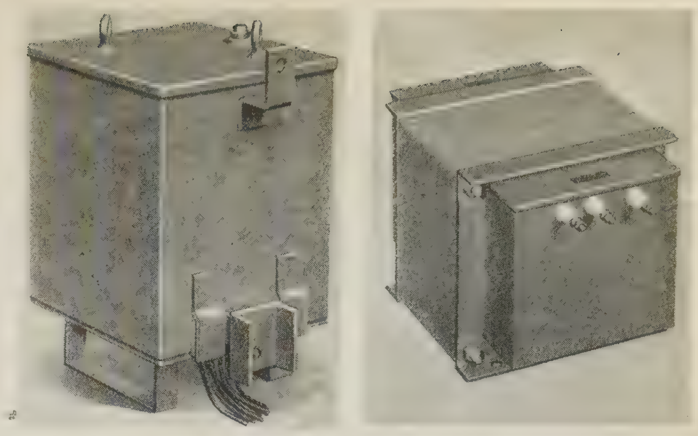


Fig. 4. Neutralizing transformers—left, for use outdoors; right, for use indoors

permanently, thereby putting the circuit out of service until the blocks can be replaced.

REMEDIAL MEASURES

Several methods have been devised over a period of years to prevent the effects of such ground potential rise. One of the most familiar in telephone practice is the use of an insulating transformer between the telephone line and the equipment in the station. For local battery telephone service, this is an effective solution, but the insulating transformer, without supplementary equipment, prevents the use of any circuit which requires the transmission of direct current over the line wires, such as the common battery telephone. Where a circuit of this type is necessary, insulated relays have been used for repeating d-c signals around the transformer.

The use of the relay protector, which is a device for short circuiting the protector gaps by means of relay contacts to prevent the permanent grounding of communication circuits, has been successful for cases where a momentary interruption during the brief interval while the fault occurs is not serious. However, on some types of circuits, such as pilot wire circuits where even a short interruption cannot be tolerated, this is not a feasible solution.

The neutralizing transformer has the advantage that it prevents even a momentary interruption of the communication circuit while the surge is present and does not limit the use of the circuit to one type of service. When suitable precautions in design are taken, it does not introduce any substantial adverse reactions to transmission over the circuit. It permits the use of either direct or alternating current, and the circuit arrangement is the same for magneto or common battery telephones, dial or manual, or for different types of supervisory control and pilot wire circuits. Most control, supervisory, and alarm circuits utilize direct current with less complication and expense than alternating current, and the neutralizing transformer, by preserving a d-c path is of great utility in this connection. The characteristics of the transformer are such as to permit satisfactory use of voice frequencies, d-c pulses, ringing current, and other low frequency currents. The various

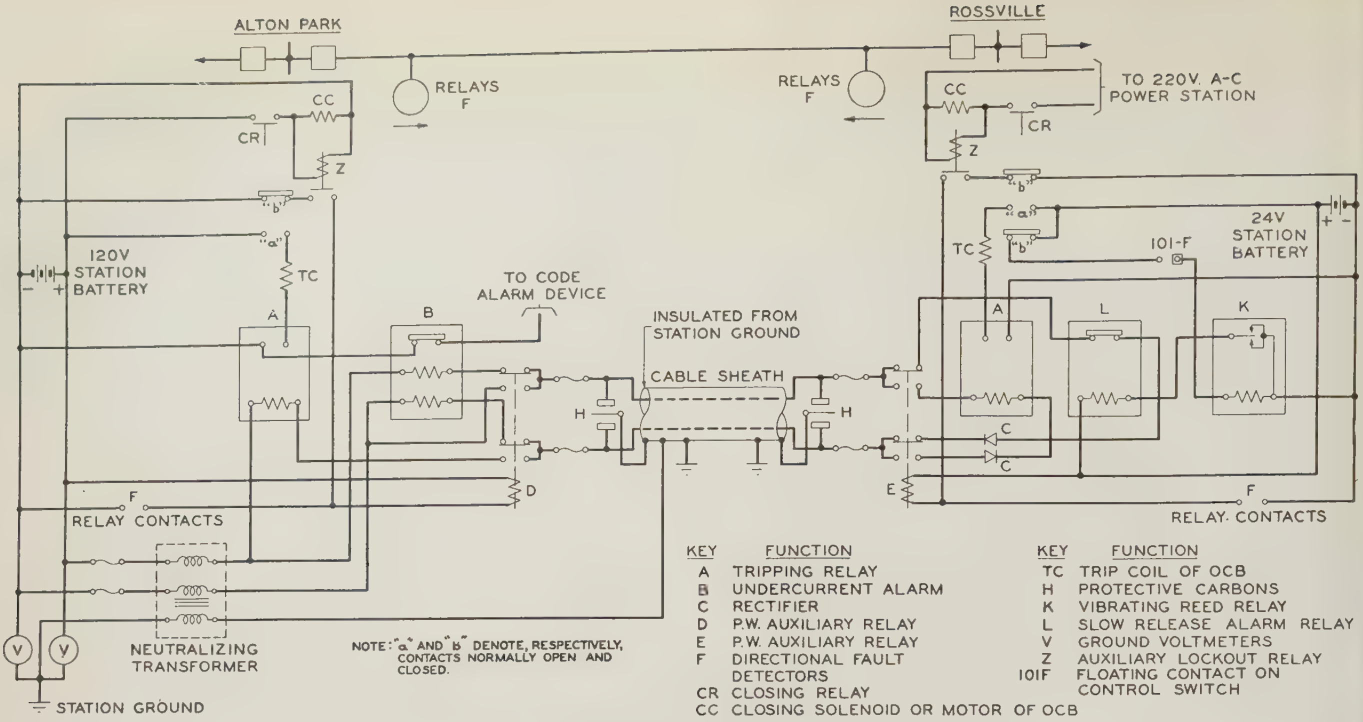


Fig. 5. Neutralizing transformer in battery leads at a substation

services may be operated over the circuit without changing the protection arrangements, and the terminal equipment can be wired and handled just as if it were at an ordinary subscriber's station.

Theory

The general principle of the neutralizing transformer as a means of inserting compensating voltages in an exposed circuit already has been referred to. Such a transformer consists of one or more secondary windings, one of which is connected in series with each communication wire to be protected, and in addition a primary winding that is connected in the circuit in such a way as to have impressed across it the disturbing voltage. By proper poling of the windings, the voltage picked up by the primary is induced in the secondary windings in the direction to oppose the disturbing voltage. A simple schematic diagram of such an arrangement is shown in figure 2 where V_0 is the disturbing voltage, V_1 the voltage across the neutralizing transformer primary winding, and V_2 the secondary winding voltages. The remanent voltage in the communication wires is the vector difference between V_0 and V_2. For complete neutralization V_2 and V_0 obviously must be equal in magnitude and opposite in phase. For the magnitude of V_2 to approach that of V_0 requires (in a transformer of 1-to-1 voltage ratio) that the voltage across the primary be as nearly as possible equal to V_0, which in turn requires that the transformer primary impedance be large compared to that of the external primary circuit. For proper phase relation between V_2 and V_0 the phase shift in the transformer due to the primary leakage reactance and resistance should be small. Lack of sufficient voltage across the primary, due to a low impedance primary, cannot be compensated for by a corresponding voltage step-up in the transformer because the increased exciting current drawn by the low impedance primary will cause objectionable phase shift in the voltage. In general, these fundamental requirements restrict the neutralizing transformer to one of relatively high primary impedance and 1-to-1 voltage ratio with close coupling and low primary resistance.

These features, required for efficient neutralizing performance are not, however, consistent from a design standpoint with those required for good transmission of communication frequencies. To obtain good transmission over these lines the secondary windings, which are in series with the lines, should be of low resistance. Similarly, the leakage reactance between the secondary windings, which also appears in series with the lines, should be low. The direct and mutual capacitances between the windings should be kept small to minimize their shunting action on the communication frequencies. It is evident from this that high winding impedances for good neutralizing action are in direct conflict with the features required for good communication transmission. The design of a successful neutralizing transformer must, therefore, be the result of a proper compromise between these divergent factors.

Neutralizing Transformer Arrangement

The method of using the neutralizing transformer on a communication circuit similar to that of figure 1 is illustrated in figure 3. The primary winding is connected between the power station ground and the cable sheath, which in turn is bonded to the central office ground, and thus has impressed across it the voltage which tends to disturb the communication wires. This voltage is induced in the secondary

windings, which are in series with the wires, in the proper direction to neutralize the disturbing voltage. The voltage in the circuit is not completely nullified, but the small remanent voltage produces no harmful effects. The protectors and fuses shown are standard telephone protection devices, supplied to guard against trouble due to accidental contact of a power wire with the cable, and against the remote possibility of failure of the transformer.

To make certain that the entire voltage to which the communication wires are subject is impressed across the primary winding, it is necessary that the cable sheath be kept free from the influence of the power station ground potential rise. This can be done by ending the cable at a point outside the field of influence and adequately insulating from ground the wires brought into the station, or, if the cable is brought into the station, by adequately insulating the sheath at the station and throughout the length within the field of influence. This same insulation, of course, also keeps power system fault current off the cable sheath and conductors.

Where the neutralizing transformer primary is grounded to the cable sheath, it is desirable that the sheath be continuous to the central office ground and bonded to it, or at least that the impedance to ground of the sheath where the primary is connected be sufficiently low to insure proper action of the transformer. In nearly all cases no special precautions will be necessary to meet the requirements.

In situations where there is no cable, where the communication circuits are brought to the power station on an open wire line, the primary can be connected to a ground located far enough from the power station to be outside its field of influence. Usually a distance of 500 or 1,000 feet is sufficient.

For power station applications, the transformers have much higher ratings than those used on the New Haven Railroad[1,2] exposure. The regions in which station ground potentials appear are relatively small, and the points at which insulation is required

Table I

60 Cycle Disturbing Voltage	Remanent Voltage (Effective)	
	0 Ohm	35 Ohms
1,000	3	3
3,000	18	21
4,000	70	81
4,500	138	160

Table II

Frequency	Approximate Transmission Loss in Decibels*
200	0.8
500	0.8
1,000	0.9
2,000	1.1
3,000	1.4

*The decibel (db) is a unit of measure of power ratios employed in communication work. Two amounts of power (W_1 and W_2) differ by N db when they are in the ratio of $10^{0.1N}$. Thus N db $= 10 \log_{10} W_1/W_2$. The transmission loss of 15 miles of copper wire of 0.104 inch diameter in dry weather is about 1 db.

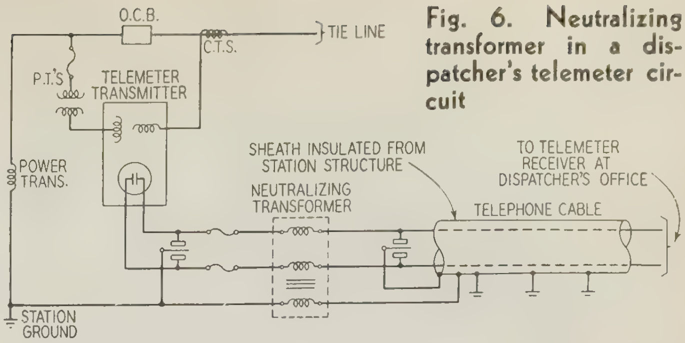


Fig. 6. Neutralizing transformer in a dispatcher's telemeter circuit

are definite; thus it is desirable and permissible to apply a single corrective device capable of handling the entire voltage. In the other situation the voltage is induced electromagnetically over many miles, and it is preferable to apply several smaller corrective devices to neutralize the voltage before it builds up to an undesirable magnitude.

Commercial Designs

Two transformers have been designed for commercial use on 2-wire circuits, one suitable for outdoor mounting and the other for use indoors. They are shown in figure 4. The first occupies a space 12 by 11 by 13 inches high and weighs about 160 pounds. The second occupies a space 12 by 9 by 9 inches high and weighs about 95 pounds. Except for differences in mounting arrangements these transformers are identical and are designed to neutralize voltages up to 4,000 volts (effective) at 60 cycles. This voltage rating is believed to be adequate for most situations. Although designed primarily for use at 60 cycles, these transformers can be used with equal effectiveness on 25 cycle voltages up to 1,500.

Neutralizing Effectiveness

These transformers are designed to be used with external primary circuit impedances between 0 and 35 ohms. As already has been shown, the degree of neutralization depends largely upon the relation between the transformer primary impedance and that of the external primary circuit. Higher values of external impedance than those for which the transformer is designed will result in correspondingly higher remanent voltages. The magnitude of the remanent voltage for these transformers with 2 values of external primary circuit impedance and for a range of disturbing voltages is given in table I. The magnetizing current with 4,000 volts across the primary winding is about 0.35 ampere. The voltage between secondary wires for all the cases in table I has been found to be less than 1 volt.

These transformers are designed with winding impedances sufficiently high to make unnecessary the use of a tuning condenser across the primary. Accordingly, the transformers respond to transient and harmonic voltages as well as to those of fundamental frequency. Tests made on a working installation indicate that in wave form the secondary voltages follow the impressed voltages very closely.

Transformers can be readily designed with more

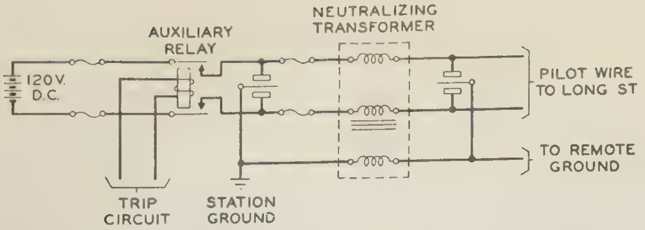


Fig. 7. Neutralizing transformer in a pilot wire circuit

than 2 secondary windings for the same voltage rating, and with characteristics generally similar to those described herein.

Effect on Communication Facilities

As pointed out previously, it is necessary to design a transformer which not only neutralizes effectively, but which has as little effect as possible on the communication facilities carried through it. The transformers just described are especially satisfactory in both these respects.

The approximate transmission loss in the voice frequency range caused by the insertion of the transformer in a 600-ohm circuit is given in table II.

These losses, in the order of 1 db, are not considered to be objectionable in the applications described here, where the transformers are installed in circuits connecting the power station with the nearest telephone exchange. For such service it is reasonable to permit larger apparatus losses than would be acceptable in important toll lines.

These specific transformers, because of the relatively high distributed capacities of their windings, are not suitable for use in circuits operating at frequencies above the voice range. However, it is possible that, by applying proper methods of loading, neutralizing transformers can be made suitable for use in carrier circuits.

The secondary windings of the transformers increase the d-c loop resistance of the circuit by about 120 ohms. This causes transmitter battery supply losses in a common battery telephone circuit in addition to the direct transmission loss and, of course, reduces the current in any signaling or control circuit that might be used. In general, however, such impairment will not be serious.

The maximum safe continuous current in each secondary winding, from a heating standpoint, is about 0.35 ampere, direct current or alternating current.

The effect of the transformers on metallic telegraphs, d-c signaling, and 20-cycle ringing is small, approximately the same as would be produced by about 1.2 miles of 19 gauge cable.

There may be situations where direct current is sent through the 2 secondary windings in parallel and in the same relative direction around the core, as in the case where a grounded d-c circuit is established over a simplex path through the center tap of a bridged retardation coil. Under such a condition a total direct current of 0.15 ampere has no appreciable effect on the effectiveness of the transformer, but larger currents cause increasing reduction in neutralizing efficiency. This limitation arises from the fact that simplex currents passing through the windings in the same direction set up a steady flux in the core that does not appear with metallic currents, since in the latter condition the currents flow in opposite directions and their magnetomotive forces cancel each other. The steady flux set up by simplex currents decreases the transformer impedance and thereby reduces the ability of the transformer to neutralize.

Specific Cases

The first trial of the neutralizing transformer on the Tennessee Electric Power Company's system was made in a circuit in which it was desired to use grounded station battery to supply 4 pilot wire relay circuits (figure 5). The relays already were insulated against ground potential rise and it had been necessary to maintain a fully insulated battery to keep all parts of the circuit insulated from ground. The transformer permitted the use of the regular station battery. This installation was made February 1, 1935. During tests on this installation, different features of the circuit performance were investigated and it was found possible to talk, ring, and dial through the transformer with complete facility even during the interval of high station ground potential rise.

The success of experimental tests made on this installation led to plans for applications of the transformers to other circuits. On May 29, 1935, one was installed in a leased wire circuit between the system load dispatcher's office and Ridgedale substation in Chattanooga for operating a torque balance telemeter which required a d-c path. Because of the high rise of ground potential occurring at Ridgedale, the engineers of the power company believed that a direct current circuit would not be satisfactory without the transformer. The circuit is shown in figure 6.

The next installation was in a remote tripping circuit between Carter Street and Long Street substations in Chattanooga (figure 7). This circuit formerly had used remote battery, and circuit failure at certain locations would operate the receiving relay and cause undesired operations of an important high-voltage oil-circuit breaker. The neutralizing transformer permitted the use of the substation storage battery which, being located at the originating end of the circuit, prevented circuit failure from causing incorrect tripping.

The application to an important dial telephone circuit at Clift Street substation in Chattanooga is

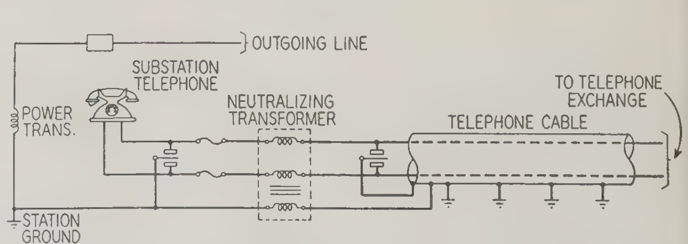


Fig. 8. Neutralizing transformer in a substation telephone circuit

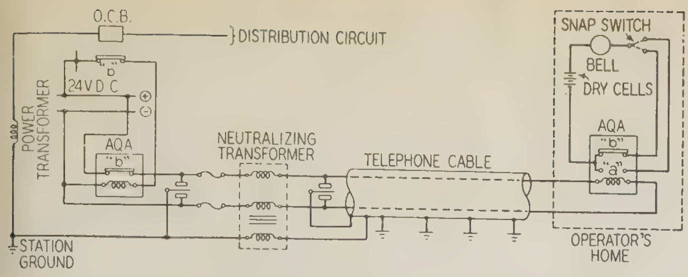


Fig. 9. Neutralizing transformer in a remote alarm circuit

shown in figure 8. The substation previously had been served by a magneto telephone circuit through an insulating transformer. While this telephone is a private branch exchange (PBX) extension, the PBX is located about a mile from the substation and the conditions were similar to dial subscriber service.

The application to a remote alarm circuit in Lawrenceburg, Tennessee, where power is supplied to a municipality is shown in figure 9. The transformer had a particular advantage in this application in that it permitted the use of the telephone company's leased service in a city where the power company has no franchise rights to string wires on the streets. It also made available a more reliable circuit. A similar application of a transformer to an alarm circuit is under way at Franklin, Tennessee.

In these installations 2 wire transformers were used. In February 1936 an experimental multiwire transformer was installed in Nashville where a leased magneto telephone circuit, an exchange dial telephone circuit, and a pilot wire relay circuit operate through the same transformer.

Operating Results

The pilot wire and telemeter installations were made prior to the 1935 lightning season, and operated satisfactorily through many cases of ground potential rise. At no time have these circuits or any circuits since installed been out of service because of protector operation or any trouble in the neutralizing transformer. Thus it is apparent from the record that the neutralizing transformer has not only been a great help in preventing long time interruptions on these circuits due to permanent grounding of protective devices, but also has been effective in preventing short time interruptions lasting for the duration of a surge.

References

1. Neutralizing Transformers and Their Use in Telephone Circuits, T. Shaw. *The Elec. Jl.*, v. 11, Nov. 1914, p. 622–8.

2. Inductive Effects of Alternating Current Railroads on Communication Circuits, H. S. Warren. A.I.E.E. Trans., v. 37, 1918, p. 503–32.

3. Discussion by H. C. Don Carlos on C. Lichtenberg's Supervisory Systems, A.I.E.E. Trans., v. 35, 1926, p. 218–19.

4. Status of Joint Development and Research on Low-Frequency Induction, R. N. Conwell and H. S. Warren. A.I.E.E. Trans., v. 50, June 1931, p. 458–69, or *B.S.T.J.*, v. 10, Apr. 1931, p. 206–30.

5. Neutralizing Disturbance Voltages in Communication Circuits, by C. A. Brigham. *Bell Lab. Record*, v. 12, June 1934, p. 311–15.

6. Pilot Wire Relay Protection, E. E. George and W. R. Brownlee. Elec. Engg. (A.I.E.E. Trans.), v. 54, Nov. 1935, p. 1262–9.

The Qualities of Incandescent Lamps (*From page 523*)

show a conventionalized distribution of American specification quality lamps. The cross-hatched bars show a corresponding distribution of imported tungsten lamps. The mean deviation of individual lives from the averages of the groups is approximately 17 per cent and 58 per cent, respectively. This detail is illuminating as a background for the simple statement that the relative average lives at specification efficiencies for these 2 products are, respectively, 1,000 hours and 181 hours.

Summary of Qualities

In table II the relative values for American specification quality lamps and for certain imported lamps are summarized in accordance with the foregoing brief presentation. In addition, reverting to the considerations advanced in the earlier part of this paper, it is calculated that the unit cost of lamps and energy for light obtained through the use of American specification quality lamps, when energy costs 4.5 cents per kilowatt hour, is about 0.406 cent per 1,000 lumen-hours. The corresponding cost to obtain the same light under the same conditions employing certain imported lamps is about 0.571 cent, or 40 per cent higher. Thus, in order to obtain a given amount of light, such as might be used in a small home, the consumer pays about $18.00 a year when using American specification quality lamps, but must pay about $25.31 when using certain imported lamps. This is the difference in the cost of light when lamps are supplied with electric energy at 4.5 cents per kilowatt-hour. For other electricity rates, the percentage differences are as indicated in figure 13.

From the foregoing discussion it must be evident that there are wide differences in quality in this type of small utilization device. In the most advanced American practice, incandescent electric lamps are the subject of sustained and highly competent research, engineering, and testing. The results attained may be measured by comparison of these products with products of similar types that are without benefit of research, engineering, and testing. Through the courtesy of organizations that possess the necessary information such a comparison is offered in this paper. It indicates that large expenditures in application of the engineering method in this field are well justified.

Table II—A Comparison of Specification Quality Lamps and Certain Imported Lamps

	Specification Quality Lamps	Certain Imported Lamps
Retail price, cents	15	5 to 10
Per cent physically defective	5	34
Reduction in value due to defects, per cent	1.1	16
Per cent average deficiency in initial efficiency at marked volts	0	15
Per cent outside federal specification tolerances	4.7	98
Average per cent deficiency in life at specification efficiency	0	74
Per cent mean reduction in efficiency during life	5	20
Cost of light in cents per 1,000 lumen-hours at 4.5 cents per kilowatt-hour	0.406	0.571
Dollars annual cost for given light output	18.00	25.31

Hydrogen Cooling—With Near-Critical Velocities

A report of experimental measurements of the heat transfer coefficients for air and hydrogen in laminar flow through rectangular ducts, and for flow conditions encountered through the use of velocities near the critical values.

By
G. W. PENNEY
ASSOCIATE A.I.E.E.

Westinghouse Elec. and Mfg. Company, East Pittsburgh, Pa.

DESIGNERS of rotating electric machines have accumulated from laboratory models and complete machines data which have enabled them to predict quite accurately the temperature rise of conventional machines. However, most of this information applies only to a given type of flow in air, and is not sufficiently fundamental to enable the prediction of the heat transfer encountered under many conditions of hydrogen cooling. The air flow in the ducts of practically all conventional rotating machines is turbulent.

Turbulent flow is used in the sense of a velocity above the critical Reynolds Number where turbulence once started in a duct tends to be maintained in contrast to velocities below the critical where a disturbance tends to damp out and the flow becomes streamline when the disturbance is removed.

For circular ducts, the Reynolds Number is $R_n = \dfrac{VD\gamma}{\mu}$

For noncircular cross sections, $R_n = \dfrac{V\gamma 4m}{\mu}$

For symbols see Appendix I.

If all units are expressed in any consistent system, R_n is dimensionless and independent of the system of units. If R_n is less than 2,000, the flow almost certainly is laminar. If R_n is more than 20,000 the flow almost certainly is turbulent. Intermediate values depend upon entrance and surface conditions. For a discussion of these factors see any good text on hydraulics.[1]

In the case of hydrogen, the kinematic viscosity $\left(\dfrac{\mu}{\gamma}\right)$ is 7 times that of air, hence the critical velocity of hydrogen is approximately 7 times that of air. In the design of hydrogen cooled machines, cases frequently arise where the velocity is less than the critical. The heat transfer coefficients have been well correlated for velocities above the critical[2,3,4,5,6,7,8], but little work has been available to give a comparison of the heat transfer in a duct where the velocity is greater than the critical with air but less than the critical with hydrogen, and many of the statements concerning the effect of turbulence appear to be based on superficial reasoning. For instance, one good text on electrical engineering makes the statement, "... for forced convection velocities below the critical are not worth using. In self-cooled direct current armatures with radial ventilating ducts, if the air velocity is below the critical the duct had better not be there." Several years ago, because of the lack of reliable data covering these conditions, the transfer of heat with laminar flow was studied theoretically, and apparatus set up to measure the heat transfer under these conditions.

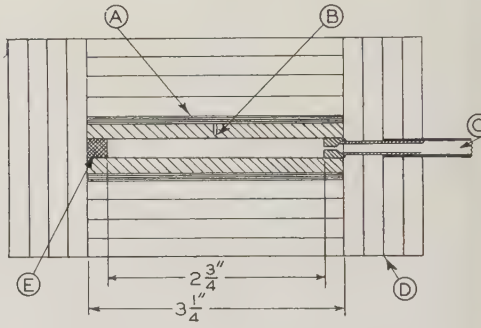
A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 19, 1936; released for publication Apr. 4, 1936.

1. For all numbered references, see list at end of paper.

Theoretical Analysis of Heat Transfer to Gases in Laminar Flow

Mathematical calculation of heat transfer to a gas in turbulent flow is not possible as yet, because the motion of the gas is not known. However, if the gas is moving in a laminar flow the necessary facts are known, provided assumptions are made as to the distribution of velocity over the cross section of the duct. In the analysis given in Appendix II, the gas is assumed to be flowing between flat parallel metal surfaces heated to a uniform temperature. Within the range of dimensions and velocities here considered the heat conducted parallel to the wall is negligible as compared with the heat conducted perpendicular to the wall. A differential equation can be set up expressing the relationship that for any differential volume of the gas the heat flow into the face nearest a metal wall will be greater than the heat conducted from the opposite face of the differential volume. The difference between these values represents the

Fig. 1. Cross section of experimental duct



A—Four layers of 0.010 inch mica; thermocouple leads between layers 1 and 2; heater wound on third layer; fourth layer separates heater from cork insulation
B—Thermocouple soldered into 0.040 inch hole drilled in steel plate; 32 couples in each plate
C—Tube for measuring pressure
D—One inch thickness of cork insulation
E—Spacing between plates changed by using 1/8, 1/4, or 3/8 inch brass spacers

heat absorbed by the differential volume of gas, and results in a temperature rise of the gas. This equation is given in Appendix II.

A solution is given, using the assumption that the velocity is uniform over the cross section of the duct, which is believed to be a good approximation for the short ducts frequently encountered on machines. Figure 3 shows a typical comparison between test results and values calculated in this manner. A solution also has been obtained assuming a parabolic distribution of velocity. This solution is difficult to evaluate for areas near the entrance, and since parabolic distribution does not exist near the entrance the values are plotted only for points some distance from the entrance.

The heat transfer constant K_v used in this paper is defined by the relation

$$K_v = \frac{W}{A \times \theta_D}$$

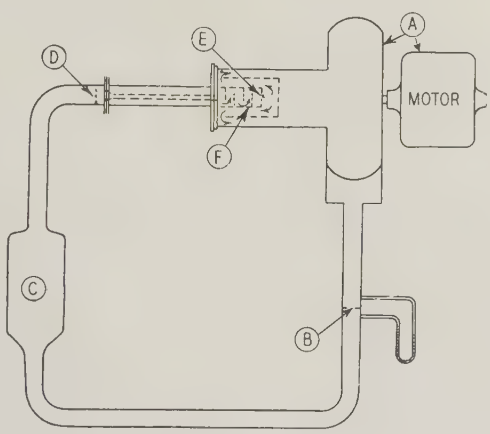
where,

W = Watts dissipated from a given section of the duct
A = Surface area of the section of the duct
θ_D = Difference between the temperature of the duct surface and average air temperature at the given cross section of the duct.

The value of K_v was computed for each inch of the duct length, and near the entrance for shorter increments of length. These values are plotted in figures

Fig. 2. Arrangement of experimental hydrogen cooling apparatus

A—Motor and fan
B—Orifiec
C—Cooler
D—Inlet couples
E—Outlet couples
F—Mixer baffles; surrounded by insulating jacket to prevent heat loss



3 and 4. The average value of K_v then was computed for the 18 inch length of duct; these averages are shown in figures 5 and 6.

Test Apparatus

The duct from which the heat transfer was measured consisted of 2 flat steel plates each $3^1/_4$ inches wide by 18 inches long by $^3/_{16}$ inch thick. These were separated by brass spacers so that the spacing between plates could be changed readily. A cross section of the duct is shown in figure 1. The temperature of the steel plates was measured by 64 thermocouples inserted in holes drilled in the plates. The duct was heated by ribbon heaters which were covered with cork one inch in thickness. The steel plates were insulated at the ends by cork gaskets. Thermocouples were located to measure the tempera-

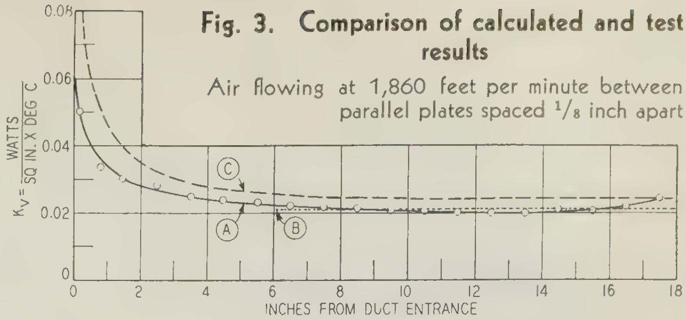


Fig. 3. Comparison of calculated and test results

Air flowing at 1,860 feet per minute between parallel plates spaced $^1/_8$ inch apart

A—Test result
B—Calculations, assuming parabolic velocity distribution
C—Calculations, assuming uniform velocity

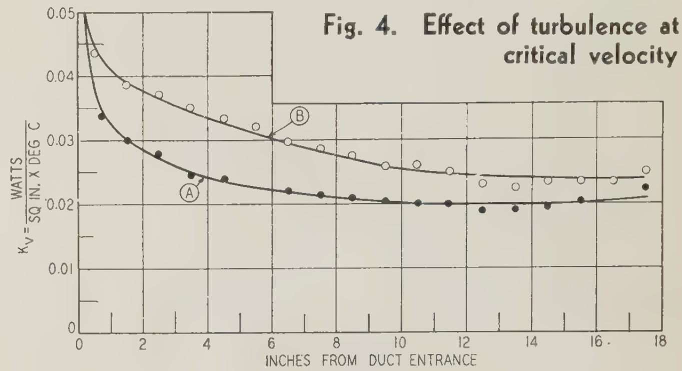


Fig. 4. Effect of turbulence at critical velocity

Air flowing between parallel plates spaced $^1/_8$ inch apart

A—Laminar flow at 1,860 feet per minute
B—Turbulent flow at 1,675 feet per minute

ture drop across the various pieces of cork. The heat flow per degree drop in temperature both for gaskets and for side insulation was calibrated before starting the test. The loss of heat then could be computed for any test from the drop in temperature across the cork.

The arrangement of the apparatus is shown in figure 2. The air was drawn through the duct by a variable speed fan equipped with a seal to prevent leakage around the shaft. The discharge of the fan was connected to a calibrated orifice, and from the orifice the air passed through a cooler and back to the heated duct. In some of the tests with air the cooler was not used, air being drawn into the heated duct from the room and discharged back into the room from the orifice. Thermocouples were provided to measure the temperature of the air entering the duct. Leaving the duct, the air passed through baffles which mixed the air before the temperature was measured by thermocouples. The volume of air could be computed either from the orifice drop or from the measured heat input to the air and the corresponding temperature rise.

In computing the heat transfer rate, the heat input to each section of the heater was measured. The stray loss could be computed from the drop in temperature across the cork. The heat conducted along the steel could be computed from the measured temperature gradient. Knowing the heat input to the heater, the heat loss through the insulation, and the heat conducted along the steel plates, the heat transferred to the air in the given section could be obtained. Starting at the duct entrance, the tem-

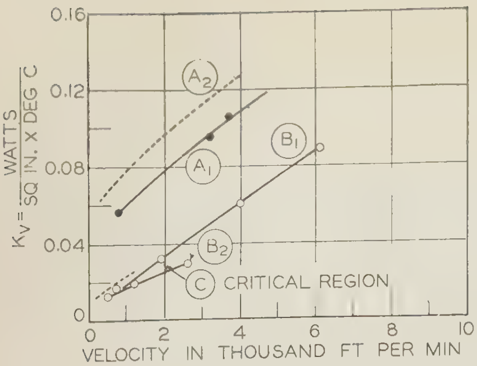


Fig. 5. Heat transfer with $^3/_8$ inch spacing between parallel plates

A_1—Hydrogen test values
A_2—Theoretical values for hydrogen, from equation 8
B_1—Air test values
B_2—Theoretical values for air, from equation 8
C—Critical region

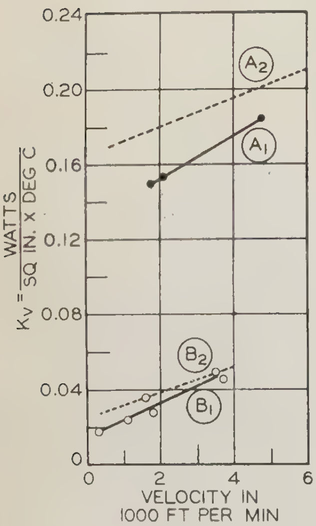


Fig. 6. Heat transfer for $^1/_8$ inch spacing between parallel plates

A_1—Hydrogen test values
A_2—Theoretical values for hydrogen from equation 8
B_1—Air test values
B_2—Theoretical values for air from equation 8

perature of the entering air was measured and the temperature rise could be computed from the watts input to the air. The value of K_v for the section calculated then was given by the watts input divided by the area and divided by the difference between surface temperature and average air temperature. Starting at the duct entrance, the value of K_v was computed for each inch of duct length. These values are plotted in figures 3 and 4.

Effect of Turbulence

Operating in air, the cooler could be removed, as previously stated, so that room air could be drawn into the heated duct and the air from the orifice discharged into the room. Then, with the fan operated at a speed to give an air velocity within the critical range, laminar flow could be obtained if the room air were quiet; or, by disturbing the air in the room such as by running an ordinary ventilating fan in the room, a turbulent flow could be obtained. The air velocity could be adjusted so that waving one's hand in front of the intake would change the flow from laminar to turbulent. Figure 4 gives curves for heat transfer constant along the length of the duct for laminar and turbulent air flow, the turbulent flow being obtained by mounting stationary baffles near the duct entrance but outside the duct; the fan speed was not changed. Laminar flow gave a velocity of 1,860 feet per minute and a pressure drop of 0.63 inch of water along a section of the duct. In turbulent flow the velocity was 1,675 feet per minute and the pressure drop 0.71 inch of water in the same section of the duct. From figure 4 it is evident that the turbulent flow at 1,675 feet per minute gave an appreciably higher heat transfer than the laminar flow at 1,860 feet per minute.

Comparison of Air and Hydrogen

From the curves given in figure 3, it is evident that the heat transfer constant K_v varies along the length of the duct. The average value of K_v was computed for each spacing and velocity for air and for hydrogen. The values thus obtained for $^3/_8$ inch spacing are plotted against velocity in figure 5. Figure 6 gives similar curves for $^1/_8$ inch spacing.

Around the critical velocity, a range of values could be obtained. The lower values seldom would be encountered in conventional rotating machinery. In the tests made the ratio of

$$\frac{\text{heat transfer constant } K_v \text{ in hydrogen}}{\text{heat transfer constant } K_v \text{ in air}}$$

varied from 1.45 to 1 to 4.69 to 1, the higher ratio being obtained at the lower gas velocities and with the narrow spacing. In other tests using higher velocities and larger spacing, the ratio has been 1.25 to 1 and even less.

Summary

For velocities up to the critical, and in some cases to 2 or 3 times the critical value, calculations assuming heat transfer by conduction alone seem to give results closely approximating the test results. Near the critical velocity the effect of turbulence is much less than frequently has been assumed.

In small ducts and at low velocities where the air flow is turbulent but hydrogen flow is laminar, the improvement in heat transfer obtained by using hydrogen is much greater than in large ducts and at high velocities, where both air and hydrogen give a turbulent flow.

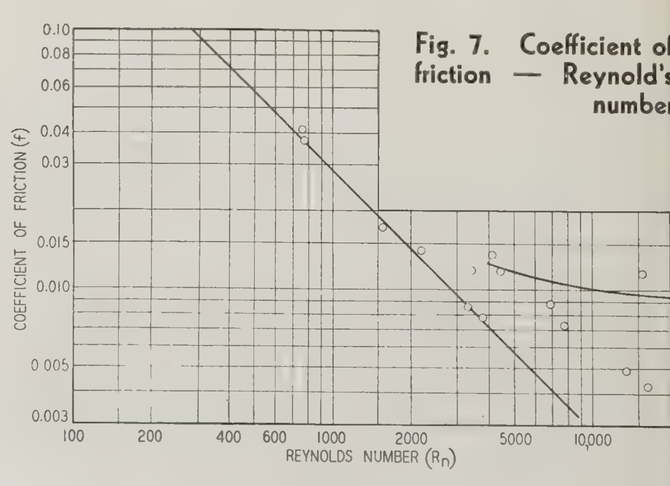


Fig. 7. Coefficient of friction — Reynold's number

Appendix I—Symbols

x = Distance measured across the duct
y = Distance measured along the duct length
a = Duct width
θ = Temperature at point x, y in the duct
θ_1 = Constant = Temperature of the duct surface
θ_D = Difference between surface temperature θ_1 and average gas temperature, at the cross section in question
K = Thermal conductivity of the gas
$K_v = \dfrac{W}{A\theta_D}$
A = Area under consideration
μ = Absolute viscosity of the gas
C = Specific heat of the gas
γ = Density of the gas
V = Velocity of the gas at point x, y
= Coefficient of friction
$\alpha = \dfrac{K}{C\gamma V}$
R_n = Reynold's Number
m = Hydraulic radius = $\dfrac{\text{cross-sectional area of duct}}{\text{perimeter of duct}}$
D = Diameter
W = Heat dissipated from area A

Appendix II—Heat Transfer in a Duct With Laminar Flow of the Cooling Gas

With laminar flow of a cooling gas, each particle is assumed to move through the duct in approximately a straight line so that heat is transferred perpendicular to the wall only by conduction.

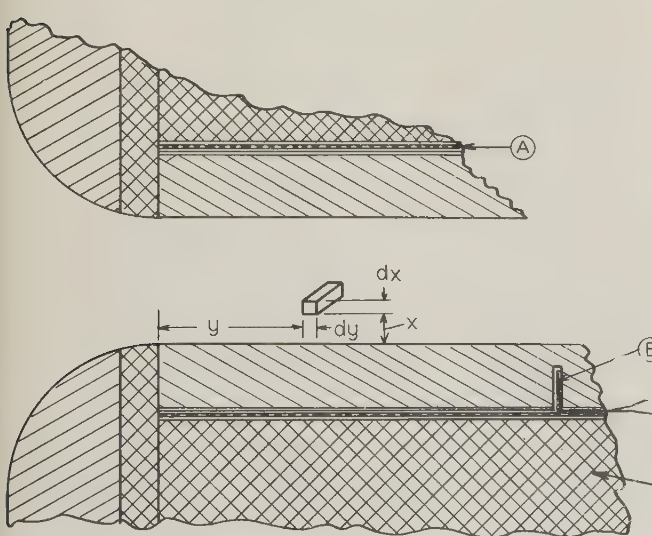


Fig. 8. Partial section at duct entrance

A—Heater B—Thermocouple C—Cork

For the range involved in this discussion, the axial conduction of heat can be neglected. A constant temperature of the duct surface is assumed.

Consider an increment of space of dimensions x, y, and unit dimension in the Z dimension. The duct is assumed to be very wide so that there will be no variation in the Z dimension.

The heat flow into the x face of the differential volume is

$$-K\frac{\delta\theta}{\delta x}\,dy$$

The heat flowing out of the second face is $-K\left(\dfrac{\delta\theta}{\delta x} + \dfrac{\delta^2\theta}{\delta x^2}\,dx\right)dy$

The difference between these values is the heat that produces a temperature rise of the gas.

The thermal capacity of the gas flowing through the differential volume per second is $C\gamma V dx$ so that,

$$-K\frac{\delta\theta}{\delta x}dy + K\left(\frac{\delta\theta}{\delta x} = \frac{\delta^2\theta}{\delta x^2}\,dx\right)dy = C\gamma V\,dx\frac{\delta\theta}{\delta y}\,dy$$

or

$$K\frac{\delta^2\theta}{\delta x^2} = C\gamma V\frac{\delta\theta}{\delta y} \tag{1}$$

Solution Assuming a Uniform Velocity

With a properly rounded entrance, the velocity over a cross section at the entrance will be substantially uniform while at considerable distance from the entrance the velocity distribution is parabolic. Inasmuch as most of the ducts considered in this discussion are relatively short, and since this is the simple solution, the velocity is first assumed to be uniform.

$$\frac{K}{C\gamma V} = \text{const} = \alpha \tag{2}$$

The differential equation becomes

$$\frac{\delta^2\theta}{\delta x^2} = \frac{1}{\alpha}\frac{\delta\theta}{\delta y} \tag{3}$$

A solution of this equation is

$$\theta = A_1 + \sum_{n=1}^{n=\infty} A_n e^{-\alpha\left(\frac{n\pi}{a}\right)^2 y}\sin\frac{n\pi}{a}x \tag{4}$$

The boundary conditions assumed are:

1. That air enters the duct at a uniform temperature, which temperature is taken as zero,

at $y = 0$, $\theta = 0$ for $1 > x > 0$

2. That the walls of the duct are assumed to be at a uniform temperature, θ_1,

$x = 0$, $\theta = \theta_1$ and at $x = a$, $\theta = \theta_1$

3. That the following form of equation 4 satisfies the boundary conditions,

$$\theta = \theta_1\left[1 - \frac{4}{\pi}\left\{e^{-\alpha\left(\frac{\pi}{a}\right)^2 y}\sin\frac{\pi x}{a} + \frac{1}{3}e^{-\alpha\left(\frac{3\pi}{a}\right)^2 y}\sin\frac{3\pi x}{a} + \frac{1}{5}e^{-\alpha\left(\frac{5\pi}{a}\right)^2 y}\sin\frac{5\pi x}{a} + \ldots\right\}\right] \tag{5}$$

4. That the heat being dissipated per unit area at any part of the surface is,

$$K\left.\frac{\delta\theta}{\delta x}\right|_{x=0} = K\theta_1\frac{4}{a}\left[e^{-\alpha\left(\frac{\pi}{a}\right)^2 y} + e^{-\alpha\left(\frac{3\pi}{a}\right)^2 y} + e^{-\alpha\left(\frac{5\pi}{a}\right)^2 y} + \ldots\right] \tag{6}$$

5. That the difference between surface temperature and average gas temperature is,

$$\theta_1 - \frac{\int_0^a \theta dx}{a} = \frac{8\,\theta_1}{\pi^2}\left[e^{-\alpha\left(\frac{\pi}{a}\right)^2 y} + \frac{1}{9}e^{-\alpha\left(\frac{3\pi}{a}\right)^2 y} + \ldots\right] \tag{7}$$

6. That the heat dissipation constant then is

$$K_v = \frac{K\dfrac{\delta\theta}{\delta x}}{\theta_1 - \dfrac{\int_0^a \theta dx}{a}} = \frac{K\pi^2\left[e^{-\alpha\left(\frac{\pi}{a}\right)^2 y} + e^{-\alpha\left(\frac{3\pi}{a}\right)^2 y} + \ldots\right]}{2a\left[e^{-\alpha\left(\frac{\pi}{a}\right)^2 y} + \frac{1}{9}e^{-\alpha\left(\frac{3\pi}{a}\right)^2 y} + \ldots\right]} \tag{8}$$

where values of K_v and y are plotted in curve 2 of figure 3, and values of K_v averaged from $y = 0$ to $y = 18$ are plotted in figures 5 and 6.

The solution assuming a parabolic distribution of velocity is worked out in a similar manner, but because of the space limitation is not given here. Curve 3 of figure 3 gives some values of the solution for parabolic velocity distribution as compared to those given by equation 8 and as compared to test values.

References

1. Applied Hydraulics, H. Addison (a book), chapter 4. John Wiley & Sons, New York, N. Y.

2. Heat Transmission, W. H. Adams (a book), chapters 4, 6, and 7. McGraw-Hill Book Company, New York, N. Y.

3. The Basic Laws and Data of Heat Transmission, W. J. King. *Mech. Engg.*, v. 54, June 1932, p. 411–4 and 426.

4. Heat Transfer in Stream-Line Flow, T. B. Drew. *Industrial and Engg. Chemistry*, v. 24, Feb. 1932, p. 152–7.

5. Heat Transfer to Liquids in Viscous Flow, C. G. Kirkbridge and W. L. McCabe. *Industrial and Engg. Chemistry*, v. 23, 1931, p. 625–31.

6. Heat Transmission to Oil Flowing in Pipes, T. K. Sherwood, D. D. Kiley and G. E. Mangsen. *Industrial and Engg. Chemistry*, v. 24, 1932, p. 273–7.

7. Free and Forced Convection of Heat in Gases and Liquids, C. W. Rice. A.I.E.E. Trans., v. 42, 1923, p. 653–702.

8. Free Convection of Heat in Gases and Liquids—II, C. W. Rice. A.I.E.E. Trans., v. 43, 1924, p. 131–43.

The Magnetic Vector Potential

In this paper an attempt is made to describe the general properties of the magnetic vector potential, to make clear its relation to the fundamental equations of electromagnetic theory, and to describe its use in the derivation of general relationships for electric circuit analysis. The concept of inductance is considered and a fundamental definition for this circuit parameter is presented. By emphasizing the manner in which the magnetic vector potential may be used in 2-dimensional problems at both high and low frequencies its value as a mathematical tool is shown.

By
J. W. McRAE
ENROLLED STUDENT A.I.E.E.
Calif. Inst. of Tech., Pasadena

INCREASINGLY more accurate and convenient methods of calculation necessarily have been developed, as the scope of electrical engineering has expanded steadily the rough qualitative considerations which originally sufficed being replaced gradually by exact quantitative methods. Important contributions to this development have come, not only from the field of physics on which the whole structure of electrical engineering is based, but also from the related field of pure mathematics. By combining all these various contributions, and adopting certain significant assumptions and simplifications, it has been possible to develop the present remarkably effective methods of electrical engineering analysis. The great majority of practical problems can be handled by these methods, but occasionally there arises a particular problem, or class of problems, which requires more fundamental treatment. Then it becomes necessary to return to the basic mathematical theory of electricity and magnetism, not only in order to obtain the required solution to each particular case, but also to determine whether the usual methods can be extended to include problems of this general type.

An interesting and important example of this sort of fundamental analysis is provided by Rogowski's original consideration of the magnetic flux set up by low frequency currents flowing in 2-dimensional systems. Previous to the publication of Rogowski's work in 1910,[1] the only available methods for determining flux distribution in 2-dimensional systems had been based on the idea of the magnetic scalar potential, and this idea had been found to lose its significance in regions carrying current. Consequently, although quite accurate flux plots could be obtained for regions not including conductors, only rough approximations could be made as to the field within the conductors. Some more fundamental method was required, and Rogowski was able to show that the properties of the magnetic vector potential at low frequencies and in 2 dimensions made possible the development of the required procedure. He illustrated the general method by calculating the distribution of the leakage flux in a transformer, approximating this problem by the corresponding 2-dimensional case. Other workers have since applied the same methods to similar important practical problems, such as the calculation of the leakage flux in the slots of rotating machines.[2,3,4] In addition, it has been found possible to extend the method of graphical flux plotting to include regions carrying current, the magnetic vector potential having been used to determine the necessary new rules of procedure.

More recently another group of problems, not accurately solvable by the usual engineering methods, has been steadily increasing in importance. These problems occur in connection with very high frequency systems for which the usual electric circuit theory, based on essentially low frequency concepts, cannot be expected to yield sufficiently accurate

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 3, 1936; released for publication April 10, 1936.

1. For numbered references see list at end of paper.

results. Moreover, since this theory contains no consideration of the effect of frequency on the fundamental nature of the circuit parameters, it can give no information regarding the errors which may be introduced by its use at high frequencies. Consequently, before high frequency problems can be handled intelligently, it is necessary to examine the methods of mathematical physics which could be applied to this kind of problem. From such an examination, 2 essentially different results properly can be expected. First, it should be possible to obtain solutions to individual problems by direct applications of the fundamental methods, and second, it should be possible to consider, by the aid of these general relationships, the foundations of electric circuit theory, thus making clear the basic nature of such concepts as inductance. From this latter consideration, the proper basis would be laid for an extension of circuit theory to include some of those problems for which adequate solutions cannot be obtained by the usual low frequency methods.

For this kind of fundamental analysis the magnetic vector potential will again be found to provide an almost essential simplification of the required mathematical procedure. Therefore it may be expected to appear more frequently in the electrical engineering literature of the future, and the following discussion is presented in order to demonstrate, in general, its useful properties and sphere of application. These ideas will be illustrated, not by an application to any definite practical problem, but rather by the general derivation of the definition of inductance at low frequencies and of certain fundamental considerations applying to high frequency systems. As Carson has pointed out,[5] general considerations of this kind are valuable in that they tend to emphasize the secondary nature of the usual circuit ideas and to focus attention directly on the fundamental field quantities, whose behavior is described by the same equations for all the frequencies encountered in electrical engineering. An understanding of the use of these equations cannot fail to give a more accurate picture of the behavior of electrical systems, in general, and seems to be essential for the treatment of high frequency problems.

In this paper an attempt is made to describe the general properties of the magnetic vector potential, and to make clear its relation to the fundamental equations of electromagnetic theory. Its use in the derivation of general relationships for electric circuit analysis is described in some detail, and in so doing the fundamental ideas involved in circuit analysis are introduced and clarified. In particular, the concept of inductance is considered rather carefully and a fundamental definition for this important circuit parameter is presented. At the same time, those properties of the magnetic vector potential which make it so valuable in 2-dimensional low-frequency problems are emphasized, and the manner in which it may be used at high frequencies is considered. Thus it is shown that the magnetic vector potential is an exceedingly useful mathematical tool, of value not only in the solution of particular problems but also in the derivation of important general relationships.

Definition of Magnetic Vector Potential

According to Maxwell's development of electromagnetic theory, fundamental field quantities comprise the electric and magnetic field intensities, together with the charge and current densities, the relations between these quantities being expressed by 4 equations. In vector notation, these are written,

$$\nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{i} + \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{3}$$

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho \tag{4}$$

Two additional equations are also required in order to define the relation between the magnetic field intensity $\boldsymbol{H}$ and the magnetic induction, or flux density, $\boldsymbol{B}$, and the similar relation between the electric field intensity $\boldsymbol{E}$ and the displacement, or electric flux density, $\boldsymbol{D}$. If the various media in the system are everywhere isotropic, these relations may be stated in the following simple form:

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{5}$$

$$\boldsymbol{D} = k\boldsymbol{E} \tag{6}$$

in which μ and k are, respectively, the magnetic permeability and dielectric constant of the medium. For the purposes of this discussion, it will be assumed that the permeability μ is actually a constant, independent of the strength of the magnetic field.

The units in which the quantities involved in the above equations are to be measured are those included in the unrationalized or Gaussian system of absolute units. According to this system, the current density $\boldsymbol{i}$, the charge density ρ, and the electric field vectors $\boldsymbol{E}$ and $\boldsymbol{D}$ are each to be measured in absolute electrostatic units, while the magnetic field vectors $\boldsymbol{H}$ and $\boldsymbol{B}$ are to be measured in absolute electromagnetic units. The quantity c is introduced simply as a conversion factor between the 2 kinds of units, but it may be shown to have the dimensions of a velocity, the permeability and dielectric constant being defined as pure numerics. By adopting this group of definitions, equations of a symmetrical nature may be obtained and the factor c may be shown to be equal to the velocity of light in vacuum.

From the definitions of the vector operators, as given in standard texts on vector analysis or mathematical physics (see references at end of paper), it will be evident that Maxwell's equations are really partial differential equations, applying simultaneously to any point in the system. This method of stating the fundamental laws of electricity and magnetism is not particularly convenient for engineering use, and it would be very desirable to derive the corresponding integral solutions. In a few simple cases this can be done by writing general solutions for the differential equations, and then fitting these solutions to the given boundary conditions. For electrical networks, however, the boundary conditions are extremely complicated; in fact, they are so complicated that this procedure becomes quite

hopeless. It consequently becomes necessary to introduce the idea of the potential functions by the use of which indirect solutions may be obtained. From these solutions several important general conclusions may be derived and useful methods may be developed for the solution of particular problems.

One of the 2 potential functions is already quite familiar to electrical engineers. It is the electric scalar potential, from which it is possible to derive the electrostatic field intensity by performing the differentiations specified by the following equation:

$$\boldsymbol{E}_q = -\nabla\Phi \tag{7}$$

It should be emphasized that this gives only one component of the total possible electric field, that which depends only on the instantaneous location and magnitude of each of the charges in the system. These charges may also be in motion and thus may be partially responsible for the existence of varying magnetic fields which, according to Faraday's law, may set up induced electric fields. Such magnetically induced electric fields are not included in $\boldsymbol{E}_q$ and must be calculated separately, according to equation 2. It will be shown later that equation 2 may be considerably simplified by the introduction of the magnetic vector potential.

It is possible to give a very definite physical significance to the potential Φ in the case of electrostatic fields. Then the entire electric intensity is given by equation 7 and the potential may be interpreted as the amount of work necessary to bring a unit test charge from outside the field up to the point in question. Because of the convenient picture which this interpretation makes possible, it is often desirable to apply it to the general case in which $\boldsymbol{E}_q$ is not the only component field. Unless care is taken to differentiate between the various component fields, this procedure is likely to give erroneous conclusions, since those parts of the field arising, for example, from electromagnetic induction or electrochemical effects are not included in $\boldsymbol{E}_q$. In fact, the work done against these additional components of field is not independent of the path and consequently they cannot be considered in terms of a scalar potential. The above interpretation of potential must, therefore, be qualified to include only the work done against the component $\boldsymbol{E}_q$ of the total field. In addition, care must be taken in applying these ideas at high frequencies, in which case the effects of retardation must be considered.

Unfortunately, the magnetic vector potential, even for steady currents and consequently static magnetic fields, cannot be given the same kind of physical significance. Instead, it probably should be considered simply as a valuable mathematical tool, its definition being determined on the basis of mathematical convenience rather than direct physical significance. The particular form of the defining equations, however, must be consistent with the physical properties of the magnetic field, as expressed in Maxwell's equations. From these equations it may be noticed that the divergence of the magnetic field is everywhere zero. In physical terms, this means that there can exist no true magnetic charges, and further implies that magnetic lines of force must always form closed loops. Thus the magnetic field differs from the static electric field in several important respects, and it should not be expected that a magnetic potential of the same type as Φ can, in general, be used. Indeed, the simplest potential function entirely consistent with the properties of the magnetic field is defined by the relation

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{8}$$

in which $\boldsymbol{A}$ is the magnetic vector potential. According to this definition, the magnetic vector potential is analogous to the electric scalar potential in at least one important respect. The process by which the field is derived is again a process of differentiation. However, since the differentiations are in this case specified by the curl, it is rather difficult to form a physical picture of the relation of the vector potential to the field.

Before this relation is considered, it should be noticed that the mathematical definition of the magnetic vector potential is not yet complete. So far, only its curl has been defined and thus it might include any arbitrary function whose curl is zero. Any number of different vector functions might therefore be set up, all of which would satisfy the condition that the magnetic field must be derivable according to equation 8. In order to determine that there shall be only one vector function satisfying this condition for any particular field configuration, it is necessary to put a restriction on the divergence of $\boldsymbol{A}$. Fortunately, such a restriction cannot conflict with equation 8 which expresses the only fundamental relation between $\boldsymbol{A}$ and the electromagnetic field. Consequently, it is possible to adopt that particular form of restriction which leads to the most convenient mathematical equations and, for steady currents and consequently static magnetic fields, the most satisfactory relation is found to be:

$$\nabla \cdot \boldsymbol{A} = 0 \tag{9}$$

It will be shown later that a somewhat different restriction on the divergence of $\boldsymbol{A}$ should be adopted for alternating fields, in order to simplify the rigorous solution of the resulting equations. However, it will also be shown that equation 9 and the equations resulting from its use provide satisfactory methods of calculation, even for systems of moderately high frequency. For this reason, it is particularly desirable to review the properties at low frequencies of the magnetic vector potential which arise from equations 8 and 9.

Properties of Magnetic Vector Potential at Low Frequencies

In order to show the properties of the magnetic vector potential, as defined by the preceding equations, a theorem in vector analysis must now be introduced. Known as Stokes's theorem, it expresses a general relation between the total "flux" of the curl of a vector function through any surface and the line integral of the same vector function around the boundary of the surface. In vector notation it is written,

$$\int_S \nabla \times \boldsymbol{A} \cdot d\boldsymbol{S} = \oint \boldsymbol{A} \cdot d\boldsymbol{l} \tag{10}$$

The left side evidently implies that the normal component of the vector $\nabla \times \boldsymbol{A}$ must be summed over the surface S, while the integration on the right side must be taken once around the boundary of the same surface. Each element of the latter integral is to be formed by taking the component of $\boldsymbol{A}$ along the boundary and multiplying by the corresponding element dl; then all these elements are to be added to give the total line integral. Applying these ideas to the magnetic field, and substituting from equation 8, there results,

$$\int_S \boldsymbol{B} \cdot d\boldsymbol{S} = \oint \boldsymbol{A} \cdot d\boldsymbol{l} \tag{11}$$

From this equation it is evidently possible to express the total magnetic flux through any surface in terms of the line integral of the vector potential, the latter integral to be evaluated along the line bounding the surface. Thus, for a given tube of flux, the line integral of the magnetic vector potential taken once around any cross section of the tube must always be the same. Similarly, the total flux in the core of a transformer or in the field pole of a generator might be given in terms of the line integral of the magnetic vector potential taken once around the iron core.

Although these relations make it possible to calculate the integral of the magnetic vector potential along particular paths when the magnetic flux is already known, no method as yet has been described for the calculation of the magnetic vector potential directly from a knowledge of the distribution of current in a conducting system. Such methods may be obtained from a consideration of equation 1. Substituting equation 8 in equation 1, assuming that the "displacement current" $\frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}$ is negligible (an assumption which is strictly correct only for steady currents, but is usually sufficiently accurate for low frequencies), and applying the restriction of equation 9, there results,

$$\nabla^2 \boldsymbol{A} = -\frac{4\pi\mu}{c} i \tag{12}$$

In regions in which there is no current flowing, this becomes,

$$\nabla^2 \boldsymbol{A} = 0 \tag{13}$$

From the definition of the operator ∇^2, it is apparent that each component of the magnetic vector potential in rectangular co-ordinates must satisfy an equation analogous to Laplace's or Poisson's equations in electrostatics. The latter equation may be written,

$$\nabla^2 \Phi = -\frac{4\pi}{k} \rho \tag{14}$$

and, if k is constant throughout space, it has the solution

$$\Phi = \int \frac{\rho dv}{kr} \tag{15}$$

When the permeability μ is constant throughout space, each component of the vector potential must obey a relation analogous to equation 15 and when the components are added, it is found that

$$\boldsymbol{A} = \int \frac{\mu i dv}{cr} \tag{16}$$

Thus, each element of current contributes to the value of the magnetic vector potential at any point a component which is in the same direction as the current element and has the magnitude $\mu i dv/cr$, where r is the distance from the current element to the point in question. If the current is confined to a very thin conductor, so that its direction is everywhere the same as the axis of the conductor, equation 16 becomes

$$\boldsymbol{A} = \int \frac{\mu I_m d\boldsymbol{l}}{r} \tag{17}$$

where I_m is the total current flowing in the length $d\boldsymbol{l}$ of the conductor. In this equation, both $\boldsymbol{A}$ and I_m are to be measured in absolute electromagnetic units and consequently c does not appear. It should be noticed that when $\boldsymbol{A}$ and I are expressed in these units, the magnetic vector potential has the same dimensions as the current.

Equations 12, 13, 16, and 17 provide the required means for the calculation of the vector potential from a known distribution of current in the conductors of the system. The integral forms, equations 16 and 17, may be used only for those cases in which the magnetic permeability may be assumed to be constant throughout space. Even then the required integration may be extremely laborious, and therefore it may be desirable to adopt an alternative method based on the general solutions of equations 12 and 13. These solutions must be obtained in their general form and then must be fitted to the conditions of the problem by adjustment of the arbitrary constants. This procedure often involves mathematical difficulties, but often it will be found that the required procedure has already been developed for the solution of the corresponding problem in electrostatics. The solution of the magnetic problem will, however, be somewhat more complicated, because of the necessity of considering each of the 3 components of the vector potential.

This difficulty disappears in 2-dimensional cases, in which the magnetic vector potential is everywhere in the same direction, and is independent of one of the rectangular co-ordinates. Thus it may be treated exactly as if it were a scalar function of 2 dimensions, and the methods used in the calculation of the potential in 2-dimensional electrostatic systems may be applied directly. Moreover, in such cases the relation between the magnetic vector potential and the magnetic field may be very readily demonstrated and visualized. In this connection, Rogowski showed that the surfaces of equal magnetic vector potential coincide with the cylindrical surfaces generated by the lines of force. In addition, the difference of magnetic vector potential between 2 such surfaces is equal to the number of lines of force contained

between the 2 surfaces, per unit length of the system. Thus a plot of the lines of constant magnetic vector potential has exactly the same shape as the corresponding flux plot and the numerical values for the vector potential may be used to express the total flux, per unit length of the system, contained between any 2 lines of force. The total flux linking with each section of a conductor carrying current may, therefore, be expressed directly in terms of the magnetic vector potential and the calculation of the inductance of the system may be greatly simplified thereby.

Certain inductance formulas for special cases have been developed on this basis, but these are restricted necessarily to 2-dimensional systems and involve the tacit assumption of the validity of the usual engineering definition of inductance, based on the idea of flux linkages. This assumption may be shown to be unnecessary, and a more precise idea of the true meaning of inductance may be obtained, if the required definitions are derived directly from Maxwell's equations. Before this can be done, however, it will be necessary to consider the meaning of the term "electromotive force" when it is applied to a conducting system.

Electromotive Force in a Conductor

The electromotive force in a conductor is defined in terms of the average rate at which work must be done by the electric field in moving each of the charges comprising the total current throughout the given length of the conductor. In order to obtain an analytical expression for this idea, it will be necessary to imagine that a particular set of cross-sectional surfaces has been constructed within the conductor. These surfaces must be so constructed (see figure 1) that each element of each surface is perpendicular to the net electric field at that point. If the conductor obeys the generalized form of Ohm's law, the current density at any point will be

$$\boldsymbol{i} = \frac{\boldsymbol{E}}{r_0} \tag{18}$$

where r_0 is the specific resistance of the conducting material and $\boldsymbol{E}$ is the net electric field intensity. Therefore the flow of current will be everywhere perpendicular to each of the set of cross-sectional surfaces.

Considering the elementary volume of the conductor contained between 2 of these cross-sectional surfaces which are very close together, the total current flowing through this volume may be subdivided into a great many filaments of infinitesimal cross sections, each filament being perpendicular to the 2 bounding surfaces. Then the rate at which the electric field must do work on the charges comprising the current in any one of these filaments will be

$$\boldsymbol{E}\cdot\boldsymbol{i}dSdl = \boldsymbol{E}\cdot d\boldsymbol{I}dl \tag{19}$$

where dS is the area of the filament, dl is its length and $d\boldsymbol{I}$ is the total current which it carries. If this quantity is evaluated for each filament, and the results added, the total rate at which work is being done throughout the volume will be obtained. The length dl need not necessarily be the same for each of the current filaments in the volume, and this variation must be taken into account in the process of summation. However, if the radial component of current flow may be neglected, dl may be taken as a constant and the total rate at which work is being done by the field in the volume under consideration may be written

$$\left\{\int_0^I \boldsymbol{E}\cdot d\boldsymbol{I}\right\}dl \tag{20}$$

Then, if the total current I is the same throughout a certain length of the conductor, the total electromotive force in this length may be found by integrating equation 20 throughout the corresponding volume and dividing by the current. Thus

$$V_{ab} = \frac{1}{I}\int_a^b\left\{\int_0^I \boldsymbol{E}\cdot d\boldsymbol{I}\right\}dl \tag{21}$$

where V_{ab} is the electromotive force between the 2 surfaces, a and b, of the set described above.

It should be noticed that when the conductor has very sharp bends or sudden changes in cross section the radial component of current flow cannot be neglected, and at such points the above expressions would require a small correction. Radial components of current flow can arise also from the effect of distributed capacitance. However, the effect of such radial flow is usually negligible in comparison with the effect of distributed capacitance on the definition of electromotive force caused by the resultant variation of the total current along the length of the conductor. This variation makes it impossible to use equation 21, and instead, a new quantity, the "equivalent" electromotive force, must be defined in terms of the current crossing a pair of reference cross sections in the conductor. If I_0 is the current at the reference cross sections, a and b, the equivalent electromotive force may be written

$$V_{ab}' = \frac{1}{I_0}\int_a^b\left\{\int_0^I \boldsymbol{E}\cdot d\boldsymbol{I}\right\}dl \tag{22}$$

Since this is actually the quantity which would be measured by suitable instruments inserted between the reference cross sections, it is of great practical importance. Its significance will be more apparent following a consideration of the components into which the total electromotive force is usually resolved.

These components are obtained by considering the various possible contributions to the electric field at every point in the conductor. For this purpose, the total electric field may be divided conveniently into 2 parts: that resulting from the currents and charges in the particular conducting system under consideration, and that resulting from all other possible causes. The latter part will then include the field set up by currents and charges which exist on neighboring conductors together with the field caused by chemical, thermal, or contact electromotive forces within the conductors of the system itself, but arbitrarily excluded from explicit consideration. All these effects may be lumped

together and represented by the "impressed field" $\boldsymbol{E}_0$.

The field set up by the currents and charges on the conducting system itself may be calculated by means of Maxwell's equations, after they have been simplified by the introduction of the 2 potential functions. The required procedure may be determined by substituting for the magnetic induction in equation 2 its value in terms of the magnetic vector potential. Then, since the space differentiation represented by the curl is quite independent of the time differentiation, this equation may be written,

$$\nabla \times \left(\boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}\right) = 0 \tag{23}$$

in which the order of differentiation has been reversed. The term within the brackets evidently represents an electric field whose curl is everywhere zero. Thus it may be immediately recognized as the field resulting from a distribution of charges and consequently derivable from the scalar potential

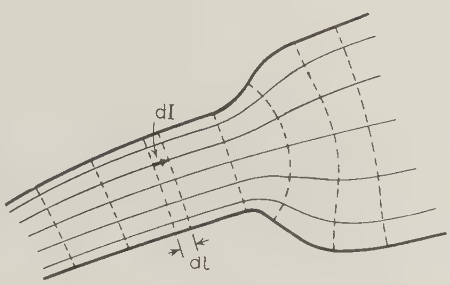


Fig. 1. Longitudinal cross section of a conductor, showing the lines of current flow and members of the set of cross-sectional surfaces (dotted lines)

Φ according to equation 7. The total electric field resulting from the charges and currents in the system is therefore given by

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\Phi \tag{24}$$

in which the first term on the right side represents the field due to the varying magnetic field and the second term that due to the charges. Introducing the impressed field $\boldsymbol{E}_0$, the total electric field in the conductor at any point becomes

$$= \boldsymbol{i}r_0 = \boldsymbol{E}_0 - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\Phi \tag{25}$$

Thus, with the aid of the 2 potentials, $\boldsymbol{A}$ and Φ, a relation has been derived by means of which the electric field intensity at any point in a conductor may be calculated, at least theoretically. No assumptions have been necessary in the derivation of this relation; it is an accurate and complete description of the field existing in any conductor which obeys the generalized form of Ohm's law. It gives much more information regarding the induced electric field than is given by the usual engineering relations for electromagnetic induction which can yield only the value of the induced electromotive force in a closed conductor, and cannot be used to determine the electric field induced at any particular point in the conductor. The value of the electromotive force induced in any conductor, either closed or open, can be obtained immediately by substituting equation 25 in equations 21 or 22. When this substitution is made, the various component electromotive forces, applied, induced, and resulting from charges, may be recognized. In particular, the induced electromotive force in a conductor with negligible distributed capacitance is seen to be

$$V_I = -\int_a^b \frac{1}{cI}\left\{\int_0^I \frac{\partial \boldsymbol{A}}{\partial t}\cdot d\boldsymbol{I}\right\}dl \tag{26}$$

and, at low frequencies, this may be expressed in terms of the coefficients of inductance.

Inductance at Low Frequencies

The assumption of low frequency must be made in order that the various component magnetic fields may be assumed to be directly proportional to, and everywhere in phase with, the various currents which produce them. Further assumptions which must be made for the same reason are, first, that the distribution of current in the conductors of the system is fixed and independent of frequency, and second, that the magnetic permeability is everywhere constant and independent of the magnetic field intensity. If these assumptions are made, the magnetic vector potential at every point in the system may be calculated according to the low frequency equations already given and, further, each component of this potential will be directly proportional to the current which sets it up. Thus, if there are n meshes, carrying n different currents, the vector potential anywhere in the system will be given by

$$\boldsymbol{A} = (\boldsymbol{A}_1 + \boldsymbol{A}_2 + \ldots + \boldsymbol{A}_n) = (\boldsymbol{k}_1 I_1 + \boldsymbol{k}_2 I_2 + \ldots + \boldsymbol{k}_n I_n) \tag{27}$$

in which the symbols $\boldsymbol{k}$ are vector factors which relate the scalar magnitude of each current to the magnitude and direction of the magnetic vector potential which this current sets up at any point. Substituting in equation 26, there results

$$V_I = -\frac{dI_1}{dt}\int_a^b \frac{1}{cI_1^2}\int_0^{I_1} \boldsymbol{A}_1\cdot d\boldsymbol{I}_1 dl_1 - \frac{dI_2}{dt}\int_a^b \frac{1}{cI_1 I_2}\int_0^{I_1} \boldsymbol{A}_2\cdot d\boldsymbol{I}_1 dl_1 - \ldots \tag{28}$$

in which the integrals may be written in the following general form:

$$L_{1h} = \int_a^b \frac{1}{cI_1 I_h}\int_0^{I_1} \boldsymbol{A}_h\cdot d\boldsymbol{I}_1 dl_1 \tag{29}$$

Obviously, this expression provides a definition for the ideas of self- and mutual inductance; when the subscript h is put equal to 1, it gives the self-inductance of circuit 1, and when the subscript has any other value between 1 and n, it gives the mutual inductance between circuit 1 and the circuit corresponding to the subscript.

It may not be immediately evident that equation 29 expresses, in special cases, exactly the same relations as are usually involved in the definition of inductance. However, this may be shown readily. For instance, if the conductor is very thin, the electric field may be assumed to be uniform over each cross

section of the conductor and the averaging process indicated by the inner integral may be dispensed with. Then equation 29 becomes

$$L_{1h} = \frac{1}{cI_h}\int_a^b A_h \cdot dl_1 \tag{30}$$

For a closed conducting circuit, this reduces to the usual "flux linkage" form after applying Stokes's theorem and substituting the magnetic flux density for the curl of the magnetic vector potential. It should be emphasized, however, that the flux linkage definition has no significance for other than closed circuits, and hence it strictly cannot be applied to circuits including capacitors. This point is, of course, trivial for power frequencies and frequencies even moderately high, but it should be kept in mind at very high frequencies.

The idea of fractional flux linkages, so useful in 2-dimensional calculations, may also be derived from equation 29. In such cases, the cross-sectional surfaces required for the definition of electromotive force become a set of planes, each of which is perpendicular to the axis of the conductors. Also, the magnetic vector potential, in such systems, must always be directed parallel to the same axis. Consequently, all the elements of current in the conductors must be parallel to the magnetic vector potential and equation 30 may be evaluated readily.

For an infinitely long conductor of circular cross section and with a coaxial return circuit, the required computation may be illustrated easily. Suppose that a plane strip of unit width has been constructed, the boundaries of which consist of parallel radial lines extending from the axis of the conductor to the return circuit, as shown in figure 2. Then by Stokes's theorem, the flux threading this strip between any 2 radii will be given by the line integral of the magnetic vector potential taken around the boundary of this portion of the strip. But, since the magnetic vector potential is perpendicular to the sides of the strip, the only contribution to this integral will be given by those portions of the boundary parallel to the axis at the 2 radii. Thus the flux per unit length of the system, which exists between any 2 radii, is equal to the difference of the magnetic vector potential between the same 2 radii. If the magnetic vector potential is set equal to zero at the radius of the return path, the total flux linking with a current element at any radius is therefore directly equal to the magnetic vector potential. In this case the averaging process involved in the inner integral of equation 29 is evidently equivalent to the more usual method using fractional flux linkages. The same ideas can be extended to other 2-dimensional cases and it has even been shown that the useful concept of geometric mean distance in 2 dimensions can be derived from equation 29. These points have been discussed in papers by Stevenson and Park[3] and by Robertson and Terry.[4]

It also should be mentioned that the substitution of the integral expressions for the potential functions in equation 25 leads to an integral equation from which it is sometimes possible to derive both the inductance and the a-c resistance of 2-dimensional conductors. In such cases, the impressed field $\boldsymbol{E}_0$ is assumed to be uniform over each cross section and the distribution of charges may be neglected. Integral equations of this form, although not written specifically in terms of the magnetic vector potential, have been used by Curtis[6] for the determination of the current distribution, inductance, and a-c resistance of 2 parallel cylindrical conductors of circular cross section. A slightly different method of attack, based directly on the properties of the magnetic vector potential, was used by Lettowsky[7] in determining the skin effect in cylindrical conductors of elliptical cross section.

Magnetic Vector Potential at High Frequencies

At high frequencies equation 25, giving the various components of the electric field in terms of the potentials, is still valid, but different methods must be used for the calculation of the potentials themselves. The restriction that the divergence of $\boldsymbol{A}$ shall be zero is no longer most convenient and instead the following restriction is adopted:

$$\nabla \cdot \boldsymbol{A} = -\frac{\mu k}{c}\frac{\partial \Phi}{\partial t} \tag{31}$$

Using this new restriction, and not neglecting the "displacement current," it now becomes possible to derive, from Maxwell's equations, the so-called "inhomogeneous wave equations," which are written,

$$\nabla^2\Phi - \frac{\mu k}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{k}\rho \tag{32}$$

$$\nabla^2\boldsymbol{A} - \frac{\mu k}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} = -\frac{4\pi\mu}{c}\boldsymbol{i} \tag{33}$$

A rigorous solution of these equations is rather difficult, but it may be shown that the "retarded potentials" provide the required solutions. These are

$$\Phi = \int \frac{\rho\left(t - \frac{r}{c}\right)}{r}\,dv \tag{34}$$

$$\boldsymbol{A} = \int \frac{\boldsymbol{i}\left(t - \frac{r}{c}\right)}{r}\,dv \tag{35}$$

in which Φ and ρ are in electrostatic units, while $\boldsymbol{A}$ and $\boldsymbol{i}$ are in electromagnetic units. It has also been assumed that μ and k are everywhere equal to unity, an assumption which is justified by the fact that magnetic materials and dielectrics are usually avoided in systems for very high frequencies.

The functional notation used in these 2 equations means that, in calculating the effect of a given element of current or charge on the potentials at a distant point, it is necessary to take into account the time necessary for the propagation of the effect from the element to the point in question. This time is given by the quantity r/c. Consequently, if the potentials are desired at a given point and time t, the magnitude of each element of current or charge must be taken as that existing, not at the time t, but at an earlier time $(t - r/c)$. Evidently, this

retardation in the magnetic and electric fields will make impossible the assumption that the potentials are everywhere in phase with the charges or currents by which they are set up, as was done in the low frequency case. In fact, this assumption actually is never completely accurate except in systems of steady current. In all other cases, such an assumption implies that the frequency is so low that steady current ideas can be used without introducing appreciable error.

Calculations for High Frequency Circuits

Having determined the proper expression for the potential functions in the case of alternating currents, it now becomes possible to write the integral expression for the electromotive force in a conductor

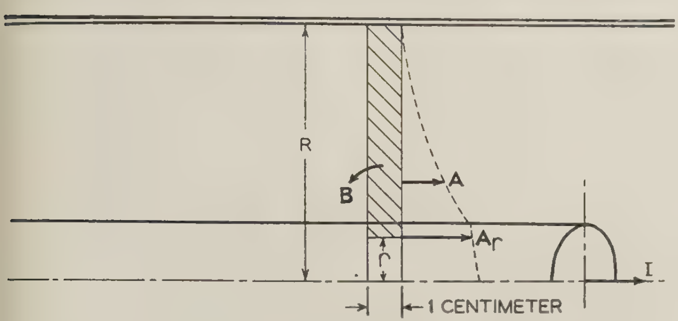


Fig. 2. Illustration of the relations between the field and the magnetic vector potential surrounding a long cylindrical conductor with coaxial return circuit

in a more accurate form. For this purpose, it will be necessary to use equation 22 for the "equivalent" electromotive force, since, in general, the current in a conductor will vary, in phase and magnitude, from one cross section to another. Using this expression, and assuming that the conductor is very thin, the following equation may be derived:

$$V_{ab}' = \frac{1}{i_0}\int_a^b E_0 i\cdot dl - \frac{1}{i_0}\int_a^b \frac{1}{c}\frac{\partial}{\partial t}\left\{\int \frac{i'\left(t-\frac{r}{c}\right)}{r}\,dl'\right\} i\cdot dl - \frac{1}{i_0}\int_a^b \frac{\partial}{\partial l}\left\{\int \frac{q'\left(t-\frac{r}{c}\right)}{r}\,dl'\right\} i\cdot dl \qquad (36)$$

where i_0 is the instantaneous current at the reference cross sections a and b, i' and q' are, respectively, the current and charge per unit length in the conductor element dl', i is the current in the element dl, and r is the distance between the elements dl' and dl, the current being in electromagnetic units, all other quantities in electrostatic units. The inner integration of each double integral is to be taken over all the conductors in the system while the second integration is to be taken over the length ab of the conductor in which the electromotive force is desired.

At low frequencies, it was possible to simplify the corresponding integral expression by introducing the 3 circuit parameters and thus to develop the usual circuit theory. On the basis of this theory, the currents in the various conductors could then be calculated from a knowledge of the applied electromotive forces. Unfortunately, the same procedure cannot be adopted at high frequencies. The integral equation 36 theoretically includes all the necessary relationships between the currents and charges and the electromotive forces, but its solution is much too complicated to give a determination of the currents from a knowledge of the electromotive forces. However, if the current distribution could be determined in some other way, this equation would make it possible to calculate the resultant electromotive forces and thus to determine, by comparison, the extent of the errors introduced by the use of the low frequency methods.

Experience has shown that a first approximation to the actual distribution of current may be obtained by the use of the low frequency concepts, such as those of distributed capacitance and inductance. That this procedure should give, in most cases, quite satisfactory results will be evident from a consideration of the magnetically-induced electromotive force in a closed circuit made up of a very thin conductor. It will be assumed that the current flowing in this loop is everywhere the same in phase and magnitude and is sinusoidal, being represented by the real part of $I\epsilon^{j\omega t}$. Then the electromotive force induced by the magnetic field may be written,

$$-\oint \frac{1}{c}\frac{\partial}{\partial t}\oint \frac{I\epsilon^{j\omega\left(t-\frac{r}{c}\right)}}{r}\,dl'\cdot dl = -j\omega\frac{I\epsilon^{j\omega t}}{c}\oint\oint\frac{1}{r}\left(\cos\frac{\omega r}{c} - j\sin\frac{\omega r}{c}\right)dl'\cdot dl \qquad (37)$$

The retardation in the magnetic field therefore results in the induction of 2 component electromotive forces, one in quadrature with the current and the other in phase opposition to the current. When the phase angle $\omega r/c$ is everywhere small, the former component can be calculated quite accurately by the low frequency methods, and the error involved in this procedure can be determined by expanding the cosine of $\omega r/c$ into a power series.

The existence of the second term in the integral is not predicted in any way by the usual circuit theory and the electromotive force which it represents evidently results from the radiation of energy from the system. This electromotive force will, under the above assumptions, be quite small in comparison with the quadrature component, as may be shown by expanding the sine of $\omega r/c$ into the corresponding power series. When this is done, the integral of the second term becomes

$$-\frac{\omega^2}{c^2}I\epsilon^{j\omega t}\oint\oint\left(1-\frac{\omega^2 r^2}{6c^2}+\frac{\omega^4 r^4}{120c^4}+\dots\right)dl'\cdot dl \qquad (38)$$

Because of the rapid convergence of the series within the brackets, this may be evaluated by integrating term by term and then summing the resultant series of integrals. In the first integration, the first term in the series will disappear since this integration involves the summation of all the vector elements dl' which comprise the closed circuit. The second term, which includes the factor ω^4/c^4, will become therefore the largest term in the final series. In most cases, this factor will be exceedingly small and

consequently the inphase component of induced electromotive force will be very small in comparison with the quadrature component. Thus the quadrature components of electromotive force may be used to give a good approximation to the actual current distribution in the conducting system, and these components may be calculated with reasonable accuracy from low frequency formulas. Moullin has demonstrated the accuracy of this procedure for certain specific cases.[8,13] The results obviously will be particularly accurate if $\omega r/c$, which is equal to $2\pi r/\lambda$, is everywhere small or, in other words, if the largest dimension of the system is small compared to the wavelength λ of the corresponding radiation.

Once the current distribution has been determined, the last 2 integrals of equation 36 may be used to determine the electromotive forces which would arise from this distribution of current and charge. As Moullin[8] has pointed out, if these electromotive forces do not agree sufficiently closely with those involved in the calculation of the current distribution, a slight change may be made in the expressions for the current distribution and the integrals of equation 36 recalculated. This process of successive approximations certainly would be laborious, and probably would involve the use of graphical integration, but it at least provides a possible solution to a rather difficult problem.

However, in most high frequency problems it is not desirable to adopt such methods in an attempt to obtain a completely consistent solution. Instead, it is generally sufficient to use the low frequency methods in the determination of the current distribution and then to calculate the increase in equivalent resistance caused by the effect of radiation. This may be determined by calculating the inphase component of the electromotive force resulting from the last 2 integrals of equation 36. In this calculation, a relation between the current and charge distribution on a conductor would be useful, and such a relation may be derived by taking the divergence of the first of Maxwell's equations, remembering that the divergence of the curl of a vector function is always zero. Substituting equation 4, and reducing to the case of a thin conductor, there results,

$$\frac{\partial i'}{\partial l} = -\frac{1}{c}\frac{\partial q'}{\partial t} \tag{39}$$

where the symbols have the same meaning as in equation 36. Consequently, when the currents are sinusoidal in time, the last 2 integrals of equation 36 may be combined conveniently into a single integral expressed in terms of the current distribution along the conductor. By the use of such an integral, the inphase component of electromotive force set up, for instance, by the currents and charges on an antenna may be determined, the usual assumption in that case being that the current is sinusoidally distributed along the conductor, this distribution being identical with that obtained by approximate low frequency calculations. When this component of the electromotive force is divided by the current at the reference cross section, usually a current loop, the equivalent "radiation resistance" of the antenna is obtained. This method was used by Pistolkors, who extended it to the case of multiple antenna ar- rays and open wire tuned transmission lines, thu providing a basis for further work by other investi gators.[10,11] Sterba and Feldman[12] have give formulas for the equivalent radiation resistance o tuned transmission lines with various terminations each of these formulas having been derived by methods, that described above and that using th Poynting vector. In the case of a small loop, eithe solidly closed or terminated on the plates of a ca pacitor, Moullin[8,13] has used the above methods t determine the radiation resistance and the necessar corrections to the inductance and capacitance.

It should be noticed that calculations of th radiation resistance based on the use of the idea o flux linkages, such as those given by Brainerd[1 and King,[15] can apply only to closed conductin circuits which carry the same current through ever cross section. In all other cases, the inphase com ponent of the electromotive force resulting from th distribution of charges on the conductors must b considered by some method such as that describe above.

References

1. Über die Streuung des Transformators, W. Rogowski. *Elek. Zeits.* v. 31, Oct. 13, 1910, p. 1033, and Oct. 20, 1910, p. 1069.

2. Fundamental Theory of Flux Plotting, A. R. Stevenson, Jr. *Gen Elec. Rev.*, v. 29, Nov. 1926, p. 797–804 (contains essentials of Rogowski's development).

3. Graphical Determination of Magnetic Fields, A. R. Stevenson, Jr. and R. H. Park. *Gen. Elec. Rev.*, v. 31. Feb. 1928, p. 99–109; and v. 31, Mar 1928, p. 153–64.

4. Analytical Determination of Magnetic Fields, B. L. Robertson and I. A. Terry. A.I.E.E. Trans., v. 48, Oct. 1929, p. 1243–59.

5. Electromagnetic Theory and the Foundations of Electric Circuit Theory, J. R. Carson. *Bell. Sys. Tech. Jl.*, v. 6, Jan. 1927, p. 1–17.

6. An Integration Method of Deriving the Alternating-Current Resistance and Inductance of Conductors, H. L. Curtis. U. S. Bur. of Stds *Scientific Papers*, v. 16, 1920, p. 93.

7. Skineffekt in Zylindrischen Leitern mit Elliptishem Querschnitt F. Lettowsky. *Arch. für Elek.*, v. 29, Aug. 1935. p. 556.

8. The Theory and Practice of Radio Frequency Measurements (a book), E. B. Moullin. J. B. Lippincott Co., second edition, 1931, p. 1–41.

9. The Radiation Resistance of Beam Antennas, A. A. Pistolkors. I.R.E *Proc.*, v. 17, March 1929, p. 562.

10. Circuit Relations in Radiating Systems and Applications to Antenna Problems, P. S. Carter. I.R.E. *Proc.*, v. 20, June 1932, p. 1004.

11. Rechnerische Ermittlung der Impedanz von Antennen, J. Labus. *Hochfrequenztechn. u. Elektroakustik*, v. 41, Jan. 1933, p. 17.

12. Transmission Lines for Short-Wave Radio Systems, E. J. Sterba and C. B. Feldman. I.R.E. *Proc.*, v. 20, July 1932, p. 1163.

13. The Fields Close to a Radiating Aerial, E. B. Moullin. Cambridge Phil. Soc. *Proc.*, v. 25, 1929, p. 491.

14. Inductance at High Frequencies and Its Relation to the Circuit Equations, J. G. Brainerd. I.R.E. *Proc.*, v. 22, 1934, p. 395.

15. The Telegraphists Equations at Ultra-High Frequencies, R. King. *Physics*, v. 6, April 1935, p. 121.

The following are recommended as useful texts:

16. Vector Analysis, J. G. Coffin. John Wiley & Sons, Inc., New York, N. Y., second edition, 1911.

17. Higher Mathematics for Engineers and Physicists, I. S. and E. S. Sokolnikoff. McGraw-Hill Book Co., New York, N. Y., 1934, p. 329–364.

18. The Classical Theory of Electricity and Magnetism, M. Abraham and R. Becker; translation by J. Dougall. Blackie & Son, Ltd., 1932.

19. Principles of Mathematical Physics, W. V. Houston. McGraw-Hill Book Co., New York, N. Y., 1934, p. 75–102 and 210–243.

20. The Electromagnetic Field, M. Mason and W. Weaver. Univ. of Chicago Press, 1929.

21. Electromagnetic Problems in Electrical Engineering, B. Hague. Oxford Univ. Press, 1929.

Laboratory Studies of Conductor Vibration

The results of wind tunnel studies of the vibration in specimen lengths of 5 types of transmission line conductors are presented in this paper. Measurements of the frequency and amplitude of vibration for various wind velocities, power required to vibrate the conductor in still air, and amount of energy taken from the wind by the vibrating conductor are reported for various conditions.

By
JOSEPH S. CARROLL
ASSOCIATE A.I.E.E.
Stanford Univ., Calif.

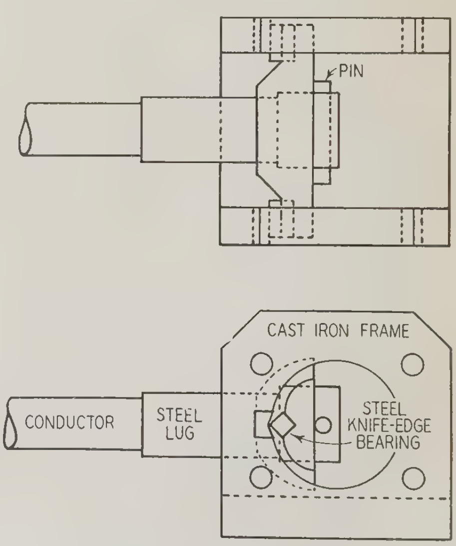
TROUBLES resulting from the vibration of transmission line conductors by transverse winds have been and still are a source of worry to the transmission engineer. The work reported in this paper was undertaken primarily to study some of the fundamental principles involved in this vibration. Different types of conductors were used and different mountings of these conductors were made primarily for the purpose of studying some of the factors involved in the problem; no attempt will be made to compare the conductors of different manufacturers and nothing in this paper is intended to be used to show the advantages or disadvantages of any particular manufacturer's cable.

This study was made in the laboratory, where 57 foot specimens were vibrated in the wind tunnel. The relation of the frequency and amplitude of vibration with the velocity of air passing the conductor was determined for different types of conductors. The amount of energy taken out of the wind by these vibrating conductors was measured for different conductors, frequencies, and amplitudes of vibration. By means of an electrical drive these conductors were vibrated in still air and the energy absorbed by the conductor as the result of internal damping was measured for various conditions and types of conductors.

Vibration of Conductors in the Wind Tunnel

The wind tunnel used in this study is a wooden structure approximately 100 feet long, 21 feet wide, and 15 feet high in over-all dimensions. Movement of air in the wind tunnel was forced by a large ventilating blower driven by a 75 horsepower motor. The outlet of this blower was joined by a funnel connection to a large pressure chamber 50 feet long, 15 feet high, and 15 feet wide. Air flowed from this chamber through adjustable orifices into smaller plenum chambers which were located along one side of the large pressure chamber, and was discharged from these plenum chambers through the main orifice where the conductor specimen was mounted. This orifice was 2 feet high and 50 feet long, the edges extending out parallel to the flow of the air for a distance of about 3 feet. The air from the main orifice was then returned to the fan intake. This air circuit was completely inclosed, thus eliminating trouble that might be caused by outside wind conditions. Adjustments were made in the plenum chamber orifices until a uniform flow of air was obtained throughout the length of the main orifice, the wind velocity being controlled by throttling the intake of the blower. The conductor specimens were mounted horizontally in the middle of the orifice and about 6 inches from the outer edge of the plenum chambers, and were a few feet longer than the orifice to allow the end supports to be clear of the opening, to rest directly on the ground, and in no way be attached to the wind tunnel itself where vibrations might be picked up. The particular type of end support used was the result of considerable study and experience, part of which will be described later in this paper. Ends of the conductor specimens were soldered into steel lugs about 6 inches long which were then secured to knife-edge

Fig. 1. Top and side views of knife-edge dead-end support for conductor



supports as shown in figure 1. As the conductor is made to vibrate the knife-edges rock back and forth on their seats with no bending of the cable at the point of support. By methods described later, it was found that the energy losses at these knife-edge supports were extremely small and could be neglected for all measurements involved in this work. The cast iron frame of these knife-edge supports rested on a wooden A frame approximately 5 feet

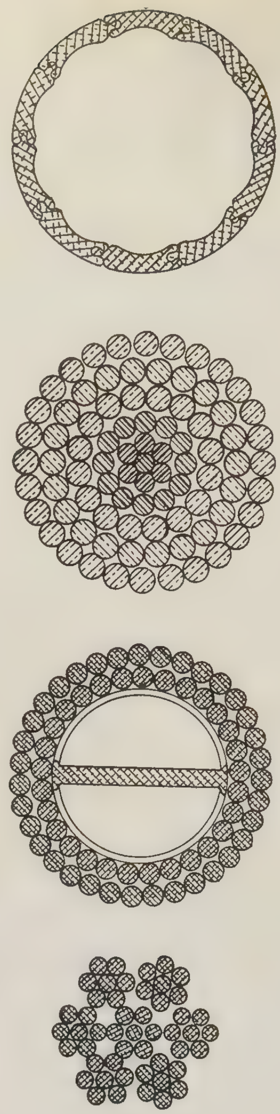
A paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Feb. 29, 1936; released for publication April 1, 1936.

high. The horizontal component of the load was taken by means of a steel cable fastened to the cast iron frame and extending back to an anchor buried securely in the ground, making an angle with the horizontal of approximately 15 degrees. A bell crank arrangement was mounted at one end between the cast iron knife-edge support and the anchor to keep constant tension on the conductor over a wide temperature range.

Conduct of Test

With the specimen mounted in place, the blower was started and its intake orifice almost completely closed. This opening was increased in such a manner as to increase the wind velocity in small steps at the main orifice. For each increment of wind velocity, the amplitude of vibration of the conductor was measured carefully and the number of loops of vibration of the conductor recorded. It was difficult to get the 57 foot specimen to vibrate in a single loop because of the extremely low wind velocity required. For further description of what takes place, reference may be made to the group of curves for the 1.0 inch type *E* conductor in figure 2, which is taken from the thesis prepared for the degree of electrical engineer by D. B. Fish and F. E. Gregory. A similar group of curves appears in an article by these men, "Characteristics and Energy Relations of Conductor Vibration" (*Elec. West*, v. 74, May 1935, p. 22–3).

With a wind velocity of 0.92 mile per hour the conductor vibrated to an amplitude of 1.07 centimeters. As the wind velocity was increased the amplitude of vibration of the conductor increased and reached a maximum of 2.16 centimeters with a wind velocity of 1.3 miles per hour. Further increase in wind velocity decreased the amplitude of vibration. As a matter of fact, this decreasing curve overlaps the beginning of the curve of vibration in 3 loops. In some instances it was necessary to damp out one of the overlapping frequencies in order to obtain the desired frequency. This was done by touching the conductor very gently at the node of the desired vibration. This procedure was continued until the frequency of vibration became sufficiently great that the amplitude of vibration was too small to be of any importance. The 4 sets of curves in figure 2 show the comparison not only of 3 different types of conductors but also 2 different diameters of the same type of conductor. These curves are fairly self-explanatory.



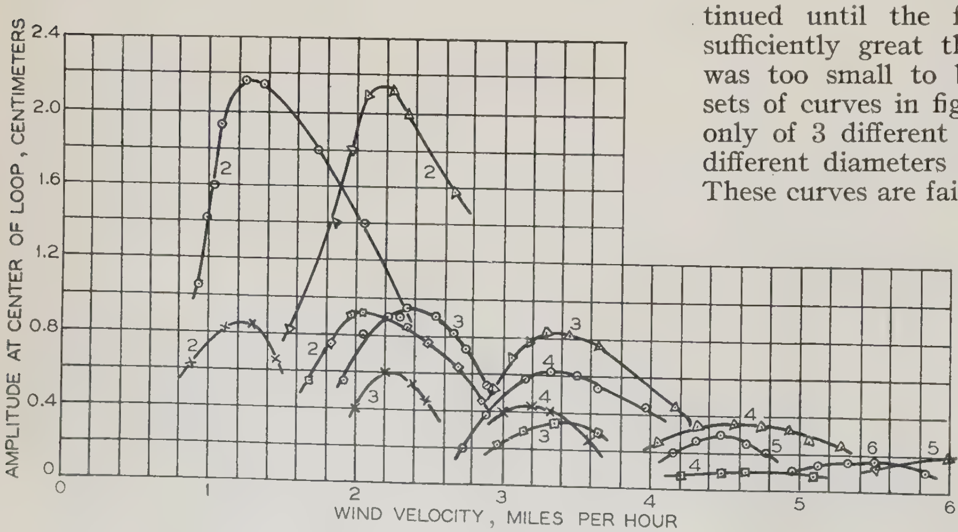
1.4 inch type *C* hollow copper conductor, made up of 10 segments with 28 inch right hand lay, and weighing 1.57 pounds per foot

1.4 inch type *D* aluminum conductor steel reinforced, with 14.5 inch left hand lay for outer layer and a weight of 1.852 pounds per foot

1.4 inch type *E* copper conductor of twisted I beam construction, with 10.9 inch right hand lay for the outer layer and a weight of 2.667 pounds per foot. The 1.0 inch type *E* conductor was o similar construction but with 28 wires having approximately 11 inch lay in the outer layer and 22 wires in the inner layer, and a weight of 1.612 pounds per foot

0.91 inch type *G* rope lay copper conductor of 7 large strands with righ hand lay and 15 inch pitch, each made up of 7 0.101 inch wires with left hand lay and 5 inch pitch, and weighing 1.54 pounds per foot

Fig. 3. Cross sections of conductors used in vibration tests



□—1.4 inch type C, 5,000 pounds tension; △—1.4 inch type E, 8,600 pounds tension
⊙—1.0 inch type E, 5,000 pounds tension; X—0.91 inch type G, 5,000 pounds tension

Fig. 2. Curves showing relation between amplitude of vibration and wind velocity for conductors in wind tunnel

All test specimens 57.3 feet long and mounted on knife-edge supports. Numbers on curves are loops of vibration

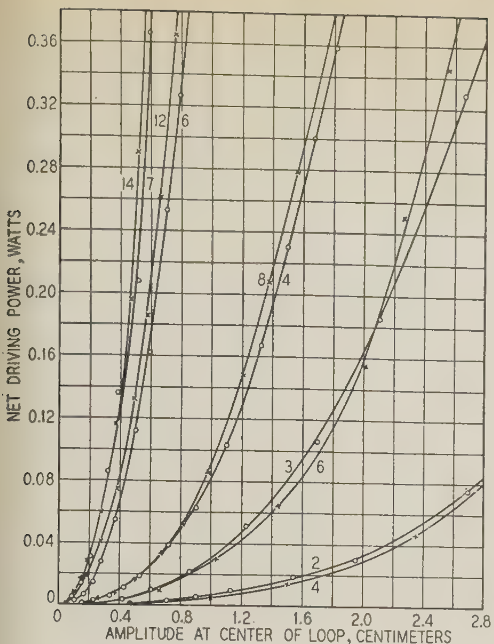
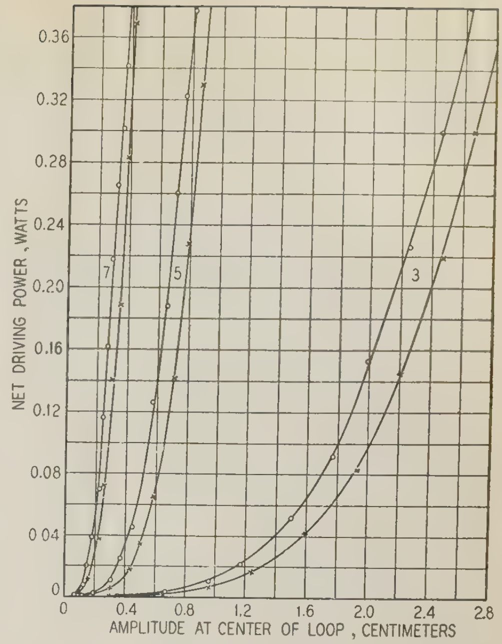


Fig. 4 (left). Curves showing power required to vibrate 1.4 inch type D conductor

Conductor dead-ended over suspension clamps and under 11,000 pounds tension. Curves through crosses are for 164 foot length, and curves through circles are for 82 foot length; numbers on curves are loops of vibration

Fig. 5 (right). Curves showing power required to vibrate 1.4 inch type C conductor

Conductor length 83.5 feet, dead-ended on knife edges, and under 5,000 pounds tension. Curves through crosses are for 6,000 pounds tension, and curves through circles are for 4,000 pounds tension; numbers on curves are loops of vibration



The results check reasonably well with the formula giving the frequency of air eddies in cycles per second on the side of the conductor, which is $f = Ku/d$ where K is 0.195 for cables in air streams, u is the velocity of the wind in centimeters per second, and d is the diameter of conductor in centimeters. For 2 loops on the 1.4 inch type E conductor the frequency calculated using this formula is 5.4 cycles per second and the observed value was 5.5 cycles per second. As the ratio of tension to weight per foot of all 4 conductors was held practically constant, the frequency of vibration of the 1.0 inch type E conductor was practically the same for the same number of loops as that of the 1.4 inch type E conductor, we would expect from the above formula that the 1.0 inch conductor would vibrate at a lower wind velocity than the 1.4 inch. This checks very well with the curves.

From the general shape of the curves the lack of sharp resonance can be seen, especially at the higher frequencies. The broad resonance curve, of course, is the result of high internal damping.

From tests that will be described later, it was found that the energy required to vibrate the 1.0 inch type E conductor in still air was practically the same as that required to vibrate the 0.91 inch type G conductor. Since the latter conductor did not vibrate to as great an amplitude in the wind as the 1.0 inch it would appear that the roughness of the surface of the type G conductor reduced the amount of energy it could take out of the wind.

A word might be said here concerning some of the factors which determine the maximum amplitude of vibration that a conductor can attain when vibrating in the wind. For a given conductor with a definite fixed tension and resonant wind velocity for a given frequency, the amplitude of vibration that the conductor will build up will depend primarily on 2 factors. One of these factors is the internal loss of the conductor, that is, the energy required to bend it back and forth in the vibrating loops. The other factor is the amount of energy which the conductor can take out of the wind while it is vibrating. When the conductor has reached its maximum amplitude of vibration in the wind, the internal losses or the energy necessary to bend the cable back and forth as it vibrates just equal the amount of energy taken out of the wind. Below this amplitude the energy taken out of the wind is always greater than the internal losses of the cable. The forms of these 2 energy curves will be considered in subsequent sections of this paper.

Measurement of Energy Required to Vibrate Conductors in Still Air

The method of driving the conductors and the method of measuring the power required to vibrate them are discussed in a paper by J. S. Carroll and J. A. Koontz, Jr.* For all of the tests reported here the a-c generator was used as a source of low frequency for the drive coils, the generator in this case being driven through a gear reduction by a small d-c motor. A slightly different wattmeter connection was used for this work than was described in the paper referred to because of the extremely small amounts of power being measured. However, the fundamental principle of the wattmeter connection was the same, and will not be discussed further here.

As previously mentioned, the knife-edge support was found to be the best method of terminating the conductors. This support was used throughout all the tests reported here except those for the type D conductor, the results of which are plotted in figure 4. At the time these tests were made it was not deemed advisable to cut the long specimen of this conductor for these tests, and a suitable short clamp for mounting on the knife-edges had not been developed. This conductor was supported 2 feet off the floor by means of a 6 inch by 6 inch wooden post at either end. On top of this wooden post rested an ordinary suspension clamp. The conductor rested in these clamps and was fastened down to the bottom

* Published in this issue, pages 490–3.

of the steel column of the building by means of suitable clamps, turnbuckles, and clevises. The main section of the conductor was horizontal. The angle from the support down to the anchor was approximately 15 degrees. It was known from pre-

type *C* conductor. It may be noted that with an increase in the tension the power required to vibrate the conductor to the same amplitude decreases. This is so for the 3 frequencies used. Since the increase in tension would cause an increase in fre-

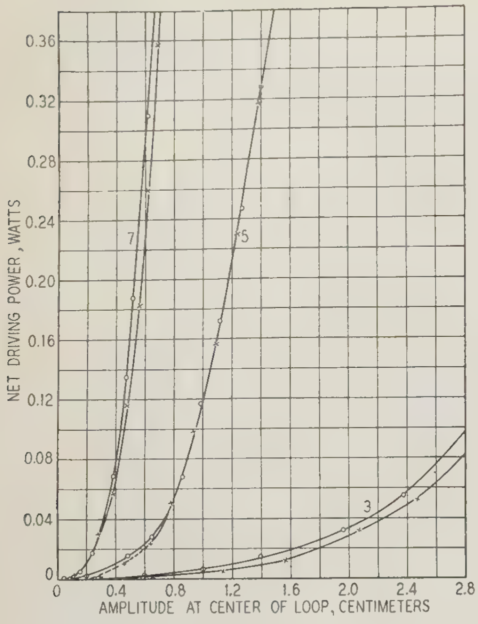
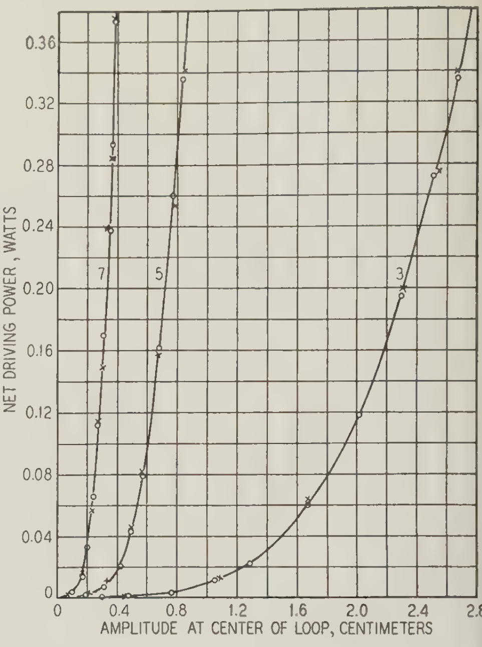


Fig. 6 (left). Curves showing power required to vibrate 1.4 inch type E conductor

Conductor length 83.5 feet, dead-ended on knife edges. Curves through crosses are for 6,000 pounds tension and curves through circles are for 8,600 pounds tension; numbers on curves are loops of vibration

Fig. 7 (right). Curves of power required to vibrate 1.4 inch type C conductor, showing that slippage between segments is negligible

Conductor length 83.5 feet, dead-ended on knife edges, and under 5,000 pounds tension. Curves through circles are for normal condition, and curves through crosses after soldering strands together for 2 inches in every 18; numbers on curves are loops of vibration



vious tests that the losses in these end supports were appreciable, so that an attempt was made to separate these losses from the energy required to vibrate the conductor. Vibration loss measurements were made first on a length of 164 feet; half of this length, or 82 feet, then was mounted with the same end supports and with the same clamps that had been used on the longer lengths. Measurements were made on this length similar to those made on the longer length. With the long length vibrating in 4 loops the frequency and loop lengths are exactly the same as when the short length is vibrating in 2 loops. Therefore, for the same amplitude, the difference in the amount of power necessary to vibrate the 164 foot length and that required to vibrate the 82 foot length should give the energy required to vibrate 82 feet of this conductor. The results shown in figure 4 indicate that the loss in the end supports was so much greater than the loss in the cable that it was impossible to determine the energy required to vibrate the cable itself. It may be noticed that less energy was required to drive the 164 foot length in 4 loops than was required to drive the 82 foot length in 2 loops. This, of course, means that the mountings at the end supports for the 2 cases were different even though extreme care was taken to make them the same. It must be remembered that these differences represent extremely small amounts of power. The method of supporting the suspension clamp in this test was somewhat different from the insulator support on the transmission line where the conductor clamp is free to pivot.

Figure 5 shows the amplitude loss curves for 3 different frequencies and 2 different tensions for the

quency for the same number of loops, the difference in loss per cycle for the 2 different tensions would be appreciably greater than shown by the 2 sets of curves.

Figure 6 is the similar group of curves for the type *E* conductor. However, it may be noticed that this conductor responded differently to a change in tension. For 5 loops there was no change in the amount of energy necessary to drive the conductor at a given amplitude. For 3 and 7 loops a decrease in tension was accompanied with a decrease in the amount of power necessary to vibrate the cable at a given amplitude.

Figure 7 shows the effect of soldering the type *C* cable in such a manner as to prevent slippage of the segments. The strands of the conductor were soldered together for about 2 inches every 18 inches along the conductor. If there were slippage between the segments of this conductor while it was vibrating an additional loss might be expected. Since there is no change in loss after soldering these segments to each other it is natural to conclude that there is no such slippage between the segments. This means that the conductor as far as these vibration tests were concerned acted as though it were a solid copper tube.

Determination of Amount of Energy Taken From Wind by Vibrating Conductor

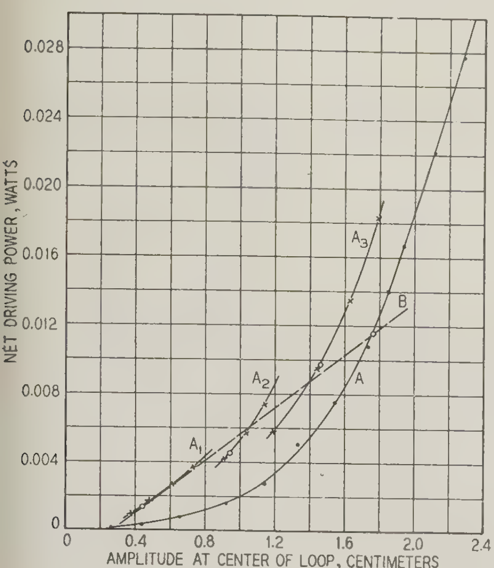
The electrical drive used to vibrate the conductors was mounted in the wind tunnel and was attached to the test specimen in such a manner that the conductor could be vibrated electrically in still air, or

could be vibrated by the wind independently of the electrical drive. With this arrangement the energy necessary to vibrate the cable at different frequencies and different amplitudes could be determined by means of the electrical drive. Without changing anything except opening up the circuit of the electrical drive coil the conductor could then be vibrated by means of the wind. Assuming that the air damping when the cable is driven electrically is negligible, which there is reason to believe is true, this arrangement serves as a substitution method for finding the energy that the vibrating conductor takes from the wind. Curve *A* in figure 8 is the amplitude-energy loss curve obtained in still air by means of the electrical drive. This cable when vibrated in the wind with no external load reached an amplitude of 1.76 centimeters (given by the circle on curve *A*.) It should be noted that the type *C* cable used in this test was a special test cable, not the same as the regular cable used in obtaining the data of figure 2. Therefore, under these conditions the energy taken out of the wind was just equal to the internal losses of the cable, which were 0.0116 watt. Another coil was mounted on the cable at the opposite end to the drive coil, and this coil, instead of being used as a drive coil, was used to load the cable. The coil, vibrating in a magnetic field, generated a voltage at its terminals and caused a current to flow through a suitable resistance connected to them, thereby absorbing energy from the cable. This proved to be a convenient and constant load that could be adjusted over a wide range. Curve A_3 of this same figure is the amplitude-energy loss curve obtained by means of the electrical drive with 5,760 ohms connected across the loading coil, the cable vibrating in still air. Then with all conditions undisturbed the drive coil circuit was opened and the fan started up. With the additional load, the cable built up to an amplitude of only 1.46 centimeters, shown by the circle on curve A_3. Under these conditions the cable was then taking 0.0098 watt out of the wind. This procedure was repeated for 2 other values of resistance across the load coil, giving curves A_1 and A_2. Curve *B*, drawn through the circles, represents the energy taken from the wind as a function of amplitude. By the same procedure curves of figures 9 and 10 were obtained for the type *E* cable for both 2 and 3 loops. The data given in figures 8, 9, and 10 are too meager to allow the drawing of any general conclusions. However, the following observations have been made:

1. From figures 8 and 9 it would appear that the amounts of energy taken out of the wind by the type *C* conductor and the type *E* conductor are nearly the same. The actual difference in power is small and is in the opposite direction from what one would expect.

2. Over the range shown in the curves the energy taken from the wind by the vibrating conductor is proportional to the amplitude. However, the straight line curve does not pass through the origin.

3. From figures 9 and 10 the energy taken from the wind at a given amplitude is roughly proportional to the cube of the 2 frequencies.



Curve *A*—Power-amplitude relation for unloaded conductor with electric drive

Curves A_1, A_2, and A_3—Same as curve *A* but with loadings

Curve *B*—Energy taken from wind by vibrating conductor as function of amplitude; circles indicate amplitudes of vibration in wind at different loads

Fig. 8. Curves showing energy taken from wind by 57.3 foot length of special 1.4 inch type C conductor vibrating in 2 loops at 5,000 pounds tension

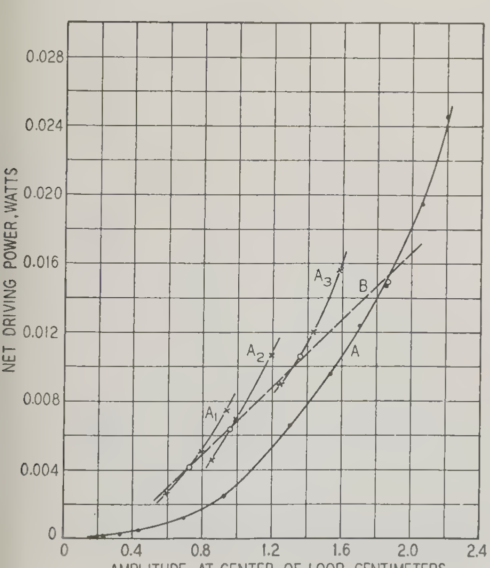
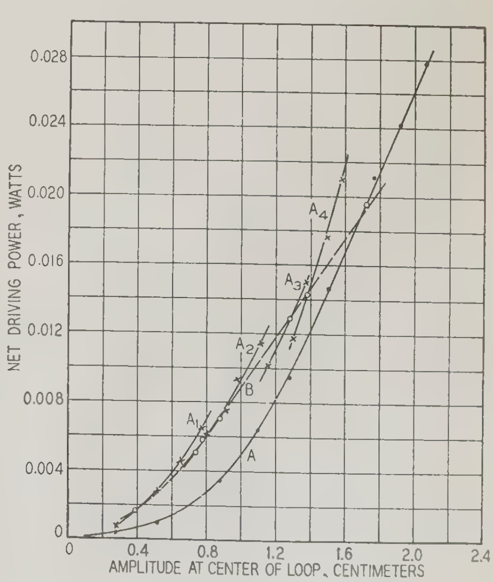


Fig. 9 (left). Energy taken from wind by 57.3 foot length of 1.4 inch type E conductor vibrating in 2 loops at 8,600 pounds tension

Curve designations same as in figure 8

Fig. 10 (right). Energy taken from wind by 57.3 foot length of 1.4 inch type E conductor vibrating in 3 loops at 8,600 pounds tension

Curve designations same as in figure 8



Synchronous Mechanical Rectifier-Inverter—II

The embodiment of the general method for obtaining successful commutation in a synchronous mechanical rectifier-inverter by the use of harmonic voltages is described in this paper, which reports the results of tests on a 50 kw experimental unit. This type of rectifier is shown to have satisfactory characteristics of power factor, regulation, overload capacity, and commutation under transient load conditions, and has good efficiency, especially at low voltages and light loads.

By
S. S. SEYFERT
MEMBER A.I.E.E.

N. S. HIBSHMAN
MEMBER A.I.E.E.

D. C. BOMBERGER
ENROLLED STUDENT A.I.E.E.

All of Lehigh Univ., Bethlehem, Pa.

A SYNCHRONOUS mechanical rectifier of inherently high efficiency and low capital cost, and adapted especially to supplying heavy direct currents at low voltage with no practical disadvantages that may not be found with other forms of rotary transforming machinery, was proposed in a paper[1] presented in 1933. Since then a self-contained unit with a nominal rating of 50 kw has been constructed and subjected to extensive tests of its practicability in the laboratories at Lehigh University. When previously reported, the only rectifier of this type that had been operated was an experimental model of limited capacity, dependent upon separately driven auxiliary generators for the neutralizing and commutating harmonic voltages essential to this method of rectification. The present machine and its accompanying circuits represent the first adaptation of the fundamental concepts of harmonic commutation to a practical rectifying unit of significant capacity.

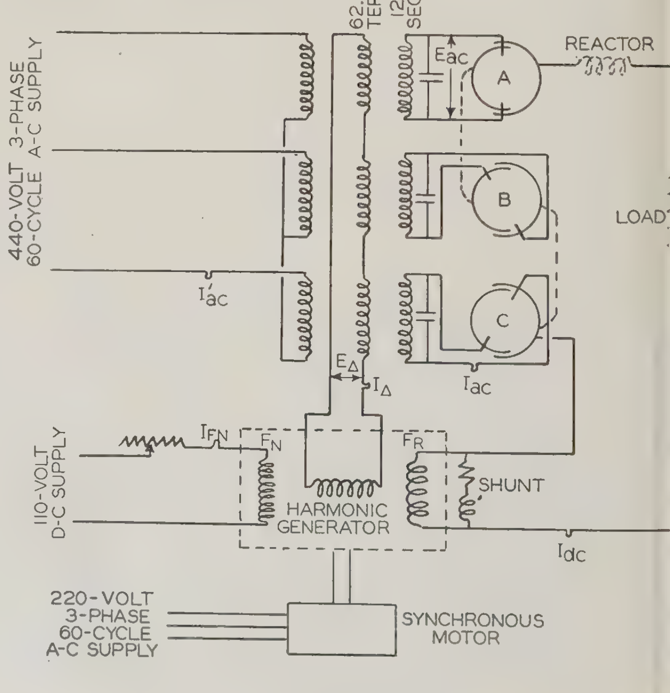
Description of Rectifier

To accomplish the conversion of 3 phase alternating current to direct current, 4 segmented rings, rotating synchronously, reverse in turn the connections of the phases of the a-c system, which act in series to produce the continuous voltage. The reversal of the connection of each phase takes plac under short circuit. To make short-circuiting pos sible, a negative sine component of the third har monic of voltage is introduced which so modifies th resultant voltage wave form of each phase as t provide a period of practically zero voltage for th duration of the short circuit. To balance the trans former and system reactance drop and reverse th current in the short-circuited phase, preparator to its reintroduction into the d-c circuit, a cosin component of the third harmonic of voltage i impressed. In the experiments described in th paper referred to in the first paragraph, the thir harmonic components of voltage were introduce into the neutral of the star connected transforme primaries which resulted in the circulation of fairly large third harmonic current in each of th supply lines. The introduction of the harmonic into the tertiary delta in the present arrangemen avoids this disadvantage.

Figure 1 shows diagrammatically the circuit ar rangement for the complete rectifying unit. Th a-c supply to be rectified is transformed to th desired voltage by a bank of 3 single phase trans formers each of which has 3 windings. The pri maries are connected in star to the incoming lin without neutral return connection to the powe source. The secondaries are connected separatel to the brushes bearing on the commutator rings The tertiary coils are placed in series as for a delt connection except that the delta is closed throug the armature of the harmonic generator. It is b this means that the 2 components of third harmoni voltage are introduced into the phase voltages.

The commutator rings are mounted on a shaf extension of the harmonic generator armature an the assembly is driven by a small a-c excited syn chronous motor as shown by figures 2 and 3. Th

Fig. 1. Diagram of connections of synchronous mechanical rectifier-inverter



A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 4, 1936; released for publication March 31, 1936.

1. For numbered references see list at end of paper.

Fig. 2 (right). View of rectifier-inverter set, showing commutator, harmonic generator, and driving motor

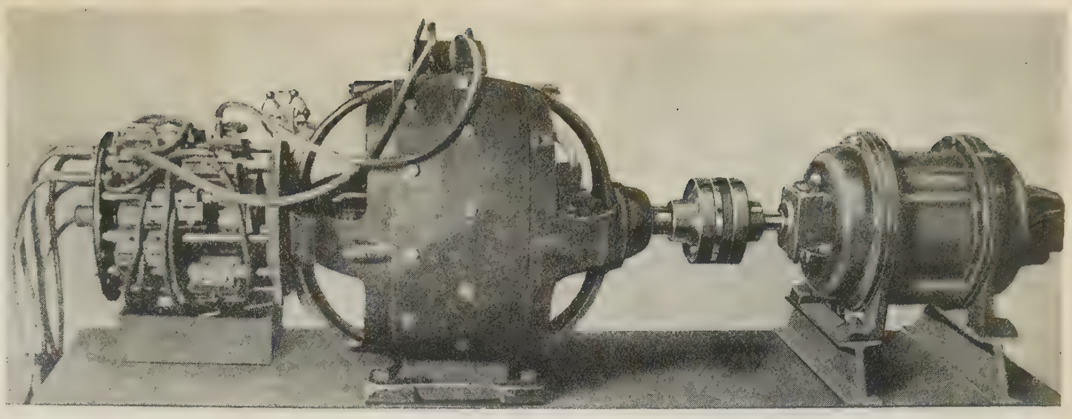


Fig. 3 (below). Rotor of single phase harmonic generator with switching rings mounted on shaft

commutator consists of 4 rings of segments and is without electrical connection of any kind except through the brushes. It is a separate unit which may be removed and replaced without disturbing any part of the circuit except for raising the brushes. It functions solely by providing periodic interconnections among the several brushes which constitute the terminals of the a-c phases and the d-c circuit.

Figure 4 shows the field structure of the harmonic generator with its 2 sets of poles, which induce the 2 components of third harmonic voltage in the single armature circuit. A section of the assembled generator and commutator appears in figure 5, which also shows some over-all dimensions. Reference to figure 1 will show how the 2 sets of poles are excited. The sine component or neutralizing voltage is induced by a field F_N with essentially constant excitation, proportional to the a-c supply voltage. The cosine component or commutating voltage is produced by F_R which is excited by the load direct current. The present harmonic generator was constructed from parts of 2 discarded d-c motors and lacks some of the qualities that should be expected of a well-designed machine, as will be pointed out.

The driving motor used is a 5 horsepower synchronous induction motor, without d-c excitation. The inherently low efficiency and large phase angle shift under load of this type of motor impose difficult operating conditions on the rectifier, but it has the virtue of simplicity.

Principles of Operation

The series operation of the 3 rectifying elements A, B, and C of figure 1 results in a continuous voltage wave form almost identical with that obtained for the current of a 6 anode mercury arc rectifier. The ratio of the continuous voltage to the effective value of one of the secondary voltages is approximately 2. This ratio is as inflexible as that of a rotary converter, being modified but slightly by the effects of voltage drops.

At no load the harmonic generator injects into the tertiary delta a third harmonic of voltage E_n called the neutralizing voltage which has a value of 50 per cent of the fundamental voltage of the 2 active transformers; this is represented as E_n in the generator vector diagram of figure 6 and as a sinusoid in figure 7. This component is generated by the field F_N and should be constant as long as the supply voltage remains constant. The limits of the commutation period of one of the phases are indicated by the lines C in figure 7.

Under load the generator injects another third harmonic of voltage E_R' into the delta which is in quadrature with E_n and whose value is determined by several factors:

1. The true commutating voltage E_R derived from the equation $\Delta I_{ac} = \frac{1}{L}\int_0^{\pi} e_r dt$ which yields $E_R(max) = \frac{\Delta I \omega_3 L}{2}$ where ΔI is the change of transformer current from say plus full load to minus full load value and $\omega_3 L$ is the triple frequency leakage reactance between the tertiary and the short-circuited secondary undergoing commutation.

2. The leakage reactance drop in the 3 transformer tertiary windings.

3. The effect of armature distortion on the poles of the neutralizing field by the armature current, which has the effect of advancing the phase of E_n thus creating a horizontal component of voltage in the diagram.

4. The effect of phase advance of the driving motor with increase of load resulting in a shift of the commutation limits to the lines C' in figure 7. In this case, the excess positive area of fundamental voltage E_1 must be overcome by an equivalent increase of E_R.

The current in the tertiary delta and generator I_a arises as a compensation for the corners of the transformer current I_{ac} in figure 7. This current, termed for brevity the delta current, is 90 degrees behind the voltage E_R' and in phase opposition to E_n; it remains closely proportional to the load.

The armature impedance drop is laid off as $Z_a I_a$ in figure 6. This vector should also include the effect of armature reaction. The vector diagram is drawn with values and angles corresponding to full load operation on the actual machine. The generator terminal voltage is E_x and $\cos\theta$ the power factor at its terminals; E_a' is the net excitation voltage and E_Q its horizontal component, while E_Q' is the value of voltage obtained from the saturation curve of the reversing end of the machine for the known

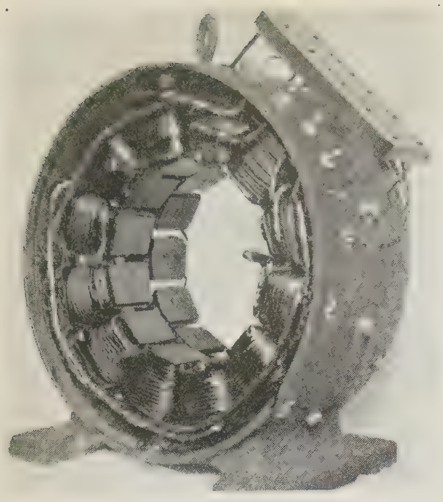


Fig. 4. Stator of harmonic generator

current in the series field. The dotted part of the diagram shows the vectors for half load operation. The approximation of this diagram may be realized inasmuch as harmonics of higher orders are found in all of the voltage and current waves.

As the delta current is in substantial phase opposition to E_n, the harmonic machine must operate as a synchronous motor when the apparatus is rectifying. In the actual machine this action in terms of kilowatts is slightly over 10 per cent of the output of the set and remains proportional thereto. This power goes first to supply certain losses in the harmonic generator, then to the losses of the synchronous driving motor and the surplus, if any is left, will appear as electrical output from the motor. In the use of the set as a rectifier, the motor carries the heaviest load at zero load on the set. During inversion the power flow reverses but has the same magnitude so that the motor must furnish not only this power but most of the losses in the harmonic generator.

Phase Shift, and Elimination of Its Effects

The variation of load explained in the preceding paragraph results in a change of the space phase of the motor rotor and of the set. Thus, during rectification, the rotor position of the set advances with increase of d-c load. Curve *D* of figure 8 shows the variation of the shaft power of the harmonic generator, the power passing through zero at a load of 60 amperes. The minimum point reached by this curve is the result of the rapid increase of the losses in the generator. A comparison of curves *B* and *C* shows how closely the electrical input to the harmonic generator may be approximated by multiplying the constant voltage E_n by the delta current.

The 5 horsepower motor used to drive the set showed a phase displacement of 24 electrical degrees over the range of load to which it was subjected. Most of the tests mentioned later were performed under this unfavorable condition. To eliminate the effects of phase shift, a d-c machine was coupled to the set. This machine was made to deliver electrical output sufficient to hold the phase angle constant as indicated by a stroboscope. This resulted in marked changes: the delta current dropped about 13 per cent, E_R' dropped from 80 to 39 volts, and the efficiency of the set increased 3 per cent.

The elimination of the disturbing effects of phase shift can be attained approximately by driving the set by a synchronous motor fully compensated for armature reaction. Such a motor would have a d-c excited field and would be provided with compensating ampere-turns behind the pole faces equivalent to the ampere-turns of the armature. For approximate compensation, the output direct current of the rectifier might be used.

Use of Capacitors

The successful operation of the set is not dependent on the use of capacitors shown connected across the secondaries in figure 1 but it has been found that they relieve the harmonic generator of its load and tend to increase the efficiency of operation. As a result of faulty design, the machine tested had an abnormally high core loss (see figure 9). The use of capacitors resulted in a reduction of delta current and in the value of E_R' required, thus greatly reducing the core loss. A small value of capacitance across the phases is advantageous as a means of absorbing surges incident to incorrect brush position or faulty adjustment of the commutating field. Most of the tests were made with a capacitance of 64 microfarads across the secondaries.

Ratings of Apparatus

Corresponding to a d-c output of 61.2 kw on a test with zero phase shift, the following kilovolt-amperes were found in the 3 transformer coils:

Secondary coil $E''I''$	= 145	$\times$ 182	=	26.4
Primary coil $E'I'$	= 290	$\times$ 88	=	25.5
Tertiary coil E_3I_3	= 72.5	$\times$ 84	=	6.1
				58.0

which is equivalent to 29 kva per transformer.

Hence the equivalent rating of the bank is $\frac{3 \times 29}{61.2}$ or 1.42 times the output kilovolt-amperes. This is comparable with the 151 per cent of transformer capacity required by the mercury arc rectifier with interphase transformer.[2]

The delta current arises from the third harmonic and its multiples found in the wave of current I_{ac} of figure 7. Analysis of this current has been made from oscillograms for 3 typical cases as shown by

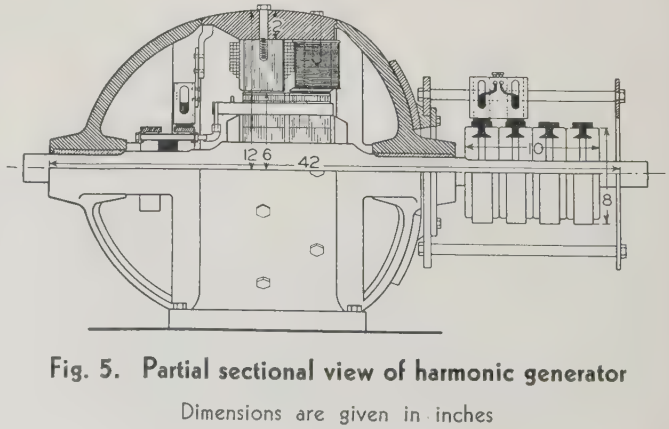


Fig. 5. Partial sectional view of harmonic generator

Dimensions are given in inches

the following tabulation. The effective value of each harmonic has been expressed in terms of the direct current.

	Fundamental	Harmonics Third	Fifth	Seventh	Ninth
1. With motor phase shift, no d-c reactance	0.885	0.254	0.128		
2. Without motor phase shift, no d-c reactance	0.895	0.176			0.018
3. Without motor phase shift, high d-c reactance	0.877	0.235	0.088	0.025	

The value of E_n in each transformer secondary should be half the fundamental voltage. For favorable conditions of operation, $E_R' = -\frac{1}{2}\, j\, E_n$ (approximately). For the case of the arrangement used in the tests, the above gives E_3 (across the delta) equal to 0.28 E_{dc}.

The component of third harmonic in the transformer current was found from the results of the test to lie between the values of case 1 and case 2 of the preceding table. Using the average of the 2 values 0.254 and 0.176 yields 0.43 I_{dc} for the delta current I_3. Substituting the full load values of E_{dc} and I_{dc}, the estimated kilovolt-ampere rating of the harmonic generator will be 7.35 as compared with 7.3 observed from the test performed at zero phase shift; this is 12 per cent of the d-c power delivered.

Shaft Power of Driving Motor

The shaft power intake of the harmonic generator varies from 1.8 kw at no load to −1.15 kw at full load when operating with free phase shift (see figure 8). When operating at zero phase shift, these values are 1.5 and −2.9 kw, respectively. The shaft rating of the motor therefore should be approximately 3 kw which is 4.9 per cent of the nominal output. The driving motor rating added to that of the generator (12 per cent) makes 16.9 per cent of the nominal d-c rating of the converter. At full load the power values in kilowatts connected with the rotor of the harmonic generator are as follows:

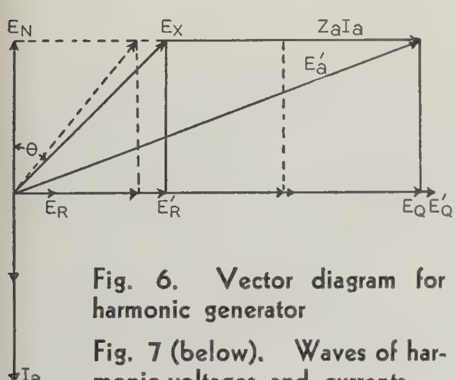
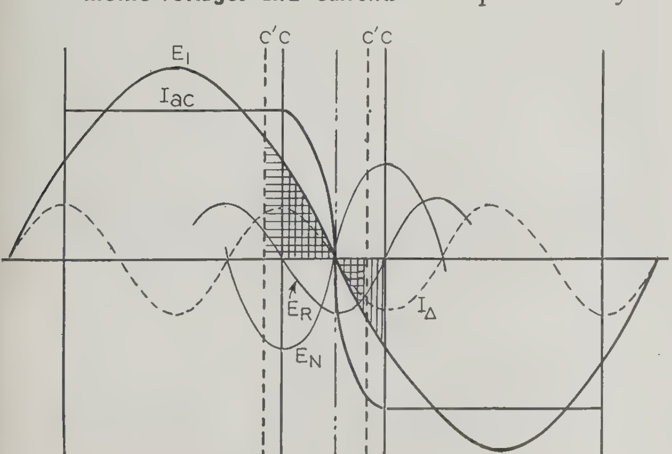


Fig. 6. Vector diagram for harmonic generator

Fig. 7 (below). Waves of harmonic voltages and currents



A—Loss in harmonic generator

B—Electrical input to harmonic generator

C—Product of neutralizing harmonic voltage and current in delta

D—Shaft power of harmonic generator

E—Subtraction of curve B from curve A

A′ and D′—Position of curves A and D when phase shift is zero

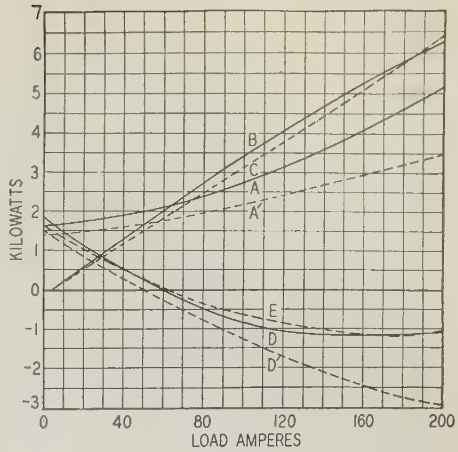


Fig. 8. Power transfers in harmonic motor-generator set

Kilovolt-amperes to harmonic generator	= 7.3	
Kilowatts to harmonic generator	= 6.54	
Losses in generator	= 3.61	(Friction and windage, 1.64; core loss, 1.8; $R_a I_a^2$, 0.170)
Shaft output of generator	= 2.93	(See curve D' of figure 8)

Performance of Experimental Set

The rectifying set described and tested was designed for a nominal capacity of 50 kw and a nominal continuous voltage of 250. Because of limitations in the transformer bank available, the set was operated at about 300 volts. The tests were carried to 91 kw, and it appears that the addition of about 50 per cent to the brush area and the redesign of the harmonic generator armature would permit continuous operation at this load.

Figure 10 shows oscillograms of continuous voltage, transformer voltage and current, harmonic voltage and current, and transformer primary line current for full load operation under conditions of free phase shift of the motor. The transformer primary line current contains a component of the fifth harmonic caused by the phase shift of the set previously discussed. Operation with zero shift eliminates this harmonic. Figure 11 is a similar oscillogram to show operation on sudden load changes. In this case the load was 500 amperes (135 kw or 250 per cent of nominal load) thrown on by a switch and tripped out by a breaker. The sudden rise of current resulted from the absence of steadying reactance in the d-c circuit. No sparking of consequence occurred during this procedure. On d-c short circuit the set performs like a rotary converter. When supplied from a substantially infinite bus, it flashes like the converter but with moderate damage to the rings. With the reactance of an additional transformer bank ahead of it and with

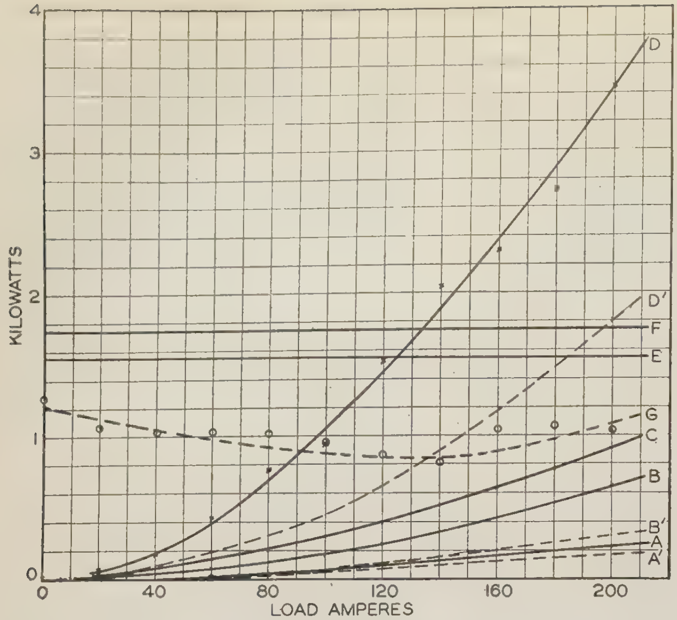


Fig. 9. Analysis of losses in harmonic motor-generator set

A—Armature copper loss
B—Reversing field copper loss
C—Brush contact loss
D—Reversing core loss
E—Journal friction and windage loss
F—Journal friction, windage, and neutralizing field loss
G—Driving motor loss
Dashed curves for zero phase shift

the protection of quick acting circuit breakers it does not flash over.

The voltage regulation of the set at nominal load was 3.13 per cent and 7.85 per cent including drop in the transformers. The primary fundamental power factor is practically 100 per cent but the existence of small components of harmonics has the effect of reducing the apparent power factor slightly.

Results of Tests

Figure 12 shows graphically the efficiency and voltage characteristics of the rectifier as determined by tests with the output absorbed by a rheostat. The efficiency characteristic curve *A* resulted from a test in which the phase angle of the synchronous driving motor was permitted to shift in response to changes in its load. This shift reaches a maximum of about 24 electrical degrees between no load and full load, calling for an additional 50 per cent of commutating harmonic voltage. When the phase shift is eliminated by loading the synchronous motor with an additional d-c generator on the shaft and holding the phase angle constant as indicated by a stroboscope, the efficiency characteristic rises to the curve *B*, and is more than 90 per cent in the range from 50 to 150 per cent load, with a maximum of 92.2 per cent. The over-all efficiency of transformers and rectifier, without phase shift compensation, is shown by curve *C*.

An analysis of the losses in the rectifier is given in figure 9. The solid curves designated by plain letters represent losses present when the synchronous driving motor phase shift is uncontrolled. The broken curves labeled with corresponding prime letters, such as D', show the considerable reduction in losses to be achieved by controlling the phase shift.

Figure 8 depicts the power transfers taking place in the harmonic generator and between it and the driving motor. Here it may be seen clearly from the proximity of curves *D* and *E* that the duty of the driving motor is limited to supplying or absorbing the difference between the harmonic machine's electrical intake and its losses. The close agreement of curves *B* and *C* confirms the previous conclusion that power action in the harmonic circuit is provided by the neutralizing voltage acting upon the delta current and that, for the most part, the harmonic machine is acting as a motor during rectification.

Since brush contact resistance plays no part in commutation, a brush high in metal content was chosen. The current density at the a-c brushes at full load is 80 amperes per square inch. On overload this figure reaches 120 amperes per square inch. The d-c brushes, which act singly part of the time, are required to handle up to twice these densities. During transients such as that illustrated by figure 11, direct currents as high as 500 amperes have been observed.

Summary of Advantages and Disadvantages

As a means for converting a-c power into d-c power this type of rectifier appears to have certain advantages over other known methods of conversion, especially below 300 volts. Points to be noted are:

1. The electrical rating of the set needs to be 12 per cent for the

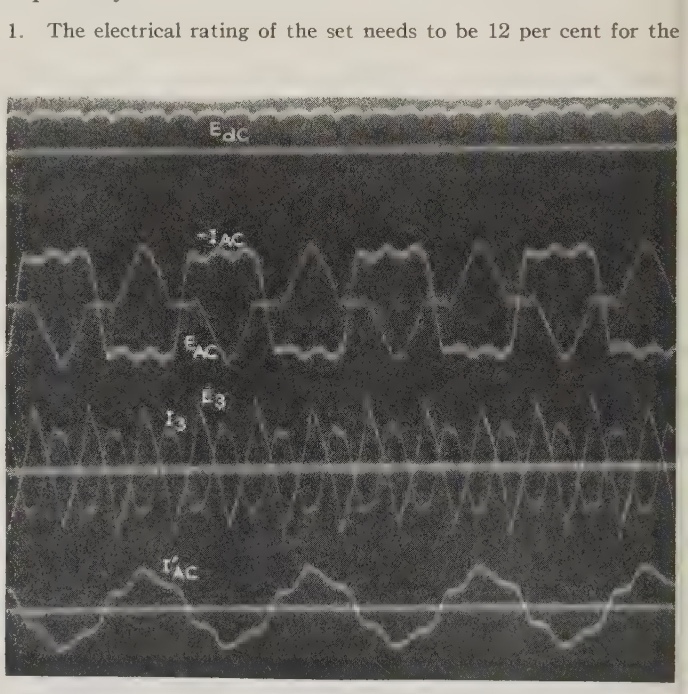


Fig. 10. Oscillogram of steady state conditions with load of 200 amperes

E_{ac} and I_{ac}—Voltage and current of one transformer secondary
E_3 and I_3—Voltage and current at harmonic generator terminals
I_{ac}'—Current in supply line

Fig. 11. Oscillogram of transient conditions with load of 500 amperes suddenly thrown on and off

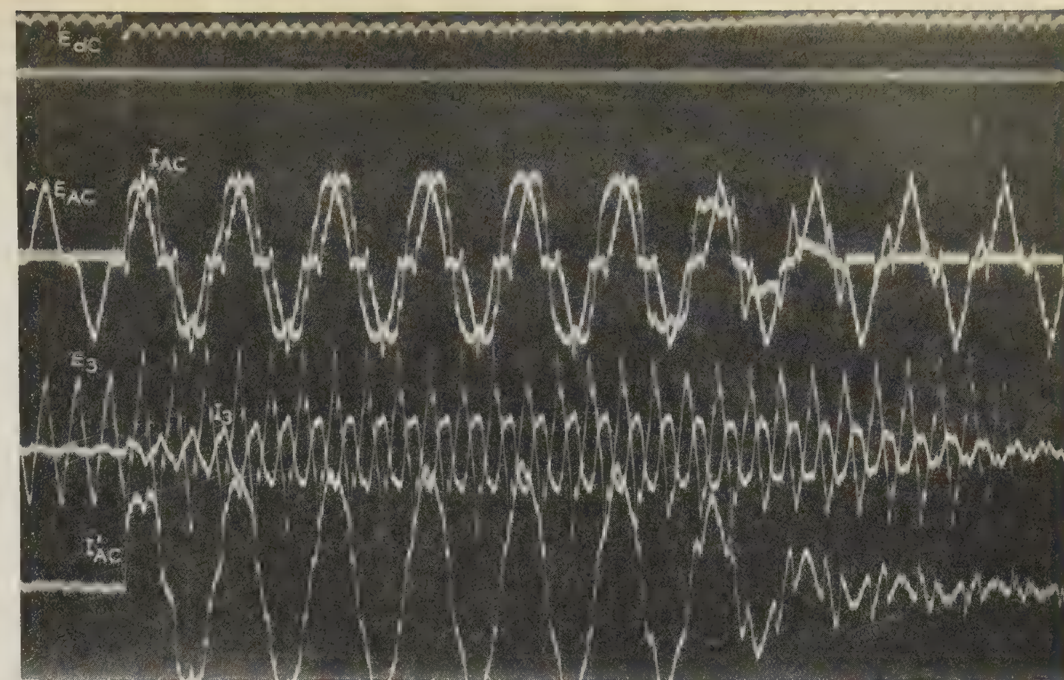
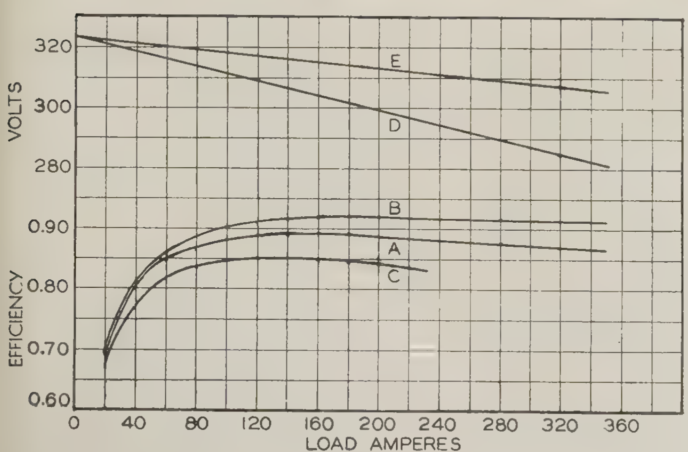


Fig. 12 (below). Efficiency and regulation of rectifier

A—Efficiency obtained from test with phase shift
B—Efficiency without phase shift
C—Efficiency including transformers
D—Continuous voltage including transformers and phase shift
E—Continuous voltage for rectifier only
Normal load 200 amperes



harmonic generator plus 5 per cent for the driving motor or a total of 17 per cent of the nominal d-c output.

2. The weight of the set should be about $^1/_3$ the weight of the equivalent rotary converter.

3. The tertiary coils required on the transformers, the reduction of power factor in the secondary, and the losses in the converter result in a 42 per cent increase in the total transformer rating over the nominal d-c output.

4. The voltage drop in the rectifier is low (3.13 per cent not including transformers).

5. The primary power factor of input to the transformers is practically unity (ignoring magnetization and small harmonic current components).

6. The efficiency runs well above that obtainable with mercury arc rectifiers converting to 110 or 220 volts direct current. The efficiency is still about 70 per cent at 10 per cent of full load, and the curve is flat over a wide range of load.

7. Since the brushes have no reversing duty, collector ring conditions obtain on the commutator which permit the use of low resistance brushes and high current densities.

8. When the brush position and the adjustment of the shunt on the commutating field are correct, the rectifier will go through every performance including heavy transient overloads without sparking.

9. The assembly is easily accessible for repairs and contains no elements strange to the electrical art.

The synchronous mechanical rectifier-inverter has the following disadvantages:

1. This rectifier has an inherently constant conversion ratio and dependence must be had on transformer taps or a-c voltage regulators to effect regulation or control of the output voltage.

2. The noise and mechanical wear of a rotating machine are present.

Prospects for Further Improvement

The machine described was designed before some of the important factors such as phase shift, armature reactance, core loss, etc., were fully appreciated in their bearing on efficiency and rating. By consideration of the work done, it is possible to predict considerable further improvements through:

1. Application of a compensated synchronous motor.

2. Reduction of the armature turns, resulting in a reactance drop equal to 50 per cent of the present value.

3. Reduction of the tooth density in the armature core or the use of alloy steel, whereby the core loss may be reduced to 500 watts or less.

No tests for inversion operation are presented at this time because the rating of the driving motor available was too small and its phase shift with load change too great. This work must await the application of a properly compensated motor.

The authors are convinced, as a result of calculations based on present design and performance, that a full-load efficiency of 94 per cent may be attained on a 50 kw rectifier of this type operating at or about 220 volts. With increase in rating and careful design, this efficiency will naturally be still further increased.

References

1. Synchronous-Mechanical Rectifier-Inverter, S. S. Seyfert. A.I.E.E. Trans., v. 52, June 1933, p. 397–407.

2. Mercury Arc Power Rectifiers (a book), Marti and Winograd McGraw Hill Book Co., New York, N. Y., 1930, p. 137–44.

News

Of Institute and Related Activities

The A.I.E.E. Summer Convention at Pasadena, California

BY VIRTUE of its location and the excellent program arranged by the summer convention committee, the summer convention at Pasadena, Calif., June 22–26, 1936, affords members unusual opportunities to see many western developments and at the same time to keep abreast of the profession. The schedule of events includes the annual business meeting and 9 technical sessions to be held during the mornings. On the afternoons the conference of officers, delegates, and members, 2 student technical sessions, and technical conferences will be held in addition to sports and other recreation. Many places of scenic or scientific interest will be visited during the convention and the large engineering projects, such as Boulder Dam, the Metropolitan Water District Aqueduct, San Francisco-Oakland Bay and Golden Gate bridges, Bonneville Dam, Grand Coulee, and Glacier Park may be visited as pre-convention or post-convention trips. Arrange your vacation plans so as to attend the summer convention at Pasadena. See the suggestion of a special train from Chicago with a tentative itinerary in ELECTRICAL ENGINEERING for April, p. 418–19, and reply promptly to the national secretary. Convention headquarters will be in the Huntington Hotel.

TECHNICAL SESSIONS

Numerous papers, dealing with a variety of subject matter embracing theory, design, and performance, will be presented and discussed at the 9 technical sessions. Subjects include some of the new apparatus and equipment now being or recently installed on west coast projects, and among the contributors may be noted many well-known eastern and western engineers. It is to be expected that the discussion of these 40-odd papers by those from both regions who are well known in the engineering profession should provide the membership with some of the latest and most valuable engineering information in several specialized fields of endeavor.

TECHNICAL CONFERENCES

Arrangements are in progress to organize and schedule during the afternoons several technical conferences or informal round-table meetings where specialists may get together and discuss certain subjects. These conferences will be entirely informal, and neither the papers and talks nor the

Schedule of Events

Monday, June 22

8:30 a.m.—Registration

10:00 a.m.—Opening of Convention—Hotel Ballroom

Address of welcome: C. W. Koiner, first chairman of the Los Angeles Section, A.I.E.E., now city manager of Pasadena. Response by Professor R. W. Sorensen, chairman, summer convention committee

At the conclusion of his remarks, Professor Sorensen will introduce President Meyer, who will preside during the remainder of the program

Annual Business Meeting

Annual report of board of directors, in abstract, by H. H. Henline, national secretary

Report of committee of tellers upon election of officers

Introduction of and response from president-elect

Presentation of prizes for papers

Presentation of Lamme Medal to Dr. Vannevar Bush

President's address by E. B. Meyer

2:00 p.m.—Conference of Officers, Delegates, and Members

(a) Section delegates

(b) Student Branch counselors

Technical Conferences

Women's lawn party and bridge tea—Huntington Hotel

8:00 p.m.—Address: SUPER POWER PARTICLES, by Dr. Robert A. Millikan, California Institute of Technology

President's reception—informal dance

Tuesday, June 23

9:00 a.m.—Electrophysics Session

Conductor Vibration Session

Women's all-day trip through Hollywood, Beverly Hills, Universal City, and to the beaches, with luncheon at Long Beach

2:00 p.m.—Conference of Officers, Delegates, and Members (continued)

Technical Conferences

Evening —Informal dance with entertainment between dance numbers—Hotel Ballroom

Wednesday, June 24

9:00 a.m.—Power Transmission and Distribution Session

Electrical Machinery Session

11:30 a.m.—Directors' luncheon meeting

Women's scheduled visits during the day to nearby gardens, the San Gabriel Mission, Huntington Library, etc., with opportunity also to visit the stores of Pasadena and Los Angeles

2:00 p.m.—Student Technical Session

Technical Conferences

4:00 p.m.—Mt. Wilson Observatory trip

Thursday, June 25

9:00 a.m.—Measurements and Selected Subjects Session

Symposium on Transformers

Illumination Session

Women's putting contest—hotel lawn

Afternoon—Golf and other sports

Evening —Convention banquet—Huntington Hotel

Production of technicolor and other motion pictures—Pasadena Civic Auditorium

Friday, June 26

9:00 a.m.—Protective Devices Session

Education Session

2:00 p.m.—Student Technical Session

6:30 p.m.—Buffet supper with entertainment in and around the swimming pool, and the award of golf prizes, etc.—Huntington Hotel gardens

Technical Program

Members who intend to discuss papers should send their names and the titles of the papers they propose to discuss to C. S. Rich, secretary, technical program committee, A.I.E.E., 33 West 39th Street, New York, N. Y., *before June 1.* This request for advance notice does not indicate that other discussions will be unwelcome, but the possibility of listing the names of some of those who will discuss papers is to be given consideration toward lending interest to the technical sessions.

In this program, reference to the issue and, in so far as possible, to the page in ELECTRICAL ENGINEERING, is given for all papers.

Tuesday, June 23

9:00 a.m.—Electrophysics

FIELDS DUE TO REMOTE THUNDERSTORMS, K. E. Gould, Bell Telephone Laboratories, Inc. Scheduled for June issue

FIELDS AND CHARGES ABOUT A CONDUCTOR, W. G. Hoover, Stanford University. May issue, p. 448–55

THE MAGNETIC VECTOR POTENTIAL, J. W. McRae, California Institute of Technology. May issue, p. 534–42

*DETERMINATION OF IMPEDANCES OF TRANSFORMER CIRCUITS, A. N. Garin and K. K. Paluev, General Electric Co. Scheduled for June issue

CURRENT AND VOLTAGE LOCI IN 3-PHASE Y-Y CIRCUITS, A. C. Seletzky, Case School of Applied Science. Sept. 1935 issue, p. 970–6

CURRENT AND VOLTAGE LOCI IN 3-PHASE DELTA-DELTA CIRCUITS, A. C. Seletzky and K. F. Sibila, Case School of Applied Science. May issue, p. 476–9

9:00 a.m.—Conductor Vibration

CABLE VIBRATION—METHODS OF MEASUREMENT, J. S. Carroll, Stanford University, and J. A. Koontz, Jr., Pacific Gas & Electric Co. May issue, p. 490–3

CABLE AND DAMPER VIBRATION STUDIES, L. A. Pipes, California Institute of Technology. Scheduled for June issue

LABORATORY STUDIES OF CONDUCTOR VIBRATION, J. S. Carroll, Stanford University. May issue, p. 543–7

VIBRATION OF CABLES AND DAMPERS, R. G. Sturm, Aluminum Company of America. Part I—May issue, p. 455–65; Part II—Scheduled for June issue

Wednesday, June 24

9:00 a.m.—Power Transmission and Distribution

FLASHOVERS ON TRANSMISSION LINES, L. V. Bewley, General Electric Co. April issue, p. 342–54

IMPEDANCE MEASUREMENTS ON UNDERGROUND CABLES, R. L. Webb and O. W. Manz, Jr., Brooklyn Edison Co. April issue, p. 359–65

SINUSOIDAL TRAVELING WAVES, Ernst Weber, Polytechnic Institute of Brooklyn, and F. E. Kulman, Brooklyn, N. Y. March issue, p. 245–51

TESTS ON OIL IMPREGNATED PAPER, H. H. Race, General Electric Co. Scheduled for June issue

A NEW UNDERSTANDING OF HIGH VOLTAGE PORCELAIN, D. H. Rowland, Locke Insulator Corp. Scheduled for June issue

9:00 a.m.—Electrical Machinery

PRESENT STATUS OF HYDROGEN COOLING FOR ROTATING ELECTRICAL MACHINERY, C. M. Laffoon, Westinghouse Electric & Mfg. Co. Scheduled for June issue

HYDROGEN COOLING—WITH NEAR-CRITICAL VELOCITIES, G. W. Penney, Westinghouse Electric & Mfg. Co. May issue, p. 530–4

SALIENT POLE SYNCHRONOUS MOTORS OPERATING OUT OF SYNCHRONISM, A. H. Lauder, General Electric Co. Scheduled for June issue

SYNCHRONOUS MECHANICAL RECTIFIER-INVERTER,—II, S. S. Seyfert, N. S. Hibshman, and D. C. Bomberger, Lehigh University. May issue, p. 548–53

STRAY LOAD TESTS ON INDUCTION MACHINES—II, T. H. Morgan and Victor Siegfried, Worcester Polytechnic Institute. May issue, p. 493–7

ANALYSIS OF UNSYMMETRICAL MACHINES, W. V. Lyon and Charles Kingsley, Jr., Massachusetts Institute of Technology. May issue, p. 471–6

Thursday, June 25

9:00 a.m.—Measurements and Selected Subjects

MEASURING EQUIPMENT FOR OIL POWER FACTOR, L. J. Berberich, Socony-Vacuum Oil Co., Inc. March issue, p. 264–8

THE SPARKLESS SPHERE GAP VOLTMETER—II, R. W. Sorensen and Simon Ramo, California Institute of Technology. May issue, p. 444–7

SOME APPLICATIONS OF INSTRUMENT TRANSFORMERS, O. A. Knopp, Pacific Gas & Electric Co. May issue, p. 480–9

EFFECT OF ELECTRIC SHOCK ON HEART, L. P. Ferris and P. W. Spence, Bell Telephone Laboratories, Inc., B. G. King and H. B. Williams, College of Physicians and Surgeons, Columbia University. May issue, p. 498–515

ELECTRIC SHOCK EFFECTS OF FREQUENCY, W. B Kouwenhoven, D. R. Hooker, and E. L. Lotz, The Johns Hopkins University. April issue, p. 384–6

9:00 a.m.—Transformer Symposium

POWER TRANSFORMERS FOR 287.5 KV SERVICE, W. G. James and F. J. Vogel, Westinghouse Electric & Mfg. Co. May issue, p. 438–44

*MODERN HIGH VOLTAGE POWER TRANSFORMERS, K. K. Paluev, General Electric Co. Scheduled for June issue

FORMULA FOR TRANSFORMER REGULATION, J. E. Clem, General Electric Co. May issue, p. 466–71

OVERREFINED OILS IN POWER TRANSFORMERS, J. G. Ford, Westinghouse Electric & Mfg. Co. April issue, p. 371–5

9:00 a.m.—Illumination

THE QUALITIES OF INCANDESCENT LAMPS, P. S. Millar, Electrical Testing Laboratories. May issue, p. 516–23, 529

*OPTICAL PRINCIPLES AND LIGHT SOURCES FOR HIGHWAY LIGHTING, C. A. B. Halvorson, General Electric Co. Scheduled for June issue

*THE NEED FOR BETTER VISIBILITY ON HIGHWAYS BY NIGHT, L. A. S. Wood, Westinghouse Electric & Mfg. Co. Scheduled for June issue

Friday, June 26

9:00 a.m.—Protective Devices

NEUTRALIZING TRANSFORMER TO PROTECT POWER STATION COMMUNICATION, E. E. George, Tennessee Electric Power Co., and R. K. Honaman, L. L. Lockrow, and E. L. Schwartz, Bell Telephone Laboratories, Inc. May issue, p. 524–29

HIGH SPEED ARC RUPTURE AT TRANSMISSION VOLTAGES, H. M. Wilcox and W. M. Leeds, Westinghouse Electric & Mfg. Co. Scheduled for June issue

287 KV SWITCHES FOR BOULDER DAM, A. J. Bowie, Bowie Switch Co. Scheduled for June issue

SPECIAL TESTS ON IMPULSE CIRCUIT BREAKERS FOR BOULDER DAM-LOS ANGELES LINE, W. F. Skeats, General Electric Co. Scheduled for June issue

A NEW, FASTER CARRIER PILOT RELAY SYSTEM, O. C. Traver and E. H. Bancker, General Electric Co. Scheduled for June issue

A NEW DISTANCE GROUND RELAY, S. L. Goldsborough and R. M. Smith, Westinghouse Electric & Mfg. Co. Scheduled for June issue

THE EFFECT OF DELTA-Y OR Y-DELTA POWER TRANSFORMERS ON THE PERFORMANCE OF DISTANCE RELAYS, Giuseppe Calabrese, The New York Edison Co., Inc. Scheduled for June issue

9:00 a.m.—Current Aspects of Engineering Education

THE YOUNG ENGINEER UNDER CHANGING CONDITIONS, R. E. Hellmund, Westinghouse Electric & Mfg. Co. April issue, p. 329–34

*SOME PROBLEMS IN ELECTRICAL ENGINEERING EDUCATION, Morland King, Lafayette College. Scheduled for June issue

PANEL DISCUSSION: OBJECTIVES AND PROGRAM OF E.C.P.D.

*These papers are under consideration for presentation at the summer convention, but up to date of going to press have not been officially placed upon the program.

Notice of Annual Meeting

The annual meeting of the American Institute of Electrical Engineers will be held at the Huntington Hotel, Pasadena, Calif., at 10 a.m. on Monday, June 22, 1936. This will constitute one session of the annual summer convention which is to be held this year in Pasadena, Calif.

At this meeting, the annual report of the board of directors, and the reports of the committee of tellers on the ballots cast for the election of officers will be presented.

Such other business, if any, as properly may come before the annual business meeting may be considered.

(Signed) H. H. HENLINE
National Secretary

A portion of the San Francisco business district as seen from one of the cat-walks near the top of one of the main suspension towers of the transbay bridge

conclusions reached are scheduled for publication, but those who attend will profit by the exchange of views. Among the subjects under consideration for conferences are high-voltage x-ray tubes, mechanical properties of electrical conductors, use of electronic tubes in industry, electrochemistry, general transformer problems, synchronous machines, and communication. More information and agenda on conferences will be announced in the June issue, if arrangements have been completed by that time.

Student Technical Sessions

In connection with the Student Branch convention 2 student technical sessions will be held, one on Wednesday afternoon, the other on Friday afternoon. At both these sessions papers will be presented by the students of the 8th and 9th Districts. Students from other Districts will, of course, be cordially welcomed and it is hoped that many from other Districts will attend.

Entertainment

In addition to the regular entertainment to be provided for the evenings, the ladies entertainment committee has arranged a variety of attractive events which are listed in the accompanying schedule.

Some may find the Little Theater in Padua Hills of unusual interest. This theater is best known for its Mexican Player group, and it has been attended by many notables. The plays are done in Spanish for people who do not understand Spanish and the group has been commented on as "the most significant folk drama in America today." Luncheon, tea, and dinners are served at reasonable prices and also Mexican dishes.

Sports

Excellent facilities for golf and tennis are available, and swimming also may be enjoyed. There are tennis courts and a swimming pool at the Huntington Hotel but no facilities for renting tennis rackets or bathing suits; therefore guests expecting to participate in these sports must bring their own.

Golf. The Annandale Golf Club, one of the most picturesque spots in Southern California, has been secured for the week of June 22, and it is there that the tournaments for the Mershon and Lee Trophies of the Institute, as well as the John B. Fisken Cup competition will be played. The competition for the John B. Fisken Cup is open only to members of the Pacific Coast Sections, and will be played on Thursday afternoon only.

Mershon trophies for both golf and tennis are now on a permanent basis, and will remain in the possession of the Institute with the names of the winners engraved upon them each year.

The date set for qualifying for the Mershon Trophy tournament is Monday, with qualifying rounds and match play Tuesday, Wednesday, Thursday, and Friday.

The Lee Trophy tournament, 36 holes of medal play, will be set for Monday and Tuesday.

The green fee for the Annandale Golf Course will be $2.00. Also, Pasadena has available 2 public courses, one the Municipal Course at Brookside, and the other a course at Altadena; the green fee for each is $1.00.

Tennis. The annual tennis tournaments and play for the Mershon tennis trophy will be held on the courts of the Huntington Hotel.

The sports prizes will be awarded at a buffet supper on Friday evening in the Huntington Hotel gardens. There also will be entertainment around the swimming pool at that time.

Future AIEE Meetings

North Eastern District Meeting,
New Haven, Conn., May 6–8, 1936

Summer Convention,
Huntington Hotel, Pasadena, Calif., June 22–26, 1936

South West District Meeting,
Dallas, Texas, Oct. 26–28, 1936

Southern District Meeting,
Birmingham, Ala., Dec. 1936

Rules on Presenting and Discussing Papers

At some of the technical sessions, a few papers may be presented only by title. This will permit the devotion of more time to discussion. At other sessions, papers will be presented in abstract, 10 minutes being allowed for each paper unless otherwise arranged, or the presiding officer meets with the authors preceding the session to arrange the order of presentation and allotment of time for papers and discussion. Authors will be notified officially in each case about one month in advance.

Any member is free to discuss any paper when the meeting is thrown open for general discussion. Usually 5 minutes are allowed to each discusser for the discussion of a single paper or of several papers on the same general subject. When a member signifies his desire to discuss several papers not dealing with the same general subject, he may be permitted a somewhat longer time.

It is preferable that a member who wishes to discuss a paper give his name in advance to the presiding officer of the session at which the paper is to be presented. Each discusser is to step to the front of the room and announce, so that all may hear, his name and professional affiliations. Three typewritten copies of discussion prepared in advance should be left with the presiding officer.

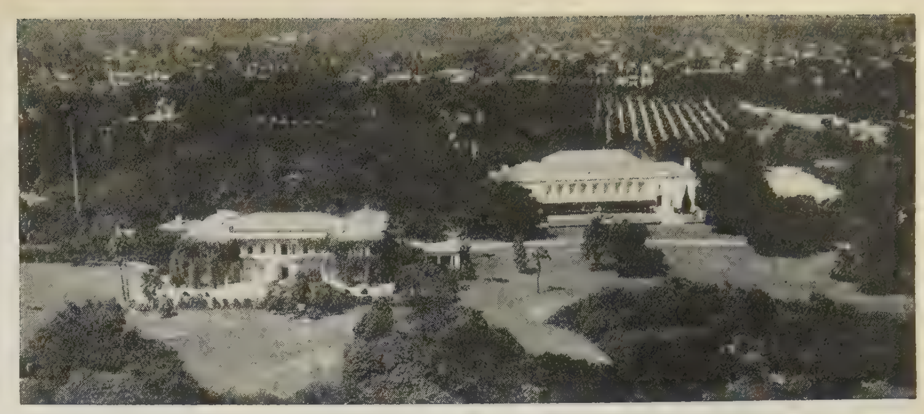
Other discussion to be considered for publication must be submitted, typed double spaced, in triplicate to C. S. Rich, secretary, technical program committee, A.I.E.E. headquarters, 33 West 39th St., New York, N. Y., on or before July 10, 1936. Discussion received after this date will not be accepted.

Pre-Convention Trip to Boulder Dam

Arrangements are being completed for a special inspection of Boulder Dam on Saturday or Sunday, June 20 or June 21, depending upon final schedules for a special train or special cars from the east to Pasadena. The Union Pacific Railroad has a branch line from Las Vegas (on its main line) to Boulder City, that will provide convenient means for routing any special train or special cars direct to Boulder City. The 7 mile journey from there to the dam can be made by busses, which at all times can accommodate up to 100. About half a day is the minimum time required to see the city and the dam. Arrangements may be made by the summer convention committee to have pictures and a description of the work done given by some of the Bureau of Reclamation engineers when the party visits Boulder Dam.

Automobiles can drive from Boulder City to Pasadena, a distance of approximately 325 miles over excellent paved roads, in about 8 hours. There is a good air conditioned hotel in Boulder City where double rooms can be rented for $5 per day

Also there are several places where meals may be obtained. At Las Vegas, which is about 25 miles from Boulder City en route to Pasadena, there are several satisfactory hotels. Visitors to Boulder Dam, who detrain at Las Vegas, may reach Boulder City by bus. It should be noted that a complete special train will not be necessary for a stopoff at Boulder Dam, as arrangements for a stopoff can be made for single cars of convention members and delegates.



The Henry E. Huntington Library and Art Gallery, about a mile from the Huntington Hotel, house many internationally famous works of art

Trips During the Convention

Mt. Wilson Observatory. A very interesting trip has been scheduled for Wednesday, June 24, at 4:00 p.m. to the Mt. Wilson Observatory of The Carnegie Institution of Washington. To get the most out of this trip, visitors should arrive at the mountain top before dark, have time for supper at the Mt. Wilson Hotel, and later enjoy the special lecture on astronomy and the work of the Observatory, have a good view of the valley before and after dark, and take a look through the 60 inch telescope. Arrangements also will be made for this trip to be taken Friday evening by those who may wish to go, but who cannot go Wednesday.

Gould Substation of the Southern California Edison Co., Ltd., is located about a quarter-mile off the Mt. Wilson road, and can be visited conveniently by those taking the Mt. Wilson trip, provided they start an hour or so ahead of the time scheduled for the Mt. Wilson trip. Carrier current pilot protection of several types and manufacture, as well as various types of capacitors for coupling to the 220 kv lines, may be inspected at this station.

The California Institute of Technology is located about a half-mile from the Huntington Hotel. Arrangements can be made at any time to visit its research laboratories—the million volt, high voltage laboratory, the Daniel Guggenheim laboratory of aeronautics, the seismological laboratory, and the astrophysics laboratories, where the famous 200 inch reflector is to be ground.

Griffith Observatory and Planetarium at Los Angeles, dedicated to the late philanthropist, Colonel Griffith J. Griffith, consists of many interesting features, not only the astronomical equipment itself but the building, its murals and science exhibits in geology, physics, chemistry, and mathematics.

Receiving Substations of the City of Los Angeles, Bureau of Power and Light: substation "B," the receiving terminus of the 287 kv transmission line from Boulder Dam; and substation "C," where erection of a 60,000 kva hydrogen cooled frequency changer will be in progress. (B and C may be visited in 4 hours).

Long Beach Steam Plant of the Southern California Edison Co., Ltd., where 415,000 kw capacity in units ranging from a 12,000 kw vertical turbine installed in 1911 to modern 100,000 kw units may be seen. The plant is arranged to burn either natural gas or oil fuel (4 hours).

National Broadcasting Company, recently completed Hollywood studio, where many NBC network programs originate (3 hours).

KFI, 50,000 watt radio broadcast transmitter at Buena Park (3 hours).

"Mutual" office, Southern California Telephone Company, where unusual and experimental telephone exchange equipment is in service (3 hours).

Dispatcher's office and communication center of the Southern California Edison Company, Ltd., can be reached in about 10 minutes' drive from the Huntington Hotel and may be visited by small groups at various times (1 hour).

Metropolitan Water District local distribution works, tunnel excavations, and lining operations (2 hours).

Trips by Arrangement

Huntington Library and Art Gallery may be visited during periods which will be announced at the convention.

San Gabriel Mission, founded in 1771 and standing today in an excellent state of preservation, can be seen in the afternoons. A huge grape vine, said to be the largest in the world, grows here. The Mission is about 3 miles from the hotel (2 hours).

Southwest Museum (3 hours).

Colorado River Aqueduct. The trip over the entire aqueduct requires about 3 days, but parts of the work can be seen in about three hours.

Additional trips can be arranged for those desiring to visit the following: oil field or refinery, rubber tire plant, automobile assembly plant, orange packing house, steel plant, and General Cable Company's "HH" cable plant.

Airplane trips for local sight seeing or other purposes can be arranged for at the nearby Grand Central Air Terminal, Glendale, Calif.

?—Special Train—?

To enable the committee to know what arrangements—if any—to make for a special train or for special cars from the East to Pasadena for the summer convention, all persons interested should communicate at once with H. H. Henline, national secretary A.I.E.E., 33 West 39th Street, New York, N. Y., stating expected number of persons in party. For tentative details of one suggested itinerary see ELECTRICAL ENGINEERING, April 1936, page 418–19.

Optional Post-Convention Trips

As alternatives to the proposed bus trip from Pasadena to San Francisco, that may be preferred by some who could leave Los Angeles as late as Monday night by train and connect with the bus party in San Francisco, Tuesday morning, June 30, the following are offered:

Saturday June 27. All day inspection trip to Metropolitan Water District aqueduct, tunnel, and reservoir works. The aqueduct will span the entire state, with its intake behind Parker Dam, about 155 miles below Boulder Dam, and its terminal distributing lines extending into each of the member cities. With a capacity of more than 1,500 cubic feet per second, about a billion gallons per day, the aqueduct is 242 miles long from its intake to its terminal storage basin, near Riverside, and also includes an initial installation of 150 miles of distributing mains, making a total length of 392 miles. In a one-day trip from Los Angeles it would be possible to visit, among other construction features, the work now in progress on Cajalco Dam and reservoir, one of the largest earth filled structures ever erected, which will have an ultimate storage capacity of 225,000 acre feet.

Construction work on the entire Metropolitan Aqueduct is in its fourth year, and the estimated cost of the project is $220,000,000. The Metropolitan Water District will be by far the largest consumer of the electric energy that is to be generated at Boulder Dam, using for pumping purposes some 36 per cent of the entire plant output.

Sunday, June 28. Electrical Engineer's Day at California Pacific International Exposition at San Diego. The exposition, with 1,400 acres of landscaped grounds, is a creation of agricultural and physical beauty in beautiful Balboa Park at San Diego, 125 miles south of Pasadena. The new Palace of Electricity and the scientific exhibits will hold much of unusual interest, including the latest in domestic household appliances. The lighting system, which literally "paints the grounds" at night, requires a total of 4,000 kw when in full operation, and comprises in itself a complete exposition of color, effect, and beauty.

Trips also may be made to Mexican Border towns.

Monday, June 29. Boat trip from Los Angeles to Catalina Island, a popular island

resort 23 miles off the mainland, where Avalon Bay offers safe bathing, glass-bottom boat trips, submarine gardens, aviary, deep sea fishing, excellent hotel accommodations, a golf course, and other attractions. Pacific Electric cars to Wilmington; boats to the island.

If some desire to make an inspection trip to the Big Creek hydroelectric plants of the Southern California Edison Co., Ltd., this can be done readily by leaving Los Angeles Sunday night on the Southern Pacific "Owl" which sets off a car at Fresno where the passengers may sleep until morning. On Monday morning the trip to the plants can be made by special bus if a sufficient number go, or by private automobiles if the party is small, returning to Fresno in the evening in time to board the "Owl" which would reach San Francisco Tuesday morning to join the main group which makes the bus trip as scheduled.

Points of Interest for the Return Trip

Those who travel to the northwest via San Francisco after the convention will find several large engineering projects of unusual interest.

San Francisco Bridges. The San Francisco-Oakland Bay Bridge, and the Golden Gate Bridge, with the longest (more than 4,000 feet) span in the world, represent the most recent and greatest feats in bridge construction.

Bonneville Project. The Bonneville power-navigation project on the Columbia River, about 42 miles east of Portland, will be at an interesting stage of construction. A sum of $32,200,000 has been allocated to this project, and the work is under construction by the corps of engineers, U.S. Army. A concrete dam of the gravity type is being built, 1,250 feet long and 170 feet high, together with a single-lift ship lock 76 x 500 x 26 feet which at ordinary low water level will have the highest lift of any lock yet constructed. Ultimate plans for power call for 10 generators, each with a capacity of 43,200 kw at a 50 foot head.

Grand Coulee Project. Also beckoning to those who route their trips through the Pacific Northwest is another tremendous government power-irrigation project, Grand Coulee, some 100 miles westerly from Spokane, Wash. To this project already has been allotted some $63,000,000 to cover its initial phases—foundation and appurtenant works—of a gravity section concrete dam that is scheduled to rise to a maximum height of some 550 feet above bedrock (approximately 350 feet from normal river level to spill crest) and have a total crest length of some 4,300 feet. Second only to Boulder Dam (727 feet) in maximum height, the Grand Coulee Dam is scheduled to require some 11,000,000 cubic yards of concrete, more than 3 times that required for Boulder Dam. Although only the initial stage now under construction is authorized, the ultimate project calls for an expenditure of more than $113,000,000. The magnitude of the undertaking and the construction problems involved can be appreciated only by those who have visited the project.

Glacier Park. Those who return to the east via Glacier National Park and who enjoy natural scenery will be well rewarded with the splendors of the northwest.

Texas Centennial. Those who return via the southwest may be interested in visiting Dallas, where the $25,000,000 Texas Centennial Exposition, open June 6 to November 29, will be in full swing. Its exhibits, displays, and special events will depict the epic history of the lone star state, as well as the many advances in art, science, commerce, industry, education, and culture. The exposition consists of some 38 buildings exhibiting the latest trends in architecture. This bids fair to be a splendid show that will rival the 1933 Century of Progress.

Panama Canal. Return trips to the east from Southern California via the Panama Canal can be arranged, as well as stopover privileges for those who wish to remain in Pasadena after the convention. Such arrangements can be made without breaking up the plans of others who may wish to go north into Oregon and Washington. Railway agents have advised that there will be no difficulty in arranging complete flexibility for small groups, or even for individuals, regarding return trips over any one of several routes.

A visit to Boulder Dam may also be listed as a post-convention trip for those who do not find opportunity to visit the Dam on the way to Pasadena.

Hotel Rates

Members should make their own hotel reservations by writing directly to the hotel of their preference

The Huntington Hotel, convention headquarters, has made a special rate for the convention of $6 per day, American plan, for 2 in a room; $7 a day for one in a room. Table d'hote meals will be available for those who are not staying at the hotel: breakfast $1, luncheon $1.25, dinner $1.50. The hotel also has an á la carte taproom where light lunches may be obtained. Students may be accommodated in the hotel annex for $3.50 per day, American plan, or they may obtain rooms only in the annex for $1.50 per day per person.

The Hotel Constance, 900 East Colorado Street, also offers special rates for those in attendance: rooms with double beds $2.50 per day for one person and $3.50 per day for 2 persons; rooms with twin beds $4.00 per day for 2 persons. All rooms have private bath. Meals: breakfast $0.25–0.65, luncheon $0.50, dinner $0.75 and $1.

The Hotel Green, corner of Raymond Avenue and Green Street, will have European plan rates during June as follows: single room without bath $2 and $2.50, double room without bath $2.50 and $3, single room with bath $2.50 and $3, double room with bath (double bed) $3.50 and $4, double room with bath (twin beds) $4 and $4.50.

The Maryland Hotel extends special convention rates, European Plan, as follows: single room with bath $3; double room with bath $5; single room without bath $2.50; double room without bath $4.

Auto camps of excellent quality also may be found in Pasadena. The Clark Auto Camp, 3019 East Colorado Street, is about $1^1/_2$ miles from the Huntington Hotel, and provides all modern conveniences at $1.50 per day.

Student accommodations will be available at the "Athenaeum," California Institute of Technology; housing—$0.75 per day; meals—$0.50 for luncheon, $0.75 for dinner.

Advance Registration

Members who will attend the convention and who will receive a return addressed advance registration card during the latter part of May should fill in and post the card promptly. This will permit the committee in charge to complete some of the advance arrangements. It will also permit badges to be made ready in advance, thus avoiding congestion at the registration desk.

Nonmembers will be charged a registration fee of $3.00; members will be charged an entertainment fee of $2.00; no charge will be made to Enrolled Students.

Members should make their own hotel reservations by writing directly to the hotel of their preference.

Committees

Members of the 1936 summer convention committee who have made the ar-

Membership—

Mr. Institute Member:

By this time you have received our letter dated April 8, 1936, asking you to send to the chairman of your Section membership committee the name of one person who, you feel, should be invited to join the Institute. We have asked you to do this several times in the past and your response has enabled the Section membership committees to bring in applications for membership which this year will result to show an increase in the membership of the Institute.

We are therefore asking your continued participation in this work.

Everett S. Lee

Chairman National Membership Committee

rangements are as follows: R. W. Sorensen, *chairman;* Fred Garrison, *assistant to chairman;* N. B. Hinson, *vice chairman;* O. W. Holden, *secretary;* J. C. Gaylord, *treasurer;* W. S. Peterson, *technical program;* G. A. Riley, *finance;* E. L. Bettannier, *transportation;* H. L. Caldwell, *housing;* R. A. Hopkins, *entertainment;* R. F. Gheen, *sports;* F. B. Doolittle, *publicity;* F. C. Lindvall, *student activities;* Mrs. Mabel MacFerran Rockwell, *ladies entertainment;* David Hall, *registration;* G. P. Garman, *inspection trips.*

Papers Solicited on D-C Stray Load Losses

The d-c subcommittee of the A.I.E.E. committee on electrical machinery earnestly requests that papers on the subject of determination of stray load losses in d-c machines be prepared and submitted to the Institute. It is the hope of the subcommittee that a session on that subject may be scheduled for the 1937 A.I.E.E. winter convention, the program for which already is being developed. Anyone interested in preparing such a paper should communicate with Scott Hancock, chairman of the subcommittee, care of Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

The subcommittee also requests that research work be started on this problem for the purpose of developing means whereby the stray load losses of d-c machines can be determined more accurately.

A.S.T.M. Annual Meeting. The American Society for Testing Materials will hold its 39th annual meeting at Chalfonte-Haddon Hall, Atlantic City, N. J., June 29–July 3, 1936. Many technical papers and reports are scheduled for presentation at the 20 sessions being planned. The program will include a symposium on X-ray crystallography and radiography, a symposium on limitations of laboratory and service tests in evaluating rubber products, several papers on spectrographic analysis, and one session on the subject of water. Other papers will deal with nonferrous metals, wire, soils, corrosion, fatigue and effect of temperature on metals, and cement and concrete, a separate session being devoted to each of these subjects.

Engineering Society of Detroit Organized. The incorporation of a new engineering society in Detroit, Mich., was authorized at a meeting of more than 50 engineers on March 17, 1936. The new society will be known as the Engineering Society of Detroit, and plans are being formulated for permanent headquarters and for meetings of officers and members. Members selected for the initial board of directors include: E. J. Burdick, W. D. Cameron, C. W. Ditchy, H. S. Ellington, M. R. Fisher, J. H. Hunt (A'07, M'13), J. W. Parker, C. R. Paton, and David Segal.

An Analysis of "Electrical Engineering" Readers

A recently compiled analysis of the readers of Electrical Engineering, embracing member and nonmember subscribers, and showing the principal occupational groups as well as the geographical distribution of the publication, is presented herewith.

The analysis shows that more readers are employed by electric power companies than any other branch of the industry, closely followed by the group engaged in electrical manufacturing. Naturally enough, the great industrial states have the heaviest representation. However, the compilation illustrates the comprehensive nation-wide distribution, as well as the extensive Canadian and other foreign circulation.

The discrepancy in the totals of the 2 analyses is due to the large student circulation of the March and April issues—more than a thousand above the average. This student circulation varies considerably, according to the academic season.

Undoubtedly the average monthly distribution in 1936 will show a marked increase over last year. Applications for admission to the Institute are particularly encouraging; in April 482 applicants were elected to membership. The analysis is of April 1, 1936.

Occupational Analysis

Occupational Divisions	Number of Readers	Per Cent of Total
Electric Light and Power Companies (including holding and management companies) Generation, transmission, and distribution of electric power—construction, operation, maintenance, commercial: engineering staffs and executives	3,728	21.5
Consulting Engineers and Large Contracting Companies Principals and engineering staffs	677	3.9
Communication Companies (telephone, telegraph, wireless) Construction, operation, maintenance, commercial; laboratories—research and development: engineering staffs and executives	1,425	8.3
Electrical Manufacturers Design, development, production, and distribution of electrical machinery and equipment: engineering staffs and executives	3,293	19.1
Industrials (factories, mines, mills, railroads) Plant operation, maintenance; production; commercial: engineering staffs and executives	1,604	9.3
Government (federal, state, municipal) Projects; bureaus; army and navy; department heads, aides, and engineering staffs	623	3.6
Educational (engineering schools) *A.* Professors and instructors	875	5.0
B. Engineering students; includes recently employed graduates and applicants for A.I.E.E. membership	2,633*	15.2
Libraries and Booksellers	431	2.5
Electrical Dealers, Contractors, Service Companies	165	0.9
Testing Laboratories, Underwriters, Financial, Attorneys, Writers, Associations	325	1.9
Public Service Commissions (members and engineering staffs)	185	1.1
Miscellaneous	368	2.1
Connections Not Reported and Unclassified	983	5.6
Total	17,315	100

*Variable with academic season; monthly average for year used here.

Geographical Distribution

Maine	40
New Hampshire	34
Vermont	29
Massachusetts	996
Rhode Island	105
Connecticut	318
New England	1,522
New York	3,696
New Jersey	1,111
Pennsylvania	1,486
Middle Atlantic	6,293
Delaware	22
Maryland	273
Dist. of Columbia	266
Virginia	156
West Virginia	92
North Carolina	123
South Carolina	83
Georgia	126
Florida	104
South Atlantic	1,245
Ohio	852
Indiana	325
Illinois	846
Michigan	496
Wisconsin	394
East North Central	2,913
Kentucky	116
Tennessee	110
Alabama	80
Mississippi	35
East South Central	341
Minnesota	152
Iowa	100
Missouri	379
North Dakota	23
South Dakota	38
Nebraska	110
Kansas	142
West North Central	944
Arkansas	36
Louisiana	89
Oklahoma	183
Texas	360
West South Central	668
Montana	57
Idaho	56
Wyoming	24
Colorado	234
New Mexico	30
Arizona	27
Utah	69
Nevada	34
Mountain	531
Washington	278
Oregon	153
California	1,063
Pacific	1,494
U.S. Possessions	82
United States Total	16,033
Canada	686
Foreign	1,789
Grand Total	18,508

World Power Conference to Be Held in Washington

A tentative program already has been formulated for the third World Power Conference to be held in Washington, D. C., September 7–12, 1936. The program will consist of discussions of the following 7 principal subjects:

1. Physical and statistical basis of the national power economy.
2. Organization of the fuel industries.
3. Organization and regulation of electric and gas utilities.
4. National and regional planning for most efficient utilization of natural resources.
5. Special problems in regional planning (water power, integration of electric and gas utility facilities).
6. Rationalization of distribution (electricity and gas, rural electrification).
7. National policies on power and resources.

These 7 principal topics will be subdivided into specific topics, and the plan is to have each nation submit a single paper on each topic. Social and economic interpretations of power development will be emphasized, the object being to interpret to laymen as well as to engineers the economic and social consequences of the utilization of power resources. The proceedings will be conducted in 4 languages: French, German, Spanish, and English.

Officers of the conference are: honorary president, Franklin D. Roosevelt, President of the United States of America; honorary vice president, Harold L. Ickes, Secretary of the Interior; chairman, William F. Durand; and director, O. C. Merrill (M'24) consulting engineer. Officers of the American committee are: chairman, Harold L. Ickes; vice chairman, William F. Durand; chairman executive committee, Morris L. Cooke, administrator, Rural Electrification Administration; and executive secretary, Joel David Wolfsohn, executive secretary, National Power Policy Committee.

Joint Summer Session on Economics Planned

The Society for the Promotion of Engineering Education has accepted the invitation of Stevens Institute of Technology to hold a summer session for teachers of economics in engineering colleges at the Stevens Engineering Camp, Johnsonburg, N. J., during the week beginning June 28. During the same week, the sixth annual economic conference for engineers, sponsored by Stevens, will be in session at the camp. Members of this separate conference, graduates of various colleges, will be permitted to attend the S.P.E.E. sessions with their former tutors as fellow students.

The S.P.E.E. daily program will include 3 daily sessions of one hour each: 2 morning sessions, one on economic theory and one on economic aspects of engineering; and one evening session, on "Economics Today." The afternoons will be free for informal conferences and extra-curricular activities such as land and water sports. The first morning sessions will consist of a series of lectures, one each day, by Horace Taylor (supervising) professor of economics, and Raymond J. Saulnier, instructor of economics, both of Columbia University. The second morning sessions will consist of 7 lectures, one each day, as follows:

1. A Possible Extension of the Principle of Increment Costs, W. D. Ennis, Stevens Institute of Technology, Hoboken, N. J.

2. Will an Investment Pay? Lawrence E. Frost (A'22, M'30), Consolidated Edison Company, New York, N. Y.

3. When Should a Machine or Structure Be Replaced? E. L. Grant, Stanford University, Calif.

4. Judgment Factors in Engineering Choice, David Levinger (M'30), Western Electric Company, New York, N. Y.

5. New Aspects of Depreciation, P. T. Norton, Virginia Polytechnic Institute, Blacksburg.

6. Planning for Economic Performance, Walter Rautenstrauch, Columbia University, New York, N. Y.

7. Current Sources of Information, W. D. Ennis, Stevens Institute of Technology, Hoboken, N. J.

The topics of discussion at the evening conferences will be announced later. The speakers are to be:

June 28. Prof. Ray B. Westerfield, Yale University, New Haven, Conn.

June 29. Norman Thomas, New York, N. Y.

June 30. Prof. Leo Wolman, Columbia University, New York, N. Y.

July 1. Prof. T. North Whitehead, Harvard University, Cambridge, Mass.

July 2. Dr. Harold G. Moulton, Brookings Institution, Washington, D. C.

July 3. William McClellan (A'04, M'09, F'12, and past-president), Potomac Light and Power Company, Washington, D. C.

July 4. E. F. Cowdrick, Standard Oil Company of New Jersey, New York, N. Y.

O. W. Eshbach (A'17, M'30) chairman of the A.I.E.E. committees on education and Student Branches, is chairman of the committee in charge of the S.P.E.E. program.

New Mercury Lamps Being Developed

Two new types of mercury lamps now are being developed in the laboratories of the General Electric Company at Nela Park, Cleveland, Ohio. They are known as the fluorescent and capillary types, the first providing a wide range of colored light, and the second—of relatively small dimensions—producing an extremely brilliant light and emitting photochemical rays. The initial development of the capillary lamp was done by the Philips Glow Lamp Works of Eindhoven, Holland. The fluorescent lamp, shown on the left, consists of a glass tube sealed at each end, within which is a trace of mercury, a small amount of argon gas at low pressure, and special fluorescent powder which clings to the inner surface. When the lamp is turned on, the argon serves as a "starter," and in a few moments a feeble blue light and a large amount of invisible ultraviolet radiation is generated. The invisible ultraviolet radiation strikes the fluorescent coating and is transformed into visible radiation of colored light, various colors being produced by different powders. The fluorescent lamp is said to give from 50 to 200 times as much colored light as an ordinary incandescent lamp for the same energy. The capillary lamp, shown on the right, can be made to emit about $3^1/_2$ times as much light for the energy consumed as the average incandescent lamp. The 2 models shown here are made of fused quartz, and they derive their name from the tiny tube or bore in each lamp. They operate at extremely high internal pressures, and like any mercury lamp require special equipment for starting and operation. The lamp to which the pencil points is a typical water-cooled model, and the other is a typical air-cooled model. It is said that the water-cooled capillary lamp, only an inch long and $^1/_8$ inch in diameter, will supply as much light as the conventional 1,000 watt filament lamp.

The Application of Statistical Methods to Industrial Standardization and Quality Control. A report (No. 600-1935) of the British Standards Institution, carrying this title has been developed under the auspices of the institution as a result of a conference arranged by the American Standards Association in 1923 between Dr. Walter A. Shewhart of the Bell Telephone Laboratories, Inc., and representatives of manufacturing industries in Great Britain, and others interested in the application of statistical methods. This report in its present form is credited very largely to the work of Dr. E. S. Pearson of University College, London. The 160-page ($5^1/_2$ by $8^1/_2$ inches)

booklet is available from the publications department, British Standards Institute, 28 Victoria Street, London, S.W.1, Eng., at the advertised price of 5s net, f.o.b. London. A copy of this report is on file at the Engineering Societies Library, 33 West 39th Street, New York, N. Y.

International Meeting of Marine Experts. The subject of safety of life at sea, which has grown in international importance during the past few years, will be the main topic of discussion when marine experts from all parts of the world convene in New York, N. Y., during the week of September 14–19, 1936. This international meeting of naval architects and marine engineers, the first to be held in the United States, is being sponsored by the Society of Naval Architects and Marine Engineers. Delegates from the foremost maritime technical societies of Great Britain, France, Italy, Germany, and Japan, as well as those from the United States, will meet in technical sessions on September 15 and 16, at which time papers dealing with safety, transatlantic liners, and propulsion will be presented by representatives of the different countries. Arrangements are being made by a committee of which Capt. Roger Williams, vice president of the Newport News Shipbuilding and Dry Dock Company, Newport News, Va., is chairman.

Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or to reject them entirely.

ALL letters submitted for consideration should be the original typewritten copy, double spaced. Any illustrations submitted should be in duplicate, one copy to be an inked drawing but without lettering, and other to be lettered. Captions should be furnished for all illustrations.

STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

Neutralizing Transformers for Pilot Wire Protection

To the Editor:

I have read with interest the correspondence of Messrs. R. W. Osborne and E. E. George in the January 1936 issue of ELECTRICAL ENGINEERING with reference to neutralizing transformers for pilot wire protection.

In view of the interest in regard to the early applications of the neutralizing transformer, your readers may like to hear more of the historical background of this device. While the applications herein mentioned do not refer to pilot wire protection, they are similar in principle and make use of the basic idea of the neutralizing transformer.

The neutralizing transformer was invented in 1906 by Prof. Chas. F. Scott, then of the Westinghouse company. Attention was focused on the need for this device at that time by the necessity of protecting communication circuits parallel to the newly electrified New York, New Haven and Hartford Railroad. In order to permit service over the New York-Boston cable, Bell System engineers applied neutralizing transformers to these circuits in October 1907, shortly after electrical operation of the railroad began. Transformers were also applied in 1908 to 2 open wire routes exposed to the electrification. These transformers were designed to prevent interference with telegraph circuits from the normal or load currents.

The Western Union Telegraph Company applied neutralizing transformers to telegraph circuits on the right-of-way of the New Haven Railroad in 1907. Subsequently, the Western Union company treated other circuits exposed to the Norfolk and Western and Virginian railway electrifications and at other locations in the same manner. More recently the Postal Telegraph Company has also made an application of the neutralizing transformer to some of its circuits.

In 1932, the Joint Subcommittee on Development and Research of the Edison Electric Institute and the Bell System made a trial installation of neutralizing transformers in New York State to determine experimentally their effectiveness in preventing the effects of induced voltages under short-circuit conditions. In this case, the application was to an open wire toll telephone line. These transformers represented a considerable modification of the devices used earlier to prevent telegraph interference.

I trust that this further light on the historical background of this device will be of interest to the readers of ELECTRICAL ENGINEERING.

Very truly yours,

R. K. HONAMAN (A'16, M'36)

Protection Methods Engineer,
Bell Telephone Laboratories,
New York, N. Y.

Editor's Note: See pages 524–9, this issue, for a paper on this subject.

Dissociation in Arc Welding

To the Editor:

Some new facts have recently appeared which bear on the problem of dissociation in welding arcs in air. It will be useful to recall that the development of the atomic hydrogen welding process (*General Electric Review*, volume 29, 1926, page 153) was based on Langmuir's theoretical and experimental studies (*Physical Review*, volume 34, 1912, page 401; *Journal* of American Chemical Society, volume 34, 1912, page 860) of the heat conduction by dissociating hydrogen. He showed that in the pure thermal conduction through a nondissociating gas film the heat flow W_C may be satisfactorily accounted for by the relation

$$W_C = S\int_{T_1}^{T_2} kdT \qquad (1)$$

where S is a shape factor, $T_2 - T_1$ is the temperature interval, and k is the kinetic theory heat conductivity at the temperature T. When, however, the gas dissociates at the hot surface, diffuses to the cooler regions, and subsequently associates, the heat flow is increased in such a manner that k of equation 1 must be replaced by a new expression:

$$k + DQ\frac{dC}{dT} = k + k_D \qquad (2)$$

Here D is the coefficient of diffusion of H (hydrogen) through H_2, Q is heat of formation of H_2 from 2H, and C is the concentration of H at the temperature T_1. Langmuir found that the diffusion heat flow W_D given by

$$W_D = S\int_{T_1}^{T_2} D\,Q\frac{dC}{dT}\,dT \qquad (3)$$

may be large compared with W_C if appreciable dissociation occurs, as in the case of hydrogen at moderate temperatures and pressures. Experiments showed the following significant values: For 2.7 per cent dissociation, only 15 per cent of the heat was carried by the diffusion of dissociated atoms; at 57 per cent dissociation, however, 80 per cent was carried by this process. At still higher temperatures where the dissociation is more complete, the pure conduction heat transfer becomes quite negligible.

Direct experimental measurement of the dissociation in N_2 (nitrogen) is made difficult by the necessarily high temperatures involved. Both theory and indirect experimental results point conclusively, however, to the presence of a high degree of dissociation in arcs in air, including welding arcs.

According to the welding literature, the temperature of welding arcs is presumed to be 4,000 degrees centigrade. At this temperature and for the values (International Critical Tables) of the dissociation potential $V_N = 11.7$ volts for the process $N_2 \rightarrow 2$ N, and $V_0 = 7.05$ volts for the case $O_2 \rightarrow 2$ O, which until recently have been accepted as correct, the degree of dissociation of welding arcs in air is practically zero for the nitrogen and only 2 per cent for the oxygen. We have recently measured (Suits, *Physics*, volume 6, 1935, page 315) the temperature of a welding arc, using a heavily coated electrode, to be 6,000 degrees Kelvin. The dissociation potentials for air have recently been measured more accurately by spectroscopic methods, and they are found to be much lower than the formerly accepted values (Johnson and Walker, *Journal* of American Chemical

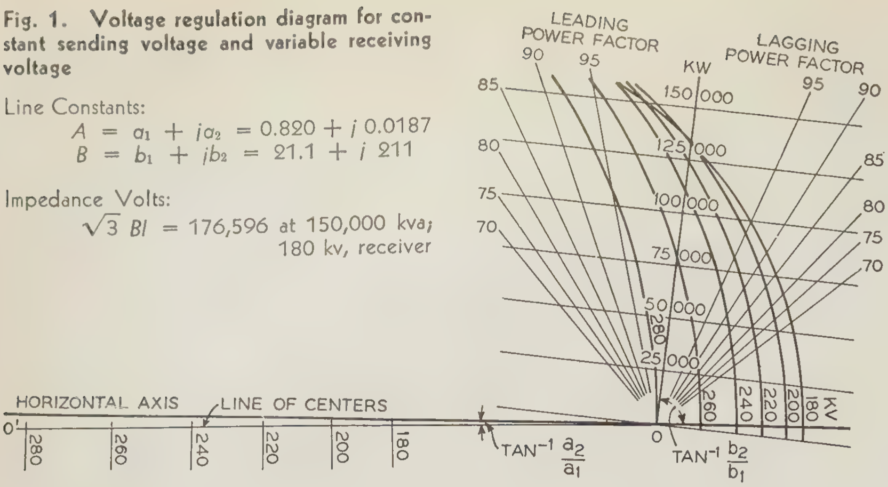


Fig. 1. Voltage regulation diagram for constant sending voltage and variable receiving voltage

Line Constants:

$A = a_1 + ja_2 = 0.820 + j\,0.0187$
$B = b_1 + jb_2 = 21.1 + j\,211$

Impedance Volts:

$\sqrt{3}\,BI = 176{,}596$ at 150,000 kva; 180 kv, receiver

Chart Values in Kilovolts for $E_s = 230$ Kv (Constant)

E_R	$\dfrac{E_R}{E_{R0}}$	$E_R\left(\dfrac{E_R}{E_{R0}}\right)A$	$E_s\left(\dfrac{E_R}{E_{R0}}\right)$
180	1.000	147.5	230
200	1.111	182	255.5
220	1.222	221	281
240	1.333	262	307
260	1.444	308	332
280	1.555	357.5	357.5

Society, volume 55, 1933, pages 179 and 191; Giauque and Clayton, *Journal* of American Chemical Society, volume 55, 1933, page 4887). The new values are $V_N = 7.9$ volts, $V_0 = 5.09$ volts. With these new data one may calculate that at welding arc temperatures the nitrogen is 75 per cent dissociated, and the oxygen is completely dissociated. The partial pressure of the atomic oxygen and atomic nitrogen is thus 0.81 atmosphere. There seems no escape from the conclusion that with this high percentage of dissociation pure thermal conduction of the type expressed by equation 1 is entirely negligible. Welding arcs in air are thus "atomic air" arcs, and the flow of heat from the arc column is almost entirely due to the diffusion of dissociated molecules. It may be shown that the shape of the volt-ampere characteristics of arcs in air, or (what is equivalent) the steady state stability, are properties determined fundamentally by the nature of the laws of heat flow from the arc column—in this case, the dissociation properties of oxygen and nitrogen.

Some interesting comparisons may be made between nitrogen and hydrogen. For example, it is well known that the coefficient of thermal conductivity k for nitrogen is only $^1/_7$th as great as for hydrogen. At arc temperatures, however, where the dissociation is nearly complete, the heat flow is proportional to VD, where V is the dissociation potential and D is the diffusion coefficient. The ratio

$$\frac{V_N D_N}{V_H D_H}$$

where the subscripts refer to nitrogen and hydrogen, is numerically equal to 1/4.5 (through an error in the choice of the numerical value of V_N, this ratio is given as 1/2.6 in a letter to the editor of the *Physical Review*, volume 47, 1935, page 975). Thus at arc temperatures, dissociation is relatively more important in nitrogen than in hydrogen.

There are many other experimental results that bear on this conclusion concerning the importance of dissociation in nitrogen arcs. For example, in arcs in argon, helium, nitrogen, and hydrogen, we have recently observed a quantity which may be termed the "transient stability," and which may be correlated with the thermal conductivity, provided dissociation is taken into account. In respect to this quantity the gases differ greatly except that helium and hydrogen are practically identical. The coefficient k for nitrogen is only $^1/_6$th as great as for helium, so that one is forced to conclude that the low thermal conductivity of nitrogen is balanced by the increased heat flow due to dissociation.

Professor Doan's experiments in argon, recently reported in Electrical Engineering (volume 54, 1935, pages 1144–9), suggest that the lack of a dissociation heat transfer in the monatomic argon is responsible for many of his observations.

Very truly yours,

C. G. Suits
(Member American Physical Society)
Research Laboratory,
General Electric Company,
Schenectady, N. Y.

A Voltage Regulation Diagram for Constant Sending Voltage

To the Editor:

Regulation diagrams based upon the $R + jX$ convention and showing receiving end load conditions for constant receiver voltage and varying sending voltage have been in use for many years. However, to the writer's knowledge, similar regulation diagrams showing receiving end load conditions for constant sending voltage and varying receiver voltage have never been published although they probably have as great a field of application.

Figure 1 shows such a diagram drawn for a long circuit for which auxiliary circuit constants have been determined. Line-to-line voltage is used in this example. The first step in the construction of this diagram is the construction of the kilowatt-reactive kilovolt-ampere grid. This is the same as in the diagram for constant receiving voltage. Voltage is the scale. Any expected value of receiving voltage, E_{R0}, is chosen, but it is usually more convenient to use the lowest value of E_R. At a distance $-E_{R0}(a_1 + ja_2)$ from the zero point of the kilowatt line, a point is located which is the center for the arc of sending voltage E_s. Any point on the arc of E_s is a receiving end load condition for $E_{R0} = 180$ kv and $E_s = 230$ kv. The arc is labeled 180 kv. Since the current varies inversely as the voltage for constant power, and since the same kilowatt-reactive kilovolt-ampere grid must be used for all values of E_R, it is necessary to modify the voltage quantities for other values of E_R. Let E_{R1}, E_{R2} ..... E_{Rn} be other values of E_R. The radii of arcs for these values of E_R will have lengths

$$E_s\left(\frac{E_{R1}}{E_{R2}}\right),\ E_s\left(\frac{E_{R2}}{E_{R0}}\right)\ \ldots .\ E_s\left(\frac{E_{Rn}}{E_{R0}}\right)$$

The centers will not coincide as in the diagram for constant receiver voltage, but will be located on line 0–0′ drawn at $\tan^{-1} a_2/a_1$ with the horizontal and at distances

$$-E_{R1}\left(\frac{E_{R1}}{E_{R0}}\right)(a_1 + ja_2),\ -E_{R2}\left(\frac{E_{R2}}{E_{R0}}\right)(a_1 + ja_2),\ \ldots .\ -E_{Rn}\left(\frac{E_{Rn}}{E_{R0}}\right)(a_1 + ja_2)$$

from 0.

The arc passing through 0 represents E_s/A, the open circuit voltage of the line. This should be used as a check upon the construction of the diagram. Diagrams for varying receiver voltage may be drawn for the other conventional methods of circuit representations. For circuits having negligible capacitance, the A constant becomes $1 + j_0$, and the B constant becomes the impedance, $R + jX$, one phase to neutral.

If the nominal Π representation is desired for circuits having moderate capacitance, the approximate A constant is used and

$$A = a_1 + ja_2 = 1 + (R + jX)\left(\frac{jb}{2}\right)$$

$$= 1 - \frac{bX}{2} + j\frac{Rb}{2}$$

in which b = capacitive susceptance, one phase to neutral, in mhos with R and X expressed in ohms. The B constant becomes the impedance as in a circuit with negligible capacitance.

Making use of the scheme described of reducing all voltage quantities to the same kilowatt-reactive kilovolt-ampere grid, it is very simple to construct diagrams which will be the exact equivalent of power circle diagrams giving both sending and receiving end load conditions. The sending end conditions are obtained from the fundamental expression $E_R = E_sD - I_sB$, which in plotting is rotated 180 degrees to fit the receiving end kilowatt-reactive kilovolt-ampere grid.

Very truly yours,

Volney J. Cissna (M'30)
U. S. Engineer Office,
Kansas City, Mo.

Personal Items

J. H. Foote (A'18, F'32) formerly supervising engineer, Commonwealth and Southern Corporation, Jackson, Mich., recently was appointed chief engineer of the Consumers Power Company, Jackson. Mr. Foote was born in Jackson in 1891, and graduated from Michigan State College with the degree of bachelor of science in civil engineering in 1914. Following a preliminary training of 3 years with several Michigan utilities, he became associated with the Consumers Power Company as assistant distribution engineer. He served in that capacity until he was promoted to electrical engineer in 1920. In 1924 he became head of the electrical engineering department of the Commonwealth Power Corporation of Michigan, Jackson, and was retained in that capacity by Stevens and Wood, Inc., successors to the Commonwealth Power Corporation of Michigan, and by Allied Engineers, Inc., successors to Stevens and Wood, Inc. In 1931 he became supervising engineer for the Commonwealth and Southern Corporation. Mr. Foote has served on the Institute's committees on power transmission and distribution (1929–31) and protective devices (1932–34). He is a member of the American Society for Testing Materials, the Edison Electric Institute, the American Association for the Advancement of Science, and the Society for the Promotion of Engineering Education.

Asger Vilstrup (A'20, M'27) acting plant manager of the British Columbia Electric Railway Company, Ltd., Vancouver, B. C., Can., has been appointed chief electrical engineer of that company. Mr. Vilstrup was born at Borris, Denmark, in 1884, and received his technical education at the college of electrical engineering in Copenhagen, Denmark. From 1903 to 1909 he was employed by electrical companies in England, including British Insulated and Helsby Cables, Ltd. In 1911 he became field draftsman and subforeman on hydroelectric construction at Lake Buntzen for the British Columbia Electric Railway Company, Ltd., and 3 years later he was made assistant engineer in the electrical department at Vancouver. In this position he was concerned with the design of substations and transmission and distribution lines, and the power system and plant conditions. In 1929 Mr. Vilstrup was appointed assistant plant manager, and since 1935 has been acting plant manager. A past-chairman (1925–26) of the Institute's Vancouver Section, Mr. Vilstrup has long taken part in its affairs, and was active in connection with the A.I.E.E. Pacific Coast convention held at Vancouver in 1932.

H. E. Clifford (A'05, F'13) dean, Harvard Graduate School of Engineering, Cambridge, Mass., has resigned, effective September 1, 1936. He has resigned also the Gordon McKay professorship of electrical engineering, and will become emeritus professor. Professor Clifford was born at Lowell, Mass., in 1866, received the degree of bachelor of science at the Massachusetts Institute of Technology in the department of electrical engineering in 1886, and attended the Harvard University graduate school for 3 years. Following the completion of his undergraduate training he became affiliated with the Massachusetts Institute of Technology as an assistant in physics. He became instructor in physics in 1888 and assistant professor in 1895. In 1902 he was appointed associate professor of theoretical electricity, and during the period 1904–09 he was professor of theoretical and applied electricity. During 3 years of that interval he acted as head of the department of electrical engineering. Professor Clifford accepted the Gordon McKay professorship of electrical engineering at Harvard University in 1909, and has served as dean of the graduate school of applied science since 1930. In addition to his regular duties he has served as a consulting engineer. He is a member of the Illuminating Engineering Society, American Academy of Arts and Sciences, American Association for the Advancement of Science, Matematico di Palermo, Tau Beta Pi, and Sigma Xi.

Herbert Vickers (M'26, F'34) head of the department of electrical and mechanical engineering, University of British Columbia, Vancouver, has submitted his resignation, effective at the end of the present scholastic term, to accept a position with a leading British firm engaged in technical electrical work. Doctor Vickers is a native (1886) of Chester, England, and graduated from the University of Liverpool with the degree of master of engineering in 1912. He received the degrees of master of science (1922) and doctor of philosophy (1923) from the University of Birmingham. Following his graduation from the University of Liverpool he served for a brief period on the electrical design staff of the Siemens Brothers Company, Stafford, England, and in 1915 he joined the design staff of the British Westinghouse Company, Manchester. In 1920, after a brief return to the Siemens Brothers Company, he became chief consulting engineer to the firm of Campbell and Isherwood, Liverpool, and served at the same time as senior lecturer at the University of Birmingham. In 1923 he became chief assistant at the Finsbury Technical College, London, and in 1924 he accepted his present position with the University of British Columbia. Doctor Vickers has contributed much to technical literature, and is the author of a widely accepted textbook on induction motors. He was chairman of the Vancouver Section of the Institute during the period 1930–31. He is a member of the Institute of Physics (British), the Physical Society of London, and the American Association for the Advancement of Science.

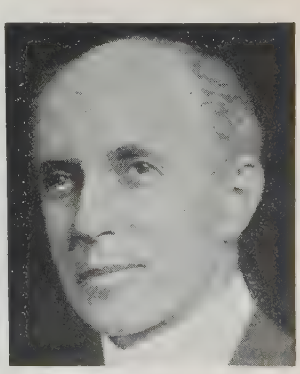
J. H. Hunt (A'07, M'13) patent section, General Motors Corporation, Detroit, Mich., was selected as one of the initial board of directors of the Engineering Society of Detroit, which was organized recently. Mr. Hunt is a native (1882) of Saranac, Mich., and graduated from the University of Michigan in 1905. In the same year he was employed as an engineering apprentice by the Western Electric Company, Chicago, Ill., and in 1906 he was made assistant designing engineer. After a year (1906–07) as instructor in electrical engineering at Washington University, St. Louis, Mo., he accepted a position as assistant professor of electrical engineering at Ohio State University; later, he held successively the positions of associate professor (1908–11) and professor (1911–12). During the year 1912–13 he served as electrical engineer for the Packard Motor Car Company, Detroit, Mich., and in 1913 he accepted a position as research engineer for the Dayton Engineering Laboratories Company, Dayton, Ohio. That company later was absorbed by the General Motors Corporation, and in 1920 Mr. Hunt became director of the electrical laboratories of the General Motors Research Corporation, Dayton. Later, the General Motors Research Corporation was moved to Detroit, and in 1928 he was transferred to the patent section of the General Motors Corporation.

F. B. Jewett (A'03, F'12, and past-president) president, Bell Telephone Laboratories and vice president, American Telephone and Telegraph Company, New York, N. Y., has been awarded one of the 2 1936

J. H. FOOTE

ASGER VILSTRUP

F. B. JEWETT



Franklin Medals of the Franklin Institute "in recognition of his many important contributions to the art of telephony, which have made conversation possible not only from coast to coast, but from this country to the other side of the world—contributions of which some were made by him alone, and some by him in collaboration with other workers in the great laboratory of research which he organized and which he has directed with such signal success." These medals have been awarded annually since 1914 from the Franklin Medal fund to those workers in physical science or technology, without regard to country, whose efforts have done most to advance a knowledge of physical science or its application. The medal will be presented to Doctor Jewett at the annual medal day exercises, to be held May 20, 1936, at the Franklin Institute Philadelphia, Pa.

W. O. Pennell (A'05, F'22) has retired as chief engineer of the Southwestern Bell Telephone Company, St. Louis, Mo. Mr. Pennell was born at Exeter, N. H., January 13, 1875. In 1896 he received the degree of bachelor of science in electrical engineering from Massachusetts Institute of Technology, and for the 2 years following taught electrical engineering at Lafayette College. He then became assistant engineer of the Bell Telephone Company of Philadelphia, holding this position until 1902, when he accepted the position of engineer with the American Telephone and Telegraph Company at Boston, Mass. The following year he went to Kansas City, Mo., as chief engineer of the Missouri and Kansas Telephone Company, predecessor of the Southwestern Bell Telephone Company. From 1912 to 1916 he was building and equipment engineer of the successor company, and from 1916 to 1918 was acting chief engineer. Mr. Pennell was appointed chief engineer in 1918. He is the author of various papers on engineering and mathematics, and the originator of several patents on widely used telephone devices. During the years 1914–17 and 1919–23 he was a member of the Institute's committee on telegraphy and telephony (now the committee on communication). His society memberships include the American Mathematical Society and the Mathematical Association of America.

P. G. Gossler (A'94, F'13, and member for life) has left the position of president of the Columbia Gas and Electric Corporation, New York, N. Y., to become chairman of the board of directors. Mr. Gossler, who was born at Columbia, Pa., August 6, 1870, was graduated from the mechanical engineering course at Pennsylvania State College in 1890, and was employed by the Edison General Electric Company and the United Electric Light and Power Company, both of New York, before he became superintendent of the Royal Electric Company, Montreal, Can., in 1895. In 1901 he transferred to the Montreal Light, Heat, and Power Company as general superintendent and engineer for 3 years, after which he returned to the United States as a vice president of J. G. White and Company. Mr. Gossler became associated with A. B. Leach and Company, New York, in 1909, where he was engaged in the management of public utilities, and while vice president of this company in 1920 became chairman of the board of the Columbia Gas and Electric Company. Two years later he became president of the latter company, from which the Columbia Gas and Electric Corporation was organized in 1926.

B. D. Hull (A'15, M'26, and past vice president) who has been engineer for Texas, Southwestern Bell Telephone Company, Dallas, has been appointed chief engineer with offices at St. Louis, Mo. He was born at Galesburg, Mich., September 12, 1882, and entered the employ of the Southwestern Bell Telephone Company in 1905, following graduation from the University of Kansas with the degree of bachelor of science in electrical engineering. Mr. Hull was employed in various phases of telephone work, and was transmission and protection engineer at St. Louis prior to his transfer to Texas in 1926 as engineer for that state. He was a member of the Institute's communication committee 1928–29, 1931–32, and 1933–35, and was a member of the technical program committee 1932–35. During the year 1928–29 he was Institute vice president for the South West District, and from 1931 to 1935 was a member of the board of directors. Mr. Hull has been particularly active in Institute affairs throughout the South West District, and it was largely through his inspiring leadership that the 3 Texas Sections (Dallas, San Antonio, and Houston) were formed.

L. V. Sutton (A'11) president and general manager of the Carolina Power and Light Company, Raleigh, N. C., has been made president of the Southeastern Electric Exchange. Mr. Sutton was born at Richmond, Va., August 6, 1889, and is a graduate of Virginia Polytechnic Institute. He was employed in the student course of the General Electric Company at Lynn, Mass., prior to employment by the Carolina Power and Light Company in 1912. After advancing to the position of assistant to general manager, Mr. Sutton left the company in 1924 to become assistant general manager of the Arkansas Central Power Company (later Arkansas Power and Light Company) at Little Rock. Three years later he became vice president and general manager of the Mississippi Power and Light Company at the time of its organization, and in 1932 he returned to the Carolina Power and Light Company, being elected president and general manager in 1933. He is a member of the Edison Electric Institute and other organizations.

F. A. Gaby (A'06, F'18) former assistant to the president, Canadian Pacific Railway Company, Montreal, has become executive vice president and director of the British American Oil Company. Mr. Gaby was born at Richmond Hills, Ontario, in 1878 and received the degree of bachelor of applied science at Toronto University. For a period of 8 years following his graduation he was engaged in electrical contracting and installation work, and in 1904 he became erecting engineer for the Canadian General Electric Company, Toronto. After a brief period (1906–07) as chief assistant electrical engineer for the city of Winnipeg, Manitoba, he became associated with the Hydro-Electric Power Commission of Ontario, Toronto, and was chief engineer of the commission from 1912 until he joined the Canadian Pacific Railway Company in 1934. He has been a member of the Institute's committee on power transmission and distribution (1914–17) and of the Edison Medal committee (1930–35).

C. W. Mier (A'25, M'31) who has been engineer for Oklahoma, Southwestern Bell Telephone Company, with headquarters at Oklahoma City, has been transferred to Dallas, Texas, as engineer for Texas. Mr. Mier was born at St. Louis, Mo., September 21, 1886, and received the degree of bachelor of science in electrical engineering from Washington University (St. Louis) in 1909. He then entered the employ of the Southwestern Bell Telephone Company at St. Louis, and in 1916 was appointed transmission engineer. In 1926 Mr. Mier became general transmission and protection engineer, and in the following year he was transferred to Oklahoma City as engineer for the state of Oklahoma. Mr. Mier has taken an active part in Institute affairs, not only as a member and past chairman (1929–30) of the Institute's Oklahoma City Section, but throughout the South West District, of which he is now secretary.

W. B. Stephenson (A'28, M'36) former general plant extension engineer, Southwestern Bell Telephone Company, St. Louis, Mo., has been transferred to Oklahoma City, Okla., as engineer for that state. Mr. Stephenson was born at Plymouth, Ind., January 16, 1891, and was graduated from Purdue University with the degree of bachelor of science in mechanical engineering in 1913. Following graduation he became a student engineer in the employ of the Southwestern Bell Telephone Company, and subsequently had various assignments, principally in traffic engineering. During 1920–21 he was equipment engineer, and from 1921 until his present appointment was general plant extension engineer. Mr. Stephenson served on the Institute's membership committee during the year 1933–34.

C. F. Kettering (A'04, F'14) vice president and director of General Motors Research Laboratory, has been awarded one of the 2 1936 Franklin Medals of the Franklin Institute "in recognition of his significant and timely contributions to the science of automotive engineering—a science out of which has grown the greatest industry in this country, the manufactured product of which has, in only a quarter of a century, changed the face of the civilized world." Doctor Kettering recently received the Washington Award for 1936. A biographical sketch of Doctor Kettering was given in Electrical Engineering for

February 1936, page 218, and the essential text of his address on the occasion of the presentation of the Washington Award was given in the April 1936 issue, pages 324–8.

H. J. McCreary (A'24) recently was appointed chief engineer of the Leich Electric Company, Genoa, Ill., manufacturers of telephone equipment. Mr. McCreary, who received the degree of bachelor of science in electrical engineering from the University of Nebraska, was employed by the Automatic Electric Company, Inc., Chicago, Ill., from 1923 to 1928 and designed and developed various items. The following year he was development engineer for the Grigsby-Grunow Company, Chicago, and then accepted the position of development engineer for the Railroad Supply Company, Chicago. Mr. McCreary has had a number of patents on communication inventions, including work in television, issued to him.

E. V. Caton (M'23) manager, Winnipeg (Manitoba) Electric Company, recently was elected vice president of the Engineering Institute of Canada. Mr. Caton, a native (1884) of Hore, England, attended Brighton College and Sheffield University, and served an apprenticeship with the Hore (England) Electric Company during the period 1902–04. After a brief association with Crompton and Company, Chalmsford, he was employed in 1907 as chief assistant in the testing department of Vickers, Ltd., Sheffield. In 1912 he accepted a position as chief engineer for the City of Winnipeg Hydro Electric System, with full charge of extensions and construction, and in 1922 he became electrical engineer for the Winnipeg Electric Company. He has been manager of that company since 1925.

C. F. Scott (A'92, F'25, HM'29, past-president and member for life) emeritus professor of electrical engineering, Yale University, New Haven, Conn., recently was elected an honorary member of the Connecticut Society of Civil Engineers. Doctor Scott, who is chairman of the Engineers' Council for Professional Development as announced in Electrical Engineering for November 1935, page 1277, has also been appointed chairman of the Connecticut state board of registration to administer the new engineers' registration law.

Herbert Hoover (HM'29) Palo Alto, Calif., recently received the medal of the League Internationale des Aviateurs upon his retirement as president of the American Relief Administration Association. The medal, for which the late King Albert of Belgium posed, was awarded to Mr. Hoover at the King's instance in 1933, to be presented as soon as an appropriate occasion should arise.

J. A. Davis, Jr. (A'30), formerly district manager, Virginia Electric and Power Company, Ashland, has been appointed assistant to the president, with offices at Richmond. Mr. Davis received the degree of bachelor of science in electrical engineering at the Virginia Polytechnic Institute in 1927, and has been associated with the Virginia Electric and Power Company continuously since his graduation. He has held successively the positions of student engineer, assistant manager of transportation, and manager of the Ashland district.

J. D. Wright (A'36) formerly assistant head of the industrial department engineering staff, General Electric Company, Schenectady, N. Y., has been appointed assistant manager of the industrial department. Mr. Wright graduated from the University of Wisconsin in 1909 with the degree of bachelor of science, and has been employed continuously by the General Electric Company since that date. He is a member of the Association of Iron and Steel Electrical Engineers and the American Iron and Steel Institute.

P. B. Stewart (A'25) who has been assistant superintendent of distribution for the Union Gas and Electric Company, Cincinnati, Ohio, has been appointed superintendent of the distribution department. Mr. Stewart was born in Peru November 15, 1900, and studied electrical engineering at the University of Cincinnati. He was employed by the Union Gas and Electric Company in 1918, and advanced through various positions, being appointed assistant superintendent of distribution in 1926.

E. R. Coop (A'26, M'33) is now distribution engineer, New England Power Engineering and Service Corporation, Providence, R. I. Formerly he was distribution engineer for the South County Public Service Company, Westerly, R. I.

H. L. Buck (A'35) who has been associated with Day and Zimmerman, Inc., Philadelphia, Pa., is now employed by the I-T-E Circuit Breaker Company, Philadelphia.

H. H. Horning (A'19) who formerly was employed by Electrad, Inc., New York, N. Y., is now district sales manager for the Imperial Electric Company, with offices at West Roxbury, Mass.

W. C. Broekhuijsen (A'23) formerly development engineer, Eastern Engineering Company, New Haven, Conn., is now with the American Machine and Foundry Company, Brooklyn, N. Y.

A. W. Bechlam (A'25) who formerly was mechanical engineer for the Solvay Process Company, Syracuse, N. Y., is now engineer for the Anaconda Copper Company, Hastings-on-Hudson, N. Y.

H. O. Hill (A'23) formerly with the McClintic-Marshall Corporation, is now assistant chief engineer for the Bethlehem Steel Company, Bethlehem, Pa.

F. D. Weber (A'31) is now with the sales department of the Louis Allis Company, Milwaukee, Wis.

R. J. Speck (A'35) is now outside plant engineer for the Southern California Telephone Company, Los Angeles.

Obituary

Herbert William Drake (A'02, F'25) equipment engineer in charge of research and development, Western Union Telegraph Company, New York, N. Y., died April 14, 1936. He was born November 24, 1878, at Leeds, England, and received his elementary education in England. From 1892 to 1897 he was employed by the Western Union Telegraph Company at Buffalo, N. Y., in the successive positions of messenger, clerk, telegraph operator, and branch office manager. He then became wire chief with the American Telephone and Telegraph Company at Buffalo until 1904, when he was transferred to New York as chief equipment man. Four years later he was made division supervisor of leased wires. In 1910 the company obtained control of the Western Union Telegraph Company, and Mr. Drake returned to the latter as assistant to engineer of equipment. His next promotion was in 1912, when he became circuit equipment engineer; in 1915 he was named apparatus engineer, a title which he held until recently. Mr. Drake was responsible for the development and design of various telephone and telegraph systems and apparatus, including protective devices and switchboards, and was granted several patents. He had been a member of the Institute's board of examiners since 1925, and was its chairman for the year 1930–31. Other committees on which he served included telegraphy and telephony (now communication) 1919–21 and 1922–31 (chairman 1927–29); meetings and papers (now technical program) 1927–29; and protective devices, 1927–31. He was a member of the New York Electrical Society and a consulting member of the telephone and telegraph section of the American Railway Association.

Marcus Bayard Butler, Jr. (M'26) employed by the Edward G. Budd Manufacturing Company, Philadelphia, Pa., died in a fire in that city on March 8, 1936. He was born at Bridgeport, Conn., October 7, 1891, and received the degree of bachelor of science in electrical engineering from Carnegie Institute of Technology in 1913. From the time of his graduation until 1917 he was employed in the electrical department of the Carnegie Steel Company at Duquesne, Pa.; from 1917 to 1919 he served in the U.S. Navy, becoming a lieutenant. Following his naval service he was manager of the Pittsburgh, Pa., district of the Chapman Engineering Company, and during 1923–24 was engaged in engineering work under C. A. Adams (A'94, F'13, past-president, and member for life). For the year 1924–25 he was chief engineer and general manager for the Boyden Neverslip Company at New Brunswick, N. J. In 1925 Mr. Butler accepted the position of chief electrical engineer of the American Chain Company, Bridgeport, Conn., which he held for 5 years. For 2 years prior to joining the Budd Manufacturing Company he was employed by the duPont Ammonia Corporation, Charleston, W. Va., as me-

chanical engineer. Mr. Butler engaged in welding research, and is credited with developing special chain welding machines and with contributions to the development of stainless steel. He was a member of the committee for welding research of the National Research Council, the American Welding Society, and the American Association for the Advancement of Science.

James E. Moravec (A'22) assistant vice president in charge of the staff department, Bell Telephone Laboratories, New York, N. Y., died March 29, 1936. Mr. Moravec was born at Chicago, Ill., July 8, 1884, and graduated from Loyola University (Chicago) in 1900. In 1902 he entered the employ of the Western Electric Company, Chicago, as a foreman's clerk, and during the following 15 years he was placed in charge of that company's manufacturing costs and accounts division. In 1918 he was assigned to the engineering department of the Western Electric Company, New York, N. Y., and as chief accountant he developed accounting methods and supervised the preparation of the information required by the government in the purchase of war equipment. In 1922 he was appointed assistant commercial manager, and in 1925, when the Bell Telephone Laboratories were established, he became commercial manager and general auditor. In 1929 he was appointed assistant vice president in charge of the staff department.

Harry U. Hart (A'07, F'13) vice president and chief engineer, Canadian Westinghouse Company, Hamilton, Ontario, died March 15, 1936. Mr. Hart was born at Covington, Ky., June 28, 1874, and attended the Marietta (Ohio) Academy, Marietta College, and the Massachusetts Institute of Technology. He became associated with the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., as an engineering apprentice in 1891, and held various engineering positions with that company thereafter until he was sent to the French Westinghouse Company, Havre, as a designing engineer in 1899. In 1903 he was transferred to the Canadian Westinghouse Company in a similar capacity. He was appointed vice president in 1929. He was a member of the Canadian Society of Civil Engineers and the Engineering Institute of Canada.

Francis Robert Healey (A'17) superintendent of the electric distribution department, Union Gas and Electric Company, Cincinnati, Ohio, died March 16, 1936. He was born at Cincinnati September 21, 1879, and became connected with the Cincinnati Edison Electric Company, predecessor of the Union Gas and Electric Company, in 1898 as a lamp trimmer. A short time later he entered the meter department, of which he subsequently became foreman, and in 1903 when the company reorganization took place, transferred to the electric distribution department, where he was employed as chief inspector 1903–05, assistant to manager 1905–09, and superintendent since 1909.

William Woodrow (A'19) instructor, communications department, New York (N. Y.) Board of Education, died March 3, 1936. Mr. Woodrow was born at Bristol, England, May 9, 1893, and attended the Merchant Venturer's Technical School at Bristol. In 1910 he was employed as an inspector for the United Electric Light and Power Company, New York, N. Y., and served that company until he became estimator and construction foreman for Hill and Company, electrical contractors, New York, in 1915. In 1916 he was appointed instructor in technical subjects in the public schools of the city of New York, and his service in that capacity was uninterrupted.

Delamore Leon Davis (A'89 and member for life) former manager, Salem (Ohio) Electric Light and Power Company, died February 13, 1936. Mr. Davis was born at Salem, January 3, 1855, and received his formal engineering training at Cornell University. He was employed as a machinist by the Buckeye Engine Company, Salem, until he became associated with the Salem Electric Light and Power Company. He served that company in several capacities, and retired as its manager in 1917. During his period of retirement he operated a small machine parts manufacturing company.

Almon Robinson (A'87 and member for life) patent consultant, Lewiston, Me., died December 23, 1934, according to word just received at Institute headquarters. Mr. Robinson was born at Lewiston, November 18, 1846, and attended the Lewiston Academy. After a brief period in Pennsylvania, where he associated himself with the steel industry, he returned to Lewiston and established a patent consulting practice. He continued that work until he retired from active business in 1931.

Membership

Recommended for Transfer

The board of examiners, at its meeting held April 23, 1936, recommended the following members for transfer to the grade of membership indicated. Any objection to these transfers should be filed at once with the national secretary.

To Grade of Fellow

Argersinger, R. E., asst. engg. mgr., Stone & Webster Engg. Corp., Boston, Mass.
Gundlach, Wm. E., chief elec. engr., The Milwaukee Elec. Ry. & Lt. Co., Milwaukee, Wis.
Hirshfeld, C. F., chief of research dept., The Detroit Edison Co., Detroit, Mich.
Swenson, G. W., prof. and head elec. engg. dept., Michigan College of Mining and Technology, Houghton, Mich.
Talbot, E. D., consulting engr., partner of John C. Larkin & Co., Syracuse, N. Y.

5 to grade of Fellow

To Grade of Member

Anderson, D. S., acting president, dean, College of Engg. and prof. of elec. engg., Tulane Univ., New Orleans, La.
Bach, Roy O., exchange transmission engr., The Pacific Tel. & Tel. Co., Seattle, Wash.
Berg, John E., asst. chief engr., General Elec. X-Ray Corp., Chicago, Ill.
Crowell, G. F., chief engr., Wisconsin Telephone Co., Milwaukee, Wis.
Diffendaffer, J. A., research and development engr., General Elec. X-Ray Corp., Chicago, Ill.
Fox, Lester H., asst. prof. of elec. engg., Mississippi State College, State College, Miss.
Greenwood, Leslie, chief elec. designer, The Harland Engg. Co., Alloa, Scotland.
Hall, A. A., prof. of elec. engg., West Virginia Univ., College of Engg., Morgantown, W. Va.
Hansen, E. B., engr., The Pacific Tel. & Tel. Co., Seattle, Wash.
Herrington, H. W., member of technical staff, Bell Tel. Labs., Inc., New York.
Jacobs, O. B., member of technical staff, Bell Tel. Labs., Inc., New York.
King, E. B., telephone engr., American Tel. & Tel. Co., New York.
Lawton, F. L., elec. engr., Saguenay Pwr. Co., Arvida, Que., Canada.
Matte, A. L., elec. engr., Bell Tel. Labs., Inc., New York.
Miller, L. E., designing engr., Reliance Elec. & Mfg. Co., Cleveland, Ohio.
Schulze, G. F., member of technical staff, Bell Tel. Labs., Inc., New York.
Warth, Stanley, division transmission engr., Southern Bell Tel. & Tel. Co., Louisville, Ky.

17 to grade of Member

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before May 31, 1936, or July 31, 1936, if the applicant resides outside of the United States or Canada.

Austin, C. P., Am. Tel. & Tel. Co., Kalamazoo, Mich.
Bates, H. H., Am. Tel. & Tel. Co., Cleveland, Ohio.
Bemis, E. W. (Member), Am. Tel. & Tel Co. New York, N. Y.
Blake, F. B. (Member), Bell Tel. Labs., Inc., New York, N. Y.
Boudreau, W. J., 80 Child St., Warren, R. I.
Brooks, E. W. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Burr, W. E., Pacific Coast Borax Co., Hinkley, Calif.
Butler, W. E., c/o R. F. Coggeshall, Intl. Gen. Elec. Co., Schenectady, N. Y.
Buttolph, L. J. (Member), Gen. Elec. Vapor Lamp Co., Hoboken, N. J.
Cantwell, J. L., Gen. Elec. Co., Pittsfield, Mass.
Chapman, H. H., Westinghouse Elec. & Mfg. Co., Chicago, Ill.
Conrad, L. L., So. Calif. Edison Co. Ltd., Alhambra, Calif.
Cooper, T. R., Pub. Serv. Comm. of West Virginia, Charleston.
DeBever, O. J., 908 Clark St., Evanston, Ill.
Dee, L. H., 94 So. Highland Ave., Ossining, N. Y.
De Laura, E. (Member), New York Quotation Co., New York.
Dissmeyer, E. F. (Member), Commonweath & Southern Corp., Jackson, Mich.
Downes, G. H. (Member), Bell Tel. Labs., New York, N. Y.
Essig, W. F., 7605—10th Ave., Brooklyn, N. Y.
Farish, E. T. (Member), Westinghouse Elec. & Mfg. Co., New York, N. Y.
Fisher, L. E. (Member), Bull Dog Electric Products Co., Detroit, Mich.
Fulton, J. R., Westinghouse Elec. & Mfg. Co., Chicago, Ill.
Goodrich, J. L., c/o Robt. H. Hoshall, Memphis, Tenn.
Hart, J. E. Jr., 3558—65th St., Woodside, N. Y.
Heins, E. F., Southwestern Bell Tel. Co., St. Louis, Mo.
Herbert, C. D., Reliance Elec. & Engg. Co. Inc., New York, N. Y.
Hewitt, C. D., New England Tel. Co., New Haven, Conn.
Holzweiss, F. L. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Honn, G. E., 420 Market St., San Francisco, Calif.
Hope, J. S., Westchester Ltg. Co., Mt. Vernon, N. Y.
Johnson, F. H., Conn. Lt. & Pwr. Co., Waterbury, Conn.
Johnson, V. E., Westinghouse Elec. & Mfg. Co., Chicago, Ill.
Jurgens, F. C., Delgado Trades Sch., New Orleans, La.
Kaegi, E. M. (Member), Allis-Chalmers Mfg. Co., Milwaukee, Wis.
Keil, M. T. (Member), Brooklyn Edison Co., Inc., N. Y.
Kelly, J. V., Central Arizona Lt. & Pwr. Co., Phoenix.
Kiefer, K. J., 7906 Laumer Ave., Cleveland, O.
Kinsey, J. F., Pub. Serv. Elec. & Gas Co., Hackensack, N. J.
Knowlton, A. D. (Member), Bell Tel. Labs., Inc., New York, N. Y.
Lambias, J. F., Jr., Brooklyn Edison Co. Inc., N. Y.
Lehane, T. J., Vapor Car Heating Co., Chicago, Ill.
Lewis, F. M. (Member), U. S. Engg. Dept., Booneville, Ore.
Mahoney, W. H., New York Quotation Co., New York, N. Y.
Malmquist, C. L., Brooklyn Edison Co. Inc., N. Y.
Marden, J. W., Westinghouse Lamp Co., Bloomfield, N. J.

Metcalf, K. B., Tennessee Valley Authority, Shelbyville, Tenn.
Meyer, F. A., Cutler Hammer Inc., Milwaukee, Wis.
Miller, N. O. C., Pioneer Placer Dredging Co., Gold Creek, Mont.
Molde, B. L., Brooklyn Edison Co. Inc., N. Y.
Moore, E. B., 275 Ocean Ave., Brooklyn, N. Y.
O'Dair, E. F., Gen. Elec. Co., New York, N. Y.
Onarheim, J. I., Allis-Chalmers Mfg. Co., Milwaukee, Wis.
Parzen, B., 1547 Walton Ave., New York, N. Y.
Paynter, W. Jr. (Member), Tennessee Valley Authority, Knoxville, Tenn.
Phillips, F. L., N. Y. Central Railroad Co., New York, N. Y.
Pike, J. J., 805—16th St., Boulder, Colo.
Plank, H. G., Pub. Utilities Comm., Manitowoc, Wis.
Potts, W. M., Pacific Coast Borax Co., Hinkley, Calif.
Powers, W. F. (Member), National Elec. Serv. Corp., New York, N. Y.
Raddin, E. H., Champion Lamp Works, Lynn, Mass.
Ray, M. P., Independent Subway System, New York, N. Y.
Reardon, John W., 13 State St., Schenectady, N. Y.
Reed. M. G. W., Intl. Nickel Co., Kirkland Lake, Ont., Can.
Richter, H. W., Intl. Business Machines Corp., Chicago, Ill.
Roe, C. H. (Member), Elec. Testing Labs., New York, N. Y.
Rusk, A. P., Pennsylvania Water & Power Co., Baltimore, Md.
Sayle, H. J., c/o Chas. F. Zweifel & Co., Inc., New York, N. Y.
Schindel, L. M., Am. Tel. & Tel. Co., Denver, Colo.
Schoettgen, A. M., I. R. T. Co., New York, N. Y.
Scutt, H. L., Leeds & Northrup Co., New York, N. Y.
Sharki, P., New York State Training School, State School, N. Y.
Shilling, C. J., Continental Steel Corp., Kokomo, Ind.
Smith, A. J., Bureau of Water & Pwr., Los Angeles, Calif.
Sylvane, A. A., Commercial Radio-Sound Corp., New York, N. Y.
Thompson, G. J., 143–12 Oak Ave., Flushing, L. I., N. Y.
Tracht, J. W., Detroit Rock Salt Co., Mich.
Traver, H. R., Oregon State Coll., Corvallis.
Van Fleet, A. K., 207 E. High St., Somerville, N. J.
Van Woerkom, J. F., 3153 Pacific Ave., Ogden, Utah.
Walker, D., 29–43 Gilmore St., E. Elmhurst, N. Y.
Waller, J. L., Hatfield Wire & Cable Co., Chicago, Ill.
Wampole, E. A., Jr., DeVry Motion Picture & Sound Equipt., Chicago, Ill.
Whaley, C. D., New York Quotation Co., New York, N. Y.
Whitford, R., ReQua Elec. Supply Co. Inc., Rochester, N. Y.
Wingo, A. C., Box N N, Boulder City, Nev.
Woolley, F., Utah Pwr. & Lt. Co., Salt Lake City.

86 Domestic

Foreign

Barnard, M. C., Metropolitan Vickers Elec. Exp. Co., Vereeniging, Transvaal, So. Africa.
Stroebel, L. H., Electricity Dept., Bedford, C. P., So. Africa.

2 Foreign

Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the addresses as they now appear on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

Beaumont, L., Box 404, Shanghai, China.
Blanc, Victor, 153 Boulevard Lefebvre, Paris, France.
Bukley, E. J., Malaja-Dmitrovka D. 8 Kv. 38, Moscow, U.S.S.R.
DeKeyser, Jacques F., 37–53—78th St., Jackson Heights, N. Y.
Gardner, Fred V., 1180 Clayton St., San Francisco, Calif.
Huang, Pienchun, Schillerstr 57, Berlin, Germany.
Johnson, James W., 3506—16th St., N. W., Washington, D. C.
Jones, Robert W., 565 Thompson Ave., Donora, Pa.
Luther, Herbert A., 50 Atwood Ave., Johnston, R. I.
Megeath, S. A., Jr., 14 North Ave , Elizabeth, N. J.
Meltvedt, Henry, 742 S. Douglas, Springfield, Mo.
Miyamoto, Tatsuo Charles, 517 M St., Sacramento, Calif.
Murray, Forrest H., 5530 Dorchester Ave., Chicago, Ill.
Patel, Ishvarlal B., 5 Second Carpenters St., Bombay, 4, India.
Ritter, Edward A., 40 Lexington St., Hamden, Conn.
Williams, Thomas J. C., 827 S. 48th St., Philadelphia, Pa.
Willson, William H., Jr., 1720—2nd Ave., Cedar Rapids, Iowa.

17 Addresses Wanted

Engineering Literature

New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, recently, are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface of the book in question.

MANUAL of ENGINEERING DRAWING for Students and Draftsmen. By T. E. French. 5 ed. N. Y. and Lond., McGraw-Hill Book Co., 1935. 481 p., illus., 9x6 in., cloth, $3.00. A college textbook, conforming to the standards of the A.S.A., based upon the "conception that drawing is a real language, to be studied and taught in the same way as any other language."

FORSCHUNGSHEFT 376. INDIZIEREN SCHNELLAUFENDER VERBRENNUNGSKRAFTMASCHINEN by E. Kallhardt; SCHWINGUNGEN von LUFTSÄULEN mit GROSSER AMPLITUDE. By C. Mayer-Schuchard. Berlin, VDI-Verlag, Jan.-Feb. 1936. 22 p., illus., 12x8 in., paper, 5 rm. Contains a report giving results of tests of optical, stroboscopic, and piezo-electric indicators on high-speed internal-combustion engines.

FORTSCHRITTE des CHEMISCHEN APPARATEWESENS. ELEKTRISCHE ÖFEN, Lieferungen 2–3. Ed. by A. Bräuer, J. Reitstötter, and H. Alterthum. Leipzig, Akademische Verlagsgesellschaft, 1935. Illus., 11x8 in., paper, 28 mks. A review of the development, construction, and uses of the electric furnace. Abstracts of German patents and a classified list of British patents are given.

INDUSTRIELLE ELEKTROWÄRME, herausgegeben von der Wirtschaftsgruppe Elektrizitätsversorgung, by Masukowitz and Knoops. Berlin, Arbeitsgemeinschaft zur Förderung der Elektrowirtschaft, 1935. 64 p., illus., 8x6 in., paper, 1 rm. Describes briefly the development of the electric furnace, economy of electric heating, types of furnaces and their applications.

ELECTROLYTIC OXIDATION and REDUCTION: Inorganic and Organic. By S. Glasstone and A. Hickling. N. Y., D. Van Nostrand Co., 1936. 420 p., illus., 9x6 in., cloth, $9.50. The theoretical aspects of these reactions are discussed, and an attempt is made to elucidate the principles of the reactions described. References to the scientific and patent literature are included.

ELEMENTARY ENGINEERING THERMODYNAMICS. By V. W. Young and G. A. Young. N. Y. and Lond., McGraw-Hill Book Co., 1936. 220 p., illus., 9x6 in., cloth, $2.50. This text presents a course which will provide the fundamental theoretical basis for an accompanying course in practical heat engineering. The treatment is brief, yet fairly comprehensive, and avoids the more complicated mathematical processes.

PROCEDURE HANDBOOK of ARC WELDING DESIGN and PRACTICE. 3 ed. Cleveland, Ohio, Lincoln Electric Co., 1935. 596 p., illus., 9x6 in., lea., $1.50. Describes various forms of arc welding processes, supplies data for welding various metals, and discusses the designing of structures and machines for arc-welded construction. New information on procedure and applications of arc welding has been included.

DIESEL ENGINES, Operation and Maintenance. By L. H. Morrison. Chicago, American Technical Society, 1936. 212 p., illus., 9x6 in., cloth, $2.25. A practical textbook intended for students and beginners and adapted for home study. The troubles in operating engines are described and methods of adjustment and repair explained.

HANDBOOK of APPLIED MATHEMATICS. By M. E. Jansson and H. D. Harper, with a section on Business Mathematics by P. L. Agnew, 2 ed. N. Y., D. Van Nostrand Co., 1936. 1010 p., illus., 8x5 in., lea., $6.00. The first section reviews the operations of arithmetic, algebra, geometry, and trigonometry; the remaining chapters show applications of mathematics in various trades.

MATHEMATICS of MODERN ENGINEERING. Vol. 1. By R. E. Doherty and E. G. Keller. N. Y., John Wiley & Sons, 1936. 314 p., illus., 9x6 in., cloth, $3.50. This book, originally developed for the advanced course in engineering of the General Electric Company, is intended to facilitate the study and use of mathematics with especial reference to its applications in engineering, and includes differential equations, determinants, Fourier series, transcendental equations, dimensional analysis, vector analysis, and Heaviside's operational calculus. Scientific approach to problems is emphasized; previous knowledge of calculus is assumed.

MEASUREMENT of INDUCTANCE, CAPACITANCE, and FREQUENCY. By A. Campbell and E. C. Childs. N. Y., D. Van Nostrand Co., 1935. 488 p., illus., 9x6 in., cloth, $12.00. The first part is based on articles written by Mr. Campbell for the "Dictionary of Physics," now brought up to date to include chapters on a-c potentiometers and frequency measurement, giving a survey of modern theoretical and practical views.

NATIONAL ASSOCIATION of RAILROAD and UTILITIES COMMISSIONERS. Proceedings of 47th Annual Convention held at Nashville, Tenn., Oct. 15–18, 1935, published by State Law Reporting Co. N. Y., 1936. 591 p., tables, 9x6 in., cloth, $6.00. The topics of the 1935 convention include, among others, the public utility holding company act of 1935, effect of recent judicial decisions on state regulation, federal communications act and jurisdiction over telephone companies, and state regulation and its results.

PHENOMENA in HIGH-FREQUENCY SYSTEMS. By A. Hund. N. Y. and Lond., McGraw-Hill Book Co., 1936. 642 p., illus., 9x6 in., cloth, $6.00. A thorough discussion of the phenomena of high frequency systems, with applications to communication problems. Experimental and theoretical reasonings are combined, and numerous references add to the value of the text.

AMERICAN SOCIETY for TESTING MATERIALS. Proceedings of the 38th Annual Meeting held at Detroit, Mich., June 24–28, 1935, v. 35, pts. 1 and 2. Phila., A.S.T.M., 1935. V. 1, 1488 p.; v. 2, 769 p., illus., 9x6 in., lea., $15.00; in cloth, $12.00; in paper, $11.00. The proceedings for 1935 in 2 volumes: 1 contains the reports of the standing committees of the association, the tentative standards that were issued or revised during 1935, and the tentative revisions of standards now under discussion; 2 contains the technical papers presented before the association.

ALTERNATING - CURRENT MACHINES. By A. F. Puchstein and T. C. Lloyd. N. Y., John Wiley & Sons, 1936. 582 p., illus., 9x6 in., lea., $5.00. A course of study within the scope of fourth-year students devoting sections to synchronous generators, transformers, polyphase and single phase induction motors, synchronous motors, alternators in parallel, synchronous converters, mercury arc rectifiers, and series and repulsion motors. Suggestions for further reading and investigation are given, and the text is so arranged that omissions can be made when necessary.

AMERICAN SOCIETY of HEATING and VENTILATING ENGINEERS GUIDE 1936 for Heating, Ventilating, Air Conditioning. N. Y., American Society of Heating and Ventilating Engineers, 1936. 1080 p., illus., 9x6 in., lea., $5.00. Includes technical data, a representative collection of manufacturers' data on equipment, and a directory of the members of the society. The new edition has been revised and chapters added on refrigeration, drying, motors and their control, railway air conditioning, and heat and fuel utilization.

Engineering Societies Library

29 West 39th Street, New York, N. Y.

MAINTAINED as a public reference library of engineering and the allied sciences, this library is a cooperative activity of the national societies of civil, electrical, mechanical, and mining engineers.

Resources of the library are available also to those unable to visit it in person. Lists of references, copies or translation of articles, and similar assistance may be obtained upon written application, subject only to charges sufficient to cover the cost of the work required.

A collection of modern technical books is available to any member residing in North America at a rental rate of five cents per day per volume, plus transportation charges.

Many other services are obtainable and an inquiry to the director of the library will bring information concerning them.

Industrial Notes

General Electric Reports Gains.—Substantial improvement in business and net earnings was reported by the General Electric Co. for the first quarter this year and the year ended March 31, as compared with the same 1935 periods. Orders received by the General Electric Company for the first quarter of 1936 amounted to $59,569,879, compared with $49,379,932 for the corresponding quarter of 1935, an increase of 21 per cent, President Gerard Swope reported recently. This was the largest quarter since the third quarter of 1931.

Rural Department for Westinghouse.—The formation of a rural electrification department, under the direction of G. A. Sawin, has been announced by the Westinghouse Electric & Mfg. Co. At present the department will be represented by two field men, B. P. Hess and R. L. Pleak, both agricultural engineers with wide experience in rural problems. The services of the new department are available for use by any central station, municipal plant, rural cooperative or others interested in the promotion of rural electric service.

Resistance Company Moves.—Announcing its fifth expansion move in 14 years the International Resistance Co. has moved to new and larger quarters at 401 North Broad St., Philadelphia, utilized exclusively for the manufacture and development of resistors and volume controls. According to President Ernest Searing, a number of new items will soon be forthcoming, including type MW power wire wound resistors—a flat, fully insulated type capable of many time-saving combinations in a single unit; new model volume controls incorporating the metallized filament type resistance element; important developments in insulated metallized resistors assuring greater heat dissipation with proportionately higher wattage ratings; and various others.

New Fuse for Use Under Oil.—The Rowan Controller Co., Baltimore, Md., has developed a new Air-Seal fuse, which has been especially designed for use under oil in Rowan oil immersed control equipment. The construction of this important improvement in fuses for such equipment is simple and the fuse has been approved by the Underwriters' Laboratories, whose tests indicated that its operating characteristics were the same as those of a standard fuse operated by air. Air-Seal fuses are supplied for 250- and 600-volt operation in all standard sizes from 30 to 400 amperes. Descriptive bulletins are available.

Ratio Differential Relay.—To provide high-speed protection for alternating current apparatus the Westinghouse Electric & Mfg. Co. has developed the type HA ratio differential relay which increases system stability limits by placing the protection of generators and transformers on the same high-speed basis now widely set for transmission lines. This relay has an operating time of one cycle and reduces considerably the chances of damage which might occur to equipment protected by slower speed relays. Because the operating current of the relay varies in proportion to the load, faulty tripping is prevented. This allows the relay to be set for close protection at normal loads. Two types are available, one for generators and motors and the other for transformers.

New Audio Transformer.—A complete new line of high fidelity audio transformers and reactors, known as Super High Fidelity—Series A, has been developed by Ferranti Electric, Inc., 130 West 42nd St., New York City. Each of these units has a frequency response of within ±1 db from 30 to 12,000 cycles. The transformer is mounted in a completely reversible through-type case; the listing is complete and includes units for practically every purpose in the electronic field. This series is said to be the lowest priced complete line of transformers employing the new self-shielding, core type construction which renders them entirely free from hum and pickup. Each unit is fitted with electrostatic shields between windings and is designed for extremely low insertion loss. Descriptive literature is available.

Expansion Plugs.—The Wrought Washer Mfg. Co., Milwaukee, Wis., announces the introduction of non-ferrous expansion plugs covering the range of metals now in demand, including brass, copper, aluminum, stainless steel, monel metal, etc. Special facilities for producing this new line have been provided and extensive tool equipment for manufacturing a full complement of sizes has been installed.

Trade Literature

Copper Conductors for Rural Lines.—Bulletin CRE-1, 100 pp. Presents the electrical and physical properties of, and sag-tension charts and line design data for, hard drawn copper conductors of the sizes and forms suitable for rural distribution lines. Outlines a method whereby the operation of conductors at any practical tension may be predetermined with working accuracy. This method, which employs the concept of the secant modulus, is new in this field. General Cable Corp., 420 Lexington Ave., New York.

Feeder Voltage Regulators.—Bulletin 2221, 4 pp. Describes type AFR-2 feeder voltage regulators for low capacity and low voltage. Construction details are illustrated and many exclusive features applying to this type of regulator are listed. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Electrical Sheets.—Bulletin, 24 pp. Describes electrical steel sheets for the laminated structures of transformers, generators, motors, and other electrical machinery. Presents the major characteristics of this material to serve as a guide for the selection of proper steels for the purpose of design. American Sheet and Tin Plate Co., Electrical Sheet Division, Frick Bldg., Pittsburgh.

Motors.—Bulletin 214, 4 pp. Describes type T heavy-duty, d-c motors for locations exposed to dirt, oil, moisture, or harmful fumes. Reliance Electric & Engg. Co., Cleveland, O.

Grinding Wheels.—Catalog, 56 pp. Describes numerous types of grinding wheels available for a wide variety of operations; lists prices. Chicago Wheel & Mfg. Co., 1101 W. Monroe St., Chicago, Ill.

Transformers.—Bulletin 1177, 12 pp. Describes high voltage power transformers, emphasizing 14 features of design. Bulletin 2220, 4 pp. Contains engineering data on the performance of high voltage power transformers under impulse conditions. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Recording Voltmeter.—Bulletin GEA-2294, 4 pp. Describes a simplified, portable strip-chart recorder for voltage-survey work. This type CD-23 voltmeter has all the inherent characteristics of the standard line of CD switchboard and portable recording instruments but it has been simplified—limited to a single range and a single speed—and thus is priced much lower. The CD-23 is for alternating current with Telechron motor clock; the CD-24 is for alternating current but is equipped with spring-clock drive. Principal applications are in the field of voltage-survey work where accurate records of circuit conditions are required. General Electric Co., Schenectady, N. Y.

Controlled Rectifier.—Bulletin 8851. Describes a new controlled rectifier providing the Ward Leonard system of control for d-c motors from an a-c power supply. A modification of the original Ward Leonard system of control for d-c motors is obtained by separately exciting the motor field by means of one part of the rectifier and supplying armature current at a predetermined and adjustable potential from another section of the controlled rectifier. Both manual regulation of the motor field current and manual and automatic regulation of the armature potential is obtained. This system of control permits a very gradual acceleration of the motor to any desired speed, maintained constant under all conditions of permissible motor loads. Ward Leonard Electric Co., Mt. Vernon, N. Y.

Thermometers.—Catalog 1250, 88 pp. Describes liquid-filled, vapor-tension, and gas-filled recording, indicating, and controlling thermometers, as well as a new, small-bulb, gas-filled thermometer. In connection with automatic temperature controllers, it gives information regarding both the electric and pneumatic types, and helpful sketches to illustrate how they are applied. Application data are given on industrial stem thermometers, resistance thermometers, humidity measuring instruments, control valves, time-temperature controllers, and the accessories that are used with thermometers on industrial applications. In order to enhance the value of the catalog as an engineering hand book, over 500 temperature recording charts are illustrated in full size. The Bristol Co., Waterbury, Conn.